Difference Cordial Labeling of Generalized Petersen Graph

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Abstract

Let G be a order- p and size- q graph. Let g be a 1-1 function from the set $V(G)$ to $\{1,2,\ldots,p\}$. Each edge uv, receives the label $|g(u) - g(v)|$. Then the function g is called a difference cordial labelling if g is $1-1$ and $|e_g(1) - e_g(0)| \le 1$ where $e_g(1)$ and $e_g(0)$ refers the number of edges having 1 as its label and not having 1 as its label respectively. If G receives labeling which is a difference cordial is difference cordial graph. In this paper, the difference cordial for generalised Petersen graph is analysed and proved the existence of difference cordial.

Keywords: Difference cordial graphs, Generalised Petersen graph.

1. INTRODUCTION

Graph labeling attracts many mathematics to contribute their research on graph theory. For more details see [2]. In 1967, Rosa introduced the labeling idea in graph theory. For all the vertices or edges or both of the graph assigning an integer under certain conditions is called labelling of graph. The idea of cordial labeling was introduced first in 1987 by cahit [3]. In [4] Ponraj, Shathish Naraynan introduced the idea of difference cordial labeling for finite undirected and simple graph. Here, we considered the graphs which are undirected, finite and simple graphs. Let G with order p and size q simple, finite, undirect and planar graph. $V(G)$ be the vertex set of G and $E(G)$ be the edge set of G. $|V|$ denotes the order of graph and $|E|$ denotes the its size. Let g be a mapping from $V(G) \to \{1,2,\ldots,p\}$ in a graph G such that each edge uv, receives the label $|g(u) - g(v)|$. Then the function g is called a difference cordial labelling if g is $1-1$ and $|e_g(1) - e_g(0)| \le 1$ where $e_g(1)$ and $e_g(0)$ refers the number of edges having 1 as its label and not having 1 as its label respectively. A graph which receives a difference cordial labeling is difference cordial graph.

2. DEFINITIONS AND NOTATIONS

2.1 Definition

For a graph G with order p and size q . Let g be a 1-1 function from the set $V(G)$ to $\{1,2,\ldots,p\}$. Each edge uv, receives the label $|g(u) - g(v)|$. Then the function g is called a difference cordial labelling if g is $1-1$ and $|e_g(1) - e_g(0)| \le 1$ where $e_g(1)$ and $e_g(0)$ refers the number of edges having 1 as its label and not having 1 as its label respectively. . If G receives labeling which is a difference cordial is difference cordial graph.

2.2 Definition

The generalized Petersen graph $Gp(n,\kappa)$ has its vertex and edge set as
$V(Gp(\eta,\kappa)) = \{x_\theta : 0 \le \theta \le \eta - 1\} \cup \{y_\theta : 0 \le \theta \le \eta - 1\}$

and $E(Gp(\eta,k)) = \{x_\theta y_\theta : 0 \le \theta \le n-1\} \cup \{x_\theta x_{\theta+1} : 0 \le \theta \le \eta-1\} \cup \{y_\theta y_{\theta+\kappa} : 0 \le \theta \le \eta-1\}$

where the subscripts are taken modulo η, for a positive integer $\eta \ge 3$ and $1 \le \kappa < \left\lfloor \frac{\eta}{2} \right\rfloor$

Nature of n	$e_g(1)$	$e_g(0)$
$\eta \equiv 1\ (mod\ 2)$	$\frac{3\eta-1}{2}$	$\frac{3\eta+1}{2}$
$\eta \equiv 0\ (mod\ 2)$	$\frac{3\eta}{2}$	$\frac{3\eta}{2}$

2.3 Definition

The graph formed by attaching a path P_m through an edge to the generalized Petersen graph $Gp(\eta,k)$ is denoted by $Gp(\eta,k) \oplus P_m$.

3. RESULTS

3.1 Theorem:

The generalized Petersen graph $Gp(\eta,k), 1 \le \kappa < \left\lfloor \frac{\eta}{2} \right\rfloor$ admits difference cordial.

Proof:

Let G be a generalized Petersen graph $Gp(\eta,\kappa)$.

$V(G) = \{x_\theta : 0 \le \theta \le \eta-1\} \cup \{y_\theta : 0 \le \theta \le \eta-1\}$ and

$E(G) = \{x_\theta y_\theta : 0 \le \theta \le \eta-1\} \cup \{x_\theta x_{\theta+1} : 0 \le \theta \le \eta-1\}$

$\cup \{y_\theta y_{\theta+\kappa} : 0 \le \theta \le \eta-1\}$, where the subscripts are taken modulo η.

$|V(G)| = 2\eta, |E(G)| = 3\eta$.

The function $f : V(G) \to \{1,2,\ldots,2\eta\}$ assigns the label as follows.

Case I: $n \equiv 1 (mod\ 2)$

$g(x_{2\theta-2}) = 4\theta - 2 \qquad 1 \le \theta \le \frac{\eta+1}{2}$

$g(x_{2\theta-1}) = 4\theta - 1 \qquad 1 \le \theta \le \frac{\eta-1}{2}$

$g(y_{2\theta-2}) = 4\theta - 3 \qquad 1 \le \theta \le \frac{\eta+1}{2}$

$g(y_{2\theta-1}) = 4\theta \qquad 1 \le \theta \le \frac{\eta-1}{2}$

Case II: $n \equiv 0 (mod\ 2)$

$g(x_{2\theta-2}) = 4\theta - 2 \qquad 1 \le \theta \le \frac{\eta}{2}$

$g(x_{2\theta-1}) = 4\theta - 1 \qquad 1 \le \theta \le \frac{\eta}{2}$

$g(y_{2\theta-2}) = 4\theta - 3 \qquad 1 \le \theta \le \frac{\eta}{2}$

$g(y_{2\theta-1}) = 4\theta \qquad 1 \le \theta \le \frac{\eta}{2}$

The above function shows that the labelling is difference cordial. The number of edges receives the label 1 and not receives the label 1 are tabulated below.

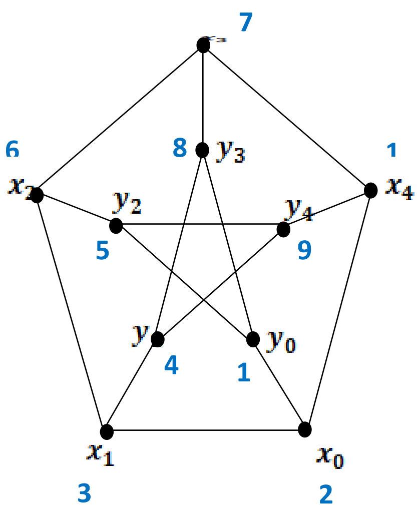
3.1.2 Table:

Value of n	g(z_1)	g(z_2)	g(z_3)	g(z_4)	g(z_5)	g(z_6)	g(z_7)	g(z_8)
1	1							
2	1	2						
3	1	3	2					
4	1	2	4	3				
5	1	2	4	3	5			
6	1	2	3	5	4	6		
7	1	2	3	5	7	6	4	
8	1	2	3	4	6	8	7	5

From the above table 3.1.1 it is observed that $|e_g(1) - e_g(0)| \leq 1$. Hence it is concluded that the generalized Petersen graph $Gp(\eta, k), 1 \leq k < \left\lfloor \frac{\eta}{2} \right\rfloor$ admits difference cordial labelling and therefore, for $1 \leq k < \left\lfloor \frac{\eta}{2} \right\rfloor$, $Gp(\eta, k)$ is difference cordial graph.

3.1.2 Example:

The Petersen graph $Gp(5,2)$ is difference cordial.



$e_g(1) = 7$ and $e_g(0) = 8$, $|e_g(1) - e_g(0)| \leq 1$.
Hence the Petersen graph $Gp(5,2)$ is difference cordial.

3.2 Theorem:

The graph $Gp(\eta, \kappa) \oplus P_\omega$ admits difference cordial.

Proof:

Let G be a $Gp(\eta,\kappa)\oplus P_\omega$ graph.

$V(G)=\{x_\theta : 0\le\theta\le\eta-1\}\cup\{y_\theta : 0\le\theta\le\eta-1\}\cup\{z_\theta : 1\le\theta\le\omega\}$

and $E(G)=\{x_\theta y_\theta : 0\le\theta\le\eta-1\}\cup\{x_\theta x_{\theta+1} : 0\le\theta\le\eta-1\}$

$\cup\{y_\theta y_{\theta+\kappa} : 0\le\theta\le\eta-1\}\cup\{x_{\eta-1}z_1\}$

$\cup\{z_\theta z_{\theta+1} : 1\le\theta\le\omega-1\}$,

where the subscripts are taken modulo η.

$|V(G)|=2\eta+\omega, |E(G)|=3\eta+\omega$.

The function $f : V(G)\to\{1,2,\dots,2n+\omega\}$

assigns the label as follows.

Case I: $n\equiv 1(\mathrm{mod}\ 2)$

$g(x_{2\theta-2})=4\theta-2 \qquad 1\le\theta\le\frac{\eta+1}{2}$

$g(x_{2\theta-1})=4\theta-1 \qquad 1\le\theta\le\frac{\eta-1}{2}$

$g(y_{2\theta-2})=4\theta-3 \qquad 1\le\theta\le\frac{\eta+1}{2}$ $g(y_{2\theta-1})=4\theta \qquad 1\le\theta\le\frac{\eta-1}{2}$

Case II: $n\equiv 0(\mathrm{mod}\ 2)$

$g(x_{2\theta-2})=4\theta-2 \qquad 1\le\theta\le\frac{\eta}{2}$

$g(x_{2\theta-1})=4\theta-1 \qquad 1\le\theta\le\frac{\eta}{2}$

$g(y_{2\theta-2})=4\theta-3 \qquad 1\le\theta\le\frac{\eta}{2}$

$g(y_{2\theta-1})=4\theta \qquad 1\le\theta\le\frac{\eta}{2}$

The following Table 3.2.1 gives the labeling of the path
$P_\omega, \omega\le 8$.

The labeling of the path P_ω are as follows

3.2.1 Table:

The labeling of the path $P_\omega, \omega>8$ are as follows

Case i: $m\equiv 0\ (\mathrm{mod}\ 4)$

$g(z_\theta)=2\eta+\theta \qquad 1\le\theta\le\frac{\omega+2}{2}$

$g\left(z_{\frac{m+2}{2}+\theta}\right)=2\eta+\frac{m+2}{2}+2\theta \qquad 1\le\theta\le\frac{\omega}{4}-1$

$g\left(z_{\frac{3m}{4}+\theta}\right)=2\eta+\frac{m+2}{2}+2\theta-1 \qquad 1\le\theta\le\frac{\omega}{4}$

Case ii: $m\equiv 1\ (\mathrm{mod}\ 4)$

$g(z_\theta)=2\eta+\theta \qquad 1\le\theta\le\frac{\omega+1}{2}$

$g\left(z_{\frac{m+1}{2}+\theta}\right)=2\eta+\frac{m+1}{2}+2\theta \qquad 1\le\theta\le\frac{\omega-1}{4}$

$g\left(z_{\frac{3m+1}{4}+\theta}\right)=2\eta+\frac{m+1}{2}+2\theta-1 \qquad 1\le\theta\le\frac{\omega-1}{4}$

***Case iii*: $m \equiv 2\ (mod\ 4)$**

$g(z_\theta) = 2\eta + \theta \qquad 1 \le \theta \le \frac{\omega+2}{2}$

$g\left(z_{\frac{m+2}{2}+\theta}\right) = 2\eta + \frac{m+2}{2} + 2\theta \qquad 1 \le \theta \le \frac{\omega-2}{4}$

$g\left(z_{\frac{3m+2}{4}+\theta}\right) = 2\eta + \frac{m+2}{2} + 2\theta - 1 \qquad 1 \le \theta \le \frac{\omega-2}{4}$

***Case iv*: $m \equiv 3\ (mod\ 4)$**

$g(z_\theta) = 2\eta + \theta \qquad 1 \le \theta \le \frac{\omega+2}{2}$

$g\left(z_{\frac{m+1}{2}+\theta}\right) = 2\eta + \frac{m+1}{2} + 2\theta \qquad 1 \le \theta \le \frac{\omega+1}{4} - 1$

$g\left(z_{\frac{3m-1}{4}+\theta}\right) = 2\eta + \frac{m+2}{2} + 2\theta - 1 \qquad 1 \le \theta \le \frac{\omega+1}{4}$

The above function shows that the labelling is difference cordial. The number of edges receives the label 1 and not receives the label 1 are tabulated below.

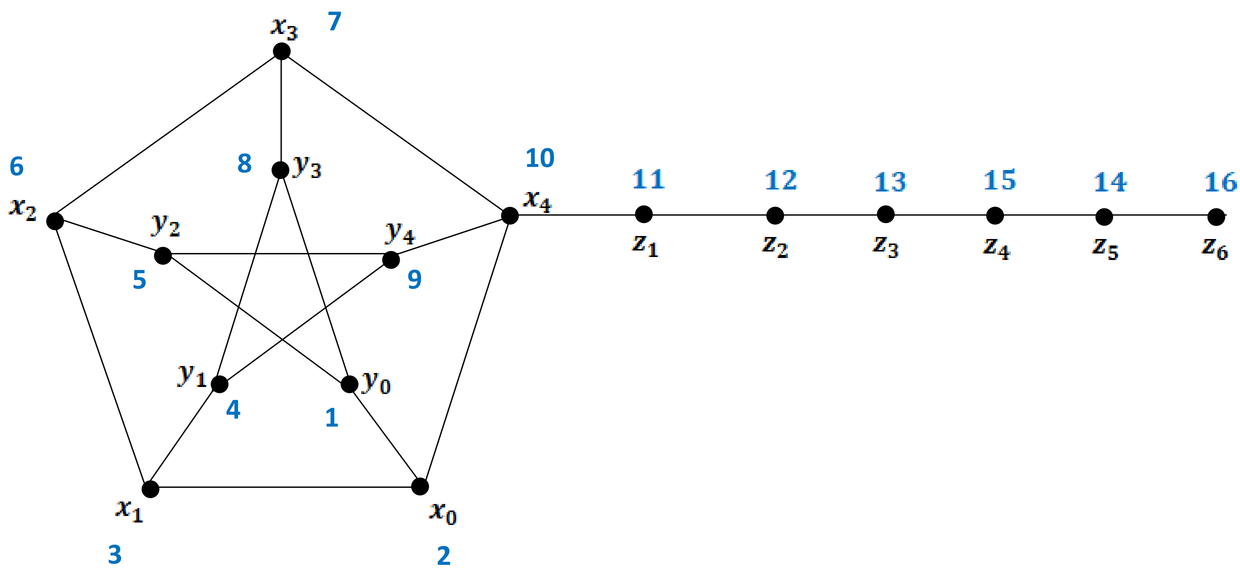
3.2.2 Table:

From the above table 3.2.2 it is observed that $|e_g(1) - e_g(0)| \le 1$. Hence it is concluded that the graph $Gp(\eta, k) \oplus P_\omega$ admits difference cordial labelling and therefore $Gp(\eta, k) \oplus P_\omega$ is difference cordial graph.

nature of n and m	$e_g(1)$	$e_g(0)$
$\eta \equiv 0\ (mod\ 2)$ $m \equiv 0\ (mod\ 2)$	$\frac{3\eta+\omega}{2}$	$\frac{3\eta+\omega}{2}$
$\eta \equiv 0\ (mod\ 2)$ $m \equiv 1\ (mod\ 2)$	$\frac{3\eta+\omega-1}{2}$	$\frac{3\eta+\omega+1}{2}$
$\eta \equiv 1\ (mod\ 2)$ $m \equiv 0\ (mod\ 2)$	$\frac{3\eta+\omega+1}{2}$	$\frac{3\eta+\omega-1}{2}$
$\eta \equiv 1\ (mod\ 2)$ $m \equiv 1\ (mod\ 2)$	$\frac{3\eta+\omega}{2}$	$\frac{3\eta+\omega}{2}$

3.2.2 Example:

The graph $Gp(5,2) \oplus P_6$ is difference cordial.



$e_g(1) = 11$ and $e_g(0) = 10$, $|e_g(1) - e_g(0)| \leq 1$.

Hence the graph $Gp(5,k) \oplus P_6$ is difference cordial.

4. CONCLUSION

Difference cordial labeling on the generalized Petersen graph $Gp(n,m)$ and the graph $Gp(5,k) \oplus P_6$ are discussed and results are presented in this paper. In future our works will be labeling on some different class of graphs.

5. REFERENCES

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