gsp$\alpha\omega$-closed sets in topological spaces

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Abstract: The aim of this paper is to introduce a new class of closed sets namely $gsp\alpha\omega$-closed sets which is obtained by generalizing gsp -closed sets via $\alpha\omega$- open sets and investigate some of their basic properties in topological spaces.

Introduction:

LEVINE [5]introduced semi-open sets in 1963. In 1986, **D.ANDRIJIEVIC[1]**introduced the notion of semi-pre- open sets in topological spaces. In 2000, the ω closed sets [9] were introduced and studied by **P.SUNDARAM** and **M.SHRIK JOHN**. **M.PARIMALA[8]**introduced the concept of $\alpha\omega$ -closed sets, and studied their properties in 2017.The aim of this paper is tointroduce a new class of closed sets namely gspα$\alpha\omega$-closed sets and investigate some of their basic properties in topological spaces.

1. PRELIMINARIES:

DEFINITION 1.1:A subset A of a space$(X,\tau)$is called a

1. semi open set if $A \subseteq cl(int(A))$
2. α-open set if $A \subseteq int\left(cl(int(A))\right)$
3. semi pre(=β)-open set if $A \subseteq cl\left(int(cl(A))\right)$
4. b-open set if $A \subseteq \left(cl(int(A))\right) \cup \left(int(cl(A))\right)$
5. regular-open set if $A = int(cl(A))$

DEFINITION 1.2:A subset A of a space (X,τ) is called

1. generalized closed (briefly g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open
2. regular-generalized closed(brieflyrg-closed)if$cl(A) \subseteq U$whenever$A \subseteq U$and U is regularopen.
3. generalized b-closed (briefly gb-closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open
4. regular-generalized b- closed(brieflyrgb-closed)if$bcl(A) \subseteq U$whenever$A \subseteq U$and$U$is

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regular- open

5. generalized semi-preregular-closed (briefly gspr-closed) if $\boldsymbol{spcl(A) \subseteq U}$ whenever $\boldsymbol{A \subseteq U}$ and $\boldsymbol{U}$ is regular open
6. generalized $\boldsymbol{\beta}$-closed (briefly g $\boldsymbol{\beta}$-closed) if $\boldsymbol{\beta cl(A) \subseteq U}$ whenever $\boldsymbol{A \subseteq U}$ and $\boldsymbol{U}$ is open.
7. $\boldsymbol{\psi\hat{g}}$-closedif $\boldsymbol{\psi cl(A) \subseteq U}$ whenever $\boldsymbol{A \subseteq U}$ andUis $\boldsymbol{\hat{g}}$-open
8. $\boldsymbol{\psi g}$-closed if $\boldsymbol{\psi cl(A) \subseteq U}$ whenever $\boldsymbol{A \subseteq U}$ and $\boldsymbol{U}$ is open
9. generalized semi-closed (briefly gs-closed) if $\boldsymbol{scl(A) \subseteq U}$ whenever $\boldsymbol{A \subseteq U}$ and U is open
10. $\hat{\eta}$*-closed if $\boldsymbol{spcl(A) \subseteq U}$ whenever $\boldsymbol{A \subseteq U}$ and $\boldsymbol{U}$ is $\boldsymbol{\omega}$-open
11. $\boldsymbol{\psi}$-closed if $\boldsymbol{scl(A) \subseteq U}$ whenever $\boldsymbol{A \subseteq U}$ and $\boldsymbol{U}$ is sg-open
12. $\boldsymbol{\omega}$(or$\boldsymbol{\hat{g}}$) closed if $\boldsymbol{cl(A) \subseteq U}$ whenever $\boldsymbol{A \subseteq U}$ and $\boldsymbol{U}$ is semi-open
13. $\boldsymbol{\alpha\omega}$-closed if $\boldsymbol{\omega cl(A) \subseteq U}$ whenever $\boldsymbol{A \subseteq U}$ and $\boldsymbol{U}$ is $\boldsymbol{\alpha}$-open
14. $\boldsymbol{g\alpha\omega}$ -closed if $\boldsymbol{cl(A) \subseteq U}$ whenever $\boldsymbol{A \subseteq U}$ and $\boldsymbol{U}$ is $\boldsymbol{\alpha\omega}$-open

2. $gsp\alpha\omega$ -CLOSED SET

DEFINITION 2.1:

A subset A of (X, τ) is called a **$gsp\alpha\omega$-closed set** if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\alpha\omega$-open in (X,, τ). The complement of **$gsp\alpha\omega$ -closed set** is **$gsp\alpha\omega$–openset.**

EXAMPLE2.2:

Let X = {a, b, c}; τ= {X,φ, {c}, {a, c}}. Closed sets are {X,φ, {b}, {a, b}}

semi-pre closed sets are (X,φ , {a}, {b}, {a, b}}. $\alpha\omega$-open sets are {X,φ , {c}, {a, c}}

$gsp\alpha\omega$ -closed sets are {X,φ , {a}, {b}, {a, b}, {b, c}}

THEOREM 2.3:

Every closed set is $gsp\alpha\omega$ -closed set

PROOF:

Let A be a closed set, $cl(A) = A$. Let $A \subseteq U, U$ be $\alpha\omega$-open

We've $spcl(A) \subseteq cl(A) \subseteq U \Rightarrow spcl(A) \subseteq U$. Hence A is $gsp\alpha\omega$ -closed set.

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.4: Let X = {a, b, c},$\boldsymbol{\tau}$= {X,$\boldsymbol{\varphi}$, {c}, {a, c}}

Closed sets are {X, φ, {b}, {a, b}. $gsp\alpha\omega$-closed sets are {X,φ, {a}, {b}, {a, b}, {b, c}}

Let A ={a}. Here, {a} is $gsp\alpha\omega$-closed set but not closed set in (X, τ)

THEOREM 2.5:

Every regular-closed set is $gsp\alpha\omega$-closed set

PROOF:

Let A be a regular-closed set. Let $A \subseteq U, U$ be $\alpha\omega$-open

But, every regular closed set is closed set$\therefore cl(A) = A$.

We've $spcl(A) \subseteq cl(A) = A \subseteq U \Rightarrow spcl(A) \subseteq U$. Hence A is $gsp\alpha\omega$-closed set

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.6:

Let X = {a, b, c}, τ= {X,φ , {b}, {c}, {b, c}}, Closed sets are {X,φ , {a}, {a, b}, {a, c}} regular-closed sets are {X,φ , {a, b}, {a, c}}, $gsp\alpha\omega$-closed sets are {X,φ , {a}, {b}, {c}, {a, b}, {a, c}}. Let A ={b}. Here, {b} is $gsp\alpha\omega$-closed set but not regular-closed set in (X, τ).

THEOREM 2.7:

Every pre-closed set is $gsp\alpha\omega$-closed set

PROOF:

Let A be a pre-closed set.Let $A \subseteq U, U$ be$\alpha\omega$-open

Since A is pre-closed, $pcl(A) = A$. We've$spcl(A) \subseteq pcl(A) = A \subseteq U \Rightarrow spcl(A) \subseteq U$

Hence A is $gsp\alpha\omega$-closed set

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.8:

Let X = {a, b, c}, τ= {X, φ, {a}, {a, c}}. Closed sets are {X,φ , {b}, {b, c}}

pre-closed sets are {X,φ , {b}, {c}, {b, c}}

$gsp\alpha\omega$ -closed sets are {X,φ , {b}, {c}, {a, b}, {b, c}} Let A = {a,b}

Here, {a, b}is$gsp\alpha\omega$-closed set but not pre-closed set in (X, τ)

THEOREM 2.9:

Every α-closed set is $gsp\alpha\omega$-closed set

PROOF:

Let A be a α-closed set. Let $A \subseteq U, U$ be$\alpha\omega$-open

Since A is α-closed, $cl\left(int\left(cl(A)\right)\right) = A$

We've $A \subseteq cl(A) \Rightarrow cl(int(cl(A))) \subseteq cl(A)$. Also, $cl(A) = A$

$$\Rightarrow cl(int(A)) \subseteq A \Rightarrow int\left(cl\left(int(A)\right)\right) \subseteq int(A) \subseteq A, [since\ int(A) \subseteq A]$$

$\Rightarrow int\left(cl\left(int(A)\right)\right) \subseteq A \subseteq U \Rightarrow spcl(A) \subseteq U$. Hence A is $gsp\alpha\omega$ -closed set

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The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.10:

Let X = {a, b, c}, τ= {X,φ , {c}, {a, c}}, Closed sets are {X,φ , {b}, {a, b}}

α-closed sets are {X,φ , {a}, {b}, {a, b}}

$gsp\alpha\omega$ -closed sets are {X,φ , {a}, {b}, {a, b}, {b, c}}. Let A = {b,c}

Here, {b, c} is $gsp\alpha\omega$-closed set but not α-closed set in (X,τ).

THEOREM 2.11:

Every semi-pre-closed set is $gsp\alpha\omega$ -closed set

PROOF:

Let A be a semi-pre-closed set. Let $A \subseteq U, U$ be $\alpha\omega$-open

Since A is semi-pre-closed, $spcl(A) = A \Rightarrow spcl(A) = A \subseteq U \therefore spcl(A) \subseteq U$

Hence A is $gsp\alpha\omega$-closed set

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.12:

Let X = {a, b, c}, τ= {X,φ , {c}, {a, c}}, Closed sets are {X,φ, {b}, {a, b}}

semi-pre-closed sets are {X,φ , {a}, {b}, {a, b}}

$gsp\alpha\omega$-closed sets are {X,φ, {a}, {b}, {a, b}, {b, c}} Let A = {b,c}

Here, {b, c} is $gsp\alpha\omega$ -closed set but not semi-pre-closed set in (X,τ)

THEOREM 2.13:

Every $g\alpha\omega$-closed set is $gsp\alpha\omega$-closed set

PROOF:

Let A be a $g\alpha\omega$ -closed set Let$A \subseteq U, U$ be $\alpha\omega$-open

Since A is $g\alpha\omega$-closed, $cl(A) \subseteq U, U$ is$\alpha\omega$-open

We've $spcl(A) \subseteq cl(A) \subseteq U \Rightarrow spcl(A) \subseteq U.$

Hence A is $gsp\alpha\omega$-closed set

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE 2.14:

Let X = {a, b, c}, τ= {X,φ , {b}, {a, b}}, Closed sets are {X,φ , {c}, {a, c}}

$\alpha\omega$-open sets are {X,φ , {b}, {a, b}}, $g\alpha\omega$ -closed sets are {X,φ , {c}, {b, c}, {a, c}}

$gsp\alpha\omega$-closed sets are {X,φ , {a}, {c}, {b, c}, {a, c}}. Let A ={a}

Here, {a} is $gsp\alpha\omega$ -closed set but not $g\alpha\omega$-closed set in (X,τ)

THEOREM 2.15:

Every $gp\alpha\omega$ -closed set is $gsp\alpha\omega$-closed set

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PROOF:

Let A be a $gsp\alpha\omega$-closed set. Let $A \subseteq U, U$ be $\alpha\omega$-open

Since A is $gp\alpha\omega$ -closed, $pcl(A) \subseteq U, U$ is $\alpha\omega$-open

We've $spcl(A) \subseteq pcl(A) \subseteq U \Rightarrow spcl(A) \subseteq U.$

Hence A is $gsp\alpha\omega$ -closed set

The converse of the above theorem need not be true as seen from the following example.

EXAMPLE-2.16:

Let X = {a, b, c}, τ= {X, , {b}, {c}, {b, c}}, Closed sets are {X,φ , {a}, {a, b}, {a, c}}

pre-closed sets are {X, , {a}, {a, b}, {a, c}}, $\alpha\omega$-open sets are {X,φ , {b}, {c}, {b, c}}

$gp\alpha\omega$-closed sets are {X,φ , {a}, {a, b}, {a, c}}

$gsp\alpha\omega$ -closed sets are {X, , {a}, {b}, {c}, {a, b}, {a, c}}. Let A ={b}

Here, {b} is $gsp\alpha\omega$-closed set but not $gp\alpha\omega$-closed set in (X, τ)

THEOREM-2.17:

If A is $\alpha\omega$-open and $gsp\alpha\omega$-closed, then A is semi-pre-closed.

PROOF:

Let $A \subseteq U, U$ be$\alpha\omega$–open. Since A is $\alpha\omega$ -open, take $A = U$ ___(1)

Also, A is $gsp\alpha\omega$-closedand open, then $A \subseteq A$and $spcl(A) \subseteq U = A$ (by (1))

$\Rightarrow A \subseteq A$and $spcl(A) \subseteq A.$ $spcl(A) = A$. Hence A is semi-pre-closed

THEOREM 2.18:

Union of two $gsp\alpha\omega$-closed sets is $gsp\alpha\omega$ -closed set

PROOF:

Let A and B be two $gsp\alpha\omega$ -closed sets in (X, τ)

Let G be any $\alpha\omega$-open set in (X, τ) such that $A \cup B \subseteq G$, then$A \subseteq G$and$B \subseteq G$

Since A and B are $gsp\alpha\omega$-closed sets, then $spcl(A) \subseteq G$ and $spcl(B) \subseteq G$

But, $spcl(A \cup B) = spcl(A) \cup spcl(B) \subseteq G$

$\Rightarrow spcl(A \cup B) \subseteq G, G$is $\alpha\omega$-openHence $A \cup B$ is $gsp\alpha\omega$-closedset

REMARK 2.19:

Intersection of two $gsp\alpha\omega$-closed sets need not be $gsp\alpha\omega$-closed sets

Forexample, X = {a, b, c}, τ= {X,φ , {c}}, Closed sets are {X,φ , {a, b}}

$gsp\alpha\omega$-closed sets are {X, φ, {b}, {c}, {a, b}, {b, c}, {a, c}}

Here, {a, b} and {a, c} are $gsp\alpha\omega$-closed sets

But {a, b} ∩{a, c} = {a} is not a $gsp\alpha\omega$ -closed set

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THEOREM 2.20:

If A is $gsp\alpha\omega$-closed set in $X$and $A \subseteq B \subseteq spcl(A)$, then B is also $gsp\alpha\omega$-closed set in X.

PROOF:

Let A be $gsp\alpha\omega$-closed set in X and $A \subseteq B \subseteq spcl(A)$. Let $B \subseteq U$ and U be $\alpha\omega$-open set in X. Since $A \subseteq B$, then $A \subseteq U$ and A is $gsp\alpha\omega$-closed set, $spcl(A) \subseteq U$

Given $B \subseteq spcl(A) \Rightarrow spcl(B) \subseteq spcl(spcl(A)) \Rightarrow spcl(B) \subseteq spcl(A) \subseteq U$

$\therefore spcl(B) \subseteq U$. Hence B is $gsp\alpha\omega$ -closed set in X.

THEOREM 2.21:

Let$A \subseteq Y \subseteq X$andsupposethatAis $gsp\alpha\omega$-closedsetinX.Then A is $gsp\alpha\omega$-closed set relative toY.

PROOF:

Let $A \subseteq Y \cap G, G$ be $\alpha\omega$–open. Since A is $gsp\alpha\omega$-closed set, then$spcl(A) \subseteq G$, whenever $A \subseteq G$, G is $\alpha\omega$-open$\Rightarrow$ $Y \cap spcl(A) \subseteq Y \cap G$.

Hence A is $gsp\alpha\omega$-closed set relative to Y.

REMARK 2.22:

The setgp*-closed set and $gsp\alpha\omega$ -closed set are independentand this can be seen from the following example.

2.23 EXAMPLE:

Let X = {a, b,c}, τ= {X, , {b}, {a, b}}

Closed sets are {X, φ,{c}, {a, c}}

gp-open sets are {X,φ , {a}, {b}, {b, c}, {a, c}}

semi-pre closed sets are {X,φ , {a}, {c}, {b, c}, {a, c}}

$\alpha\omega$-open sets are {X,φ , {b}, {a, b}}

Let $A = \{a, b\} \subseteq X, cl(A) = X \subseteq X$.

Let $= \{a\} \subseteq \{a, b\}, spcl(A) = \{a\} \subseteq \{a, b\}$.

gp*-closed sets are {X,φ , {c}, {a, b}, {a, c}}

$gsp\alpha\omega$-closed sets are {X, , {a}, {c}, {b, c}, {a, c}} .

Here, the sets {a} and {b, c} are $gsp\alpha\omega$-closed set but not gp*closed set.

Also, the set {a, b} is gp*-closed set but not $gsp\alpha\omega$-closed set.

CONCLUSION

In this paper we have introduced$gsp\alpha\omega$-closed set and studied some of their properties in topological spaces. Also we can extend the study to $gsp\alpha\omega$-continuous maps,$gsp\alpha\omega$-irresolute maps .This study can be extended to the concept of compactness, connectedness and separation axioms. Also it can be extended to spaces like Bitopology, Fuzzy and Ideal topological spaces.

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