SPHERICAL CUBIC SOFT MATRICES AND ITS APPLICATION FOR DECISION MAKING IN MEDICAL FIELD

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Abstract - ***In this paper, we introduce the notion of Spherical Cubic Soft Matrices (SCSMs) and discuss their Properties, Also develop the concepts of determinant and adjoint of cubic soft matrices. Finally we analyze the application of these matrices in decision making problem.***

Keywords: Fuzzy matrix, Spherical fuzzy Matrix, Spherical cubic soft matrix,Arithmetic operations, properties, determinant and adjoint

INTRODUCTION

The fuzzy set (FS) and interval valued fuzzy set was introduced by Zadeh[26] to model these imprecise evaluations in the decision-making process. As an extension of FS, the intuitionistic fuzzy set (IFS) is characterized by the membership degree and the non-membership degree satisfying the condition that their sum is less than or equal to 1 was developed by Atanassov [4].Fuzzy matrices were introduced for the first time by Thomason [22], who discussed theconvergence of powers of fuzzy matrix.Ragab et al[12] presented some properties on determinant and adjoint of square fuzzy matrix. Kim and Roush [10] studied the canonical form of an idempotent matrix. Hashimoto studied the canonical form of a transitive matrix. Adak et.al [1,2,3] develops some properties of generalized intuitionistic fuzzy, fuzzy block matrices and distributive lattices.

The concept of fuzzy matrices with fuzzy rows and fuzzy columns was presented by Pal [10]. Pal also discussed about the interval valued fuzzy matrices with interval valued fuzzy rows and interval valued fuzzy columns. A triangular fuzzy matrix norm was investigated by Pradhan and Pal [15,16]. Fuzzy matrices engage in recreation to a vital role in scientific development.

Madhumangal pal and SanjibMontal [11] introduced the bipolar matrix and their algebraic operations. SanjibMontal [19] introduced the bipolar matrix and their algebraic operations. Yager [23,24,25] recently proposed the concept of Pythagorean fuzzy set (PFS) Hesitant fuzzy sets can be used as a functional tool allowing many potential membership degrees of an element to a set. These fuzzy sets let several degrees of an element to be possible between zero developed by Torra [21]. A cubic set is a hybrid structure involving an internal valued fuzzy set.Jun et al., [2012] [9(a)] have introduced the concept of cubic sets and studied some fundamental properties of these structures. They have introduced classification of cubic sets as internal and external cubic sets and discussed the ideas of P-order and R-order in cubic sets as well as the operations P-union, R-union, P-intersection and R-intersection.

Neutrosophic logic and neutrosophic sets were developed by Smarandache[18] as an extension of intuitionistic fuzzy sets. A neutrosophic set is defined as the set where each element of the universe has a degree of truthiness, indeterminacy and falsity. Picture fuzzy sets were developed by Cuong [6,7,8] as a generalization of intuitionistic fuzzy set.

Picture fuzzy matrices were developed by Madhhumangal pal and Shovan Degra [20]. Spherical fuzzy sets (SFS) were introduced by Kahraman and Gündogdu [15] as an extension of Pythagorean, neutrosophic and picture fuzzy sets. The idea behind SFS is to let decision makers to generalize other extensions of fuzzy sets by defining a membership function on a spherical surface and independently assign the parameters of that membership function with a larger domain.

Muhiuddin and Al-roqi [14], introduced the notions of internal, external cubicsoft sets, P-cubic (R-cubic) soft subsets, R-union(R-intersection, P-union and Pintersection)of cubic soft sets and the complement of a cubic soft set. They investigatedseveral related properties and applied the notion of cubic soft sets toBCK/BCI-algebras.

Chinnadurai.V and Barvkavi.S,[5]introduced the notions of internal, external cubicsoftmatrices, P-cubic (R-cubic) soft matrices, R-union(R-intersection, P-union and Pintersection)of cubic soft matrices and the complement of a cubic soft matrices. They investigatedseveral related properties and applied the notion of cubic soft matrices.Sarala N, Rajkumari S. [17] developed theapplication of intuitionistic fuzzy soft matrices in Decision Making Problem by using medical diagnosis.

In this paper, we introduce notion of spherical cubic soft matrix. We defined P-(R-)order, P-(R-) union, P-(R-) intersection of spherical cubic soft matrix and their properties are discussed. Also we define the determinant and adjoint of spherical cubic soft matrix with numerical example. Finally one application is discussed under the field of medical diagnosis.

2. PRELIMINARIES

In this section, some basic definitions which are essential for further discussion and recalled .

DEFINITION 2.1

A matrix of the form$A = \left[(a_{ij})\right]_{m\times n}$ where $a_{ij} \in [0,1] 1 \le i \le m$ and $1 \le i \le n$ is called a fuzzy matrix (FM) if the a_{ij} represents the memberships of the elements to a fuzzy set.

DEFINITION 2.2

An intuitionistic fuzzy matrix (IFM) is of the form $A = \left[(a_{ij}, a_{ij}')\right]_{m\times n}$where $a_{ij}, a_{ij}' \in [0,1]$, satisfying $0 \le a_{ij} + a_{ij}' \le 1$ for all i, j, a_{ij} and a_{ij}' representmembership and non membership of an element to IFS.

DEFINITION 2.3

Let S be an initial universal set and P be a set of parameters. A cubic soft set over U is defined to be a pair $(\mathcal{F}, A)$ where F is a mapping from A to P(S) and A ⊆ P: Then the pair $(\mathcal{F}, A)$ can be represented as,

$(\mathcal{F}, A) = \{\mathcal{F}(i)/i \in A\}$, where $\mathcal{F}(i) = \left[\langle s, \widetilde{A}_i(s), \mu_i(s)\rangle / s \in S,\ \ i \in A\right]$

is a cubic soft set in which $\widetilde{A}_i(s)$ is the interval valued fuzzy set and $\mu_i(s)$ is a fuzzy set.

DEFINITION 2.4

Spherical fuzzy set (SFS), $\widetilde{A}_s$ of universe of discourse U is given by $\widetilde{A}_s = \left[\langle s, \mu\widetilde{A}_s(s), \upsilon\widetilde{A}_s(s), \pi\widetilde{A}_s(s)\rangle / s \in S, \right]$ where$\mu\widetilde{A}_s(s): U \to [0,1]$, $\upsilon\widetilde{A}_s(s): U \to [0,1]$and $\pi\widetilde{A}_s(s): U \to [0,1]$. Also $0 \le \mu^2\widetilde{A}_s(s) \le \upsilon^2\widetilde{A}_s(s) \le \pi^2\widetilde{A}_s(s) \le 1$. For each s, the numbers $\mu\widetilde{A}_s(s), \upsilon\widetilde{A}_s(s)$ and $\pi\widetilde{A}_s(s)$ are the degree of membership, non membership and hesitancy of s to $\widetilde{A}_s$

DEFINITION 2.5

Let S be an initial universal set. Then the Interval Valued Spherical Fuzzy Set (IVSFS) is defined by

$\widetilde{A_s}^I = \left[\langle \widetilde{M_s}^I(i), \widetilde{N_s}^I(i)\rangle, \widetilde{R_s}^I(i)/i \in S, \right]$ where $\widetilde{M_s}^I(i) = \left[\widetilde{M_s}^{I^L}(i), \widetilde{M_s}^{I^U}(i)\right]$, $\widetilde{N_s}^I(i) = \left[\widetilde{N_s}^{I^L}(i), \widetilde{N_s}^{I^U}(i)\right]$ and $\widetilde{R_s}^I(i) = \left[\widetilde{R_s}^{I^L}(i), \widetilde{R_s}^{I^U}(i)\right]$ also $\widetilde{M_s}^I(i): U \to [0,1]$, $\widetilde{N_s}^I(i): U \to [0,1]$ and $\widetilde{R_s}^I(i): U \to [0,1]$. Also $0 \le \left(\widetilde{M_s}^I(i)\right)^2 \le \left(\widetilde{N_s}^I(i)\right)^2 \le \left(\widetilde{R_s}^I(i)\right)^2 \le 1$.

DEFINITION 2.6

A Spherical Fuzzy Matrix (SFM) of order $(m \times n)$ is denoted by $\widetilde{A}_{s(m\times n)}$ and that of order$(m \times m)$, that is square SFM is denoted by$\widetilde{A}_{s(m\times m)}$. We conclude that the SFM is of the form$\widetilde{A}_s = \left[\left(a_{ij}, a_{ij}^{'}, a_{ij}^{*}\right)\right]_{(m\times n)}$, where $a_{ij}, a_{ij}^{'}, a_{ij}^{*}$ are the Membership, Non-membership and Hesitancy of$\widetilde{A}_s$. Also satisfies the condition$0 \le {a_{ij}}^2 + {a_{ij}^{'}}^2 + {a_{ij}^{*}}^2 \le 1$.

DEFINITION 2.7

The SFM is of the form$\widetilde{A}_s = \left[\left(a_{ij}, a_{ij}^{'}, a_{ij}^{*}\right)\right]_{(m\times n)}$, where $a_{ij}, a_{ij}^{'}, a_{ij}^{*}$ are the Membership, Non-membership and Hesitancy of$\widetilde{A}_s$.

Now $\Re_{\widetilde{A}_s} = \sqrt{1 - {a_{ij}}^2 - {a_{ij}^{'}}^2 - {a_{ij}^{*}}^2}$ is called as a refusal degree of SFM.

DEFINITION 2.8

The Interval Valued Spherical Fuzzy Matrix (IVSFM) is defined by $\widetilde{A}_s^{\ I} = \left[\left(a_{ij}, a_{ij}^{'}, a_{ij}^{*}\right)\right]_{(m\times n)}$ where $a_{ij}, a_{ij}^{'}, a_{ij}^{*}$ are the Membership, Non-membership and Hesitancy of$\widetilde{A}_s$ and satisfies the condition$0 \le {a_{ij}}^2 + {a_{ij}^{'}}^2 + {a_{ij}^{*}}^2 \le 1$. Also$a_{ij} = \left[{a_{ij}}^L, {a_{ij}}^U\right]$, $a_{ij}^{'} = \left[{a_{ij}^{'}}^L, {a_{ij}^{'}}^U\right]$and $a_{ij}^{*} = \left[{a_{ij}^{*}}^L, {a_{ij}^{*}}^U\right]$.

DEFINITION 2.9

Let $U = \{u_1, u_2, u_3, \dots \dots u_m\}$ be an initial universal set and $E = \{e_1, e_2, e_3, \dots \dots e_m\}$be a set of parameters. Let $A \subseteq E$. Then cubic soft set $(\mathcal{F}, A)$ can be expressed in matrix form as

$$A^{\boxplus} = \left[a_{ij}\right] = \left[\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}\right]$$

Such that $A^{\boxplus} = \left[a_{ij}\right] = \langle \widetilde{A}_{e_j}(u_i), \lambda_{e_j}(u_i)\rangle = \langle \widetilde{A}_{ij}^{a}, \lambda_{ij}^{a}\rangle$ which is called an $m \times n$ cubic soft matrix (CSM) of the cubic soft set$(\mathcal{F}, A)$, where $i = 1,2, \dots m$and $j = 1,2, \dots m$. Also $\widetilde{A}_{ij}^{a}$is interval valued fuzzy matrix and λ_{ij}^{a} is a fuzzy value.

3. SPHERICAL CUBIC SOFT MATRICES (SCSM)

A Spherical Cubic Soft Matrix (SCSM) of order $(m \times n)$ is denoted by $\widetilde{A}_{s(m\times n)}^{c}$ and that of order$(m \times m)$, that is square SCSM is denoted by$\widetilde{A}_{s(m\times m)}^{c}$. We conclude that the SCSM is of the form$\widetilde{A}_s^c = \left[\left(a_{ij}, a_{ij}^{'}, a_{ij}^{*}\right), \lambda_{ij}\right]_{(m\times n)}$, where $\left(a_{ij}, a_{ij}^{'}, a_{ij}^{*}\right)$ is the interval valued spherical fuzzy matrix and λ_{ij} is the fuzzy value,$a_{ij}, a_{ij}^{'}, a_{ij}^{*}$ are the Membership, Non-membership and Hesitancy of$\widetilde{A}_s^c$. Also satisfies the condition$0 \le \left({a_{ij}}^U\right)^2 + \left({a_{ij}^{'}}^U\right)^2 + \left({a_{ij}^{*}}^U\right)^2 \le 1$. Also$a_{ij} = \langle\left[{a_{ij}}^L, {a_{ij}}^U\right]\rangle$, $a_{ij}^{'} = \langle\left[{a_{ij}^{'}}^L, {a_{ij}^{'}}^U\right]\rangle$ and$a_{ij}^{*} = \langle\left[{a_{ij}^{*}}^L, {a_{ij}^{*}}^U\right]\rangle$.

Consider the SCSM$\widetilde{A}_s^c = \left[\left(a_{ij}, a_{ij}^{'}, a_{ij}^{*}\right), \lambda_{ij}\right]_{(m\times n)}$, then the **complement** of the spherical cubic soft matrix is denoted by

$\left(\widetilde{A}_s^c\right)^c = \left[\left\{1-\left[a_{ij}{}^L, a_{ij}{}^U\right], 1-\left[a_{ij}^{'}{}^L, a_{ij}^{'}{}^U\right], 1-\left[a_{ij}^{*}{}^L, a_{ij}^{*}{}^U\right]\right\}, 1-\lambda_{ij}\right]$ for all i, j.

3.1 P- UNION, P- INTERSECTION, R-UNION AND R – INTERSECTION OF SPHERICAL CUBIC SOFT MATRICES

This section, we define P- Union, P- Intersection, R-Union and R – Intersection of two Spherical cubic soft matrices and explore some properties.

Let $\widetilde{A}_s^c = \left[\left(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle\left[a_{ij}^{'}{}^L, a_{ij}^{'}{}^U\right]\rangle, \langle[a_{ij}^{*}{}^L, a_{ij}^{*}{}^U]\rangle\right), \lambda_{ij}\right]_{(m\times n)}$, and

$\widetilde{B}_s^c = \left[\left(\langle[b_{ij}{}^L, b_{ij}{}^U]\rangle, \langle\left[b_{ij}^{'}{}^L, b_{ij}^{'}{}^U\right]\rangle, \langle[b_{ij}^{*}{}^L, b_{ij}^{*}{}^U]\rangle\right), \mu_{ij}\right]_{(m\times n)}$ Then

P-UNION

P- Union of $\widetilde{A}_s^c$ and $\widetilde{B}_s^c$ denoted by $\widetilde{A}_s^c \vee_P \widetilde{B}_s^c$ is defined as $\widetilde{A}_s^c \vee_P \widetilde{B}_s^c = \widetilde{D}_s^c$ where

$\widetilde{D}_s^c = \left[\left(\langle[d_{ij}{}^L, d_{ij}{}^U]\rangle, \langle\left[d_{ij}^{'}{}^L, d_{ij}^{'}{}^U\right]\rangle, \langle[d_{ij}^{*}{}^L, d_{ij}^{*}{}^U]\rangle\right), \gamma_{ij}\right]_{(m\times n)}$ with $\widetilde{D}_s^c =$
$\left(\max\{[a_{ij}{}^L, a_{ij}{}^U], [b_{ij}{}^L, b_{ij}{}^U]\}, \max\left\{\left[a_{ij}^{'}{}^L, a_{ij}^{'}{}^U\right], \left[b_{ij}^{'}{}^L, b_{ij}^{'}{}^U\right]\right\}, \max\{[a_{ij}^{*}{}^L, a_{ij}^{*}{}^U], [b_{ij}^{*}{}^L, b_{ij}^{*}{}^U]\}\right)$ and $\gamma_{ij} = \left(\max[\lambda_{ij}, \mu_{ij}]\right)$ for all i,j.

P – INTERSECTION

P – Intersection of $\widetilde{A}_s^c$ and $\widetilde{B}_s^c$ denoted by $\widetilde{A}_s^c \wedge_P \widetilde{B}_s^c$ is defined as $\widetilde{A}_s^c \wedge_P \widetilde{B}_s^c = \widetilde{D}_s^c$ where

$\widetilde{D}_s^c = \left[\left(\langle[d_{ij}{}^L, d_{ij}{}^U]\rangle, \langle\left[d_{ij}^{'}{}^L, d_{ij}^{'}{}^U\right]\rangle, \langle[d_{ij}^{*}{}^L, d_{ij}^{*}{}^U]\rangle\right), \gamma_{ij}\right]_{(m\times n)}$ with $\widetilde{D}_s^c =$
$\left(\min\{[a_{ij}{}^L, a_{ij}{}^U], [b_{ij}{}^L, b_{ij}{}^U]\}, \min\left\{\left[a_{ij}^{'}{}^L, a_{ij}^{'}{}^U\right], \left[b_{ij}^{'}{}^L, b_{ij}^{'}{}^U\right]\right\}, \min\{[a_{ij}^{*}{}^L, a_{ij}^{*}{}^U], [b_{ij}^{*}{}^L, b_{ij}^{*}{}^U]\}\right)$ and $\gamma_{ij} = \left(\min[\lambda_{ij}, \mu_{ij}]\right)$ for all i,j.

R-UNION

R – Union of $\widetilde{A}_s^c$ and $\widetilde{B}_s^c$ denoted by $\widetilde{A}_s^c \wedge_R \widetilde{B}_s^c$ is defined as $\widetilde{A}_s^c \wedge_R \widetilde{B}_s^c = \widetilde{D}_s^c$ where

$\widetilde{D}_s^c = \left[\left(\langle[d_{ij}{}^L, d_{ij}{}^U]\rangle, \langle\left[d_{ij}'{}^L, d_{ij}'{}^U\right]\rangle, \langle[d_{ij}^{*}{}^L, d_{ij}^{*}{}^U]\rangle\right), \gamma_{ij}\right]_{(m\times n)}$ with $\widetilde{D}_s^c =$
$\left(\max\{[a_{ij}{}^L, a_{ij}{}^U], [b_{ij}{}^L, b_{ij}{}^U]\}, \max\left\{\left[a_{ij}'{}^L, a_{ij}'{}^U\right], \left[b_{ij}'{}^L, b_{ij}'{}^U\right]\right\}, \max\{[a_{ij}^{*}{}^L, a_{ij}^{*}{}^U], [b_{ij}^{*}{}^L, b_{ij}^{*}{}^U]\}\right)$ and $\gamma_{ij} = \left(\min[\lambda_{ij}, \mu_{ij}]\right)$ for all i,j.

R – INTERSECTION

R – Intersection of $\widetilde{A}_s^c$ and $\widetilde{B}_s^c$ denoted by $\widetilde{A}_s^c \wedge_R \widetilde{B}_s^c$ is defined as $\widetilde{A}_s^c \wedge_R \widetilde{B}_s^c = \widetilde{D}_s^c$ where

$\widetilde{D}_s^c = \left[\left(\langle[d_{ij}{}^L, d_{ij}{}^U]\rangle, \langle\left[d_{ij}'{}^L, d_{ij}'{}^U\right]\rangle, \langle[d_{ij}^{*}{}^L, d_{ij}^{*}{}^U]\rangle\right), \gamma_{ij}\right]_{(m\times n)}$, with the matrix

$\widetilde{D}_s^c = \left(\min\{[a_{ij}{}^L, a_{ij}{}^U], [b_{ij}{}^L, b_{ij}{}^U]\}, \min\left\{\left[a_{ij}'{}^L, a_{ij}'{}^U\right], \left[b_{ij}'{}^L, b_{ij}'{}^U\right]\right\}, \min\{[a_{ij}^{*}{}^L, a_{ij}^{*}{}^U], [b_{ij}^{*}{}^L, b_{ij}^{*}{}^U]\}\right)$
and $\gamma_{ij} = \left(\max[\lambda_{ij}, \mu_{ij}]\right)$ for all i,j.

We now develop the some interesting properties of union and intersection of Spherical soft matrices.

3.2.1 PROPOSITION

Let $\widetilde{A}_s^c$, $\widetilde{B}_s^c$ and $\widetilde{C}_s^c$ are spherical cubic soft matrices of order $m \times n$, Then the following holds.

1. If $\widetilde{A}_s^c \subseteq_P \widetilde{B}_s^c$ and $\widetilde{A}_s^c \subseteq_P \widetilde{C}_s^c$, then $\widetilde{A}_s^c \subseteq_P \widetilde{B}_s^c \wedge_P \widetilde{C}_s^c$.

2. If $\widetilde{A}_s^c \subseteq_P \widetilde{B}_s^c$ and $\tilde{C}_s^c \subseteq_P \widetilde{B}_s^c$, then $\widetilde{A}_s^c \vee_P \tilde{C}_s^c \subseteq_P \widetilde{B}_s^c$.
3. If $\widetilde{A}_s^c \subseteq_R \widetilde{B}_s^c$ and $\widetilde{A}_s^c \subseteq_R \tilde{C}_s^c$, then $\widetilde{A}_s^c \subseteq_R \widetilde{B}_s^c \wedge_R \tilde{C}_s^c$.
4. If $\widetilde{A}_s^c \subseteq_R \widetilde{B}_s^c$ and $\tilde{C}_s^c \subseteq_R \widetilde{B}_s^c$, then $\widetilde{A}_s^c \vee_R \tilde{C}_s^c \subseteq_R \widetilde{B}_s^c$.

PROOF

i). Consider the spherical cubic soft matrices $\widetilde{A}_s^c = \left[\left(\langle[{a_{ij}}^L, {a_{ij}}^U]\rangle, \langle\left[{a'_{ij}}^L, {a'_{ij}}^U\right]\rangle, \langle[{a^*_{ij}}^L, {a^*_{ij}}^U]\rangle\right), \lambda_{ij}\right]_{(m\times n)}$,
$\widetilde{B}_s^c = \left[\left(\langle[{b_{ij}}^L, {b_{ij}}^U]\rangle, \langle\left[{b'_{ij}}^L, {b'_{ij}}^U\right]\rangle, \langle[{b^*_{ij}}^L, {b^*_{ij}}^U]\rangle\right), \mu_{ij}\right]_{(m\times n)}$ and

$\tilde{C}_s^c = \left[\left(\langle[{c_{ij}}^L, {c_{ij}}^U]\rangle, \langle\left[{c'_{ij}}^L, {c'_{ij}}^U\right]\rangle, \langle[{c^*_{ij}}^L, {c^*_{ij}}^U]\rangle\right), \eta_{ij}\right]_{(m\times n)}$

Also given, $\widetilde{A}_s^c = \left[\left(\langle[{a_{ij}}^L, {a_{ij}}^U]\rangle, \langle\left[{a'_{ij}}^L, {a'_{ij}}^U\right]\rangle, \langle[{a^*_{ij}}^L, {a^*_{ij}}^U]\rangle\right), \lambda_{ij}\right]_{(m\times n)} \subseteq_P$

$\widetilde{B}_s^c = \left[\left(\langle[{b_{ij}}^L, {b_{ij}}^U]\rangle, \langle\left[{b'_{ij}}^L, {b'_{ij}}^U\right]\rangle, \langle[{b^*_{ij}}^L, {b^*_{ij}}^U]\rangle\right), \mu_{ij}\right]_{(m\times n)}$ and

$\widetilde{A}_s^c = \left[\left(\langle[{a_{ij}}^L, {a_{ij}}^U]\rangle, \langle\left[{a'_{ij}}^L, {a'_{ij}}^U\right]\rangle, \langle[{a^*_{ij}}^L, {a^*_{ij}}^U]\rangle\right), \lambda_{ij}\right]_{(m\times n)} \subseteq_P$

$\tilde{C}_s^c = \left[\left(\langle[{c_{ij}}^L, {c_{ij}}^U]\rangle, \langle\left[{c'_{ij}}^L, {c'_{ij}}^U\right]\rangle, \langle[{c^*_{ij}}^L, {c^*_{ij}}^U]\rangle\right), \eta_{ij}\right]_{(m\times n)}$

Now $\widetilde{B}_s^c \wedge_P \tilde{C}_s^c = \left\langle \begin{matrix} \min\{[{b_{ij}}^L, {b_{ij}}^U], [{c_{ij}}^L, {c_{ij}}^U]\}, \min\left\{\left[{b'_{ij}}^L, {b'_{ij}}^U\right], \left[{c'_{ij}}^L, {c'_{ij}}^U\right]\right\}, \min\{[{b^*_{ij}}^L, {b^*_{ij}}^U], [{c^*_{ij}}^L, {c^*_{ij}}^U]\}, \\ (\min[\mu_{ij}, \eta_{ij}]) \end{matrix} \right\rangle$

So clearly from straight forward, $\widetilde{A}_s^c \subseteq_P \widetilde{B}_s^c \wedge_P \tilde{C}_s^c$ holds. Similarly ii), iii) and iv) holds.

3.2.2 PROPOSITION

Let $\widetilde{A}_s^c$ be a spherical cubic soft matrices of order $m \times n$, then the following holds.

i). $\widetilde{A}_s^c \vee_P \widetilde{A}_s^c = \widetilde{A}_s^c$

ii) $\widetilde{A}_s^c \vee_R \widetilde{A}_s^c = \widetilde{A}_s^c$

iii) $\widetilde{A}_s^c \wedge_P \widetilde{A}_s^c = \widetilde{A}_s^c$

iv) $\widetilde{A}_s^c \wedge_R \widetilde{A}_s^c = \widetilde{A}_s^c$

PROOF

i) Consider the spherical cubic soft matrix $\widetilde{A}_s^c = \left[\left(\langle[{a_{ij}}^L, {a_{ij}}^U]\rangle, \langle\left[{a'_{ij}}^L, {a'_{ij}}^U\right]\rangle, \langle[{a^*_{ij}}^L, {a^*_{ij}}^U]\rangle\right), \lambda_{ij}\right]_{(m\times n)}$

Now, $\widetilde{A}_s^c \vee_P \widetilde{A}_s^c = \left[\left(\langle[{a_{ij}}^L, {a_{ij}}^U]\rangle, \langle\left[{a'_{ij}}^L, {a'_{ij}}^U\right]\rangle, \langle[{a^*_{ij}}^L, {a^*_{ij}}^U]\rangle\right), \lambda_{ij}\right] \vee_P$

$$\left[\left(\langle[{a_{ij}}^L, {a_{ij}}^U]\rangle, \langle\left[{a'_{ij}}^L, {a'_{ij}}^U\right]\rangle, \langle[{a^*_{ij}}^L, {a^*_{ij}}^U]\rangle\right), \lambda_{ij}\right]$$

$$= \left\{\begin{matrix} \left(\max\{[{a_{ij}}^L, {a_{ij}}^U], [{a_{ij}}^L, {a_{ij}}^U]\}, \max\left\{\left[{a'_{ij}}^L, {a'_{ij}}^U\right], \left[{a'_{ij}}^L, {a'_{ij}}^U\right]\right\}, \max\{[{a^*_{ij}}^L, {a^*_{ij}}^U], [{a^*_{ij}}^L, {a^*_{ij}}^U]\}\right), \\ (\max[\lambda_{ij}, \lambda_{ij}]) \end{matrix}\right\}$$

$$= \left[\left(\langle[{a_{ij}}^L, {a_{ij}}^U]\rangle, \langle\left[{a'_{ij}}^L, {a'_{ij}}^U\right]\rangle, \langle[{a^*_{ij}}^L, {a^*_{ij}}^U]\rangle\right), \lambda_{ij}\right]$$

$= \widetilde{A}_s^c$.

ii) $\widetilde{A}_s^c \vee_R \widetilde{A}_s^c = \left[\left(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle\left[a'_{ij}{}^L, a'_{ij}{}^U\right]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U]\rangle\right), \lambda_{ij}\right] \vee_R \left[\left(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle\left[a'_{ij}{}^L, a'_{ij}{}^U\right]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U]\rangle\right), \lambda_{ij}\right]$

$$= \begin{Bmatrix} \left(\max\{[a_{ij}{}^L, a_{ij}{}^U], [a_{ij}{}^L, a_{ij}{}^U]\}, \max\left\{\left[a'_{ij}{}^L, a'_{ij}{}^U\right], \left[a'_{ij}{}^L, a'_{ij}{}^U\right]\right\}, \max\{[a^*_{ij}{}^L, a^*_{ij}{}^U], [a^*_{ij}{}^L, a^*_{ij}{}^U]\}\right), \\ \left(\min[\lambda_{ij}, \lambda_{ij}]\right) \end{Bmatrix}$$

$$= \left[\left(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle\left[a'_{ij}{}^L, a'_{ij}{}^U\right]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U]\rangle\right), \lambda_{ij}\right]$$

$= \widetilde{A}_s^c$.

iii) $\widetilde{A}_s^c \wedge_P \widetilde{A}_s^c = \left[\left(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle\left[a'_{ij}{}^L, a'_{ij}{}^U\right]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U]\rangle\right), \lambda_{ij}\right] \wedge_P \left[\left(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle\left[a'_{ij}{}^L, a'_{ij}{}^U\right]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U]\rangle\right), \lambda_{ij}\right]$

$$= \begin{Bmatrix} \left(\min\{[a_{ij}{}^L, a_{ij}{}^U], [a_{ij}{}^L, a_{ij}{}^U]\}, \min\left\{\left[a'_{ij}{}^L, a'_{ij}{}^U\right], \left[a'_{ij}{}^L, a'_{ij}{}^U\right]\right\}, \min\{[a^*_{ij}{}^L, a^*_{ij}{}^U], [a^*_{ij}{}^L, a^*_{ij}{}^U]\}\right), \\ \left(\min[\lambda_{ij}, \lambda_{ij}]\right) \end{Bmatrix}$$

$$= \left[\left(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle\left[a'_{ij}{}^L, a'_{ij}{}^U\right]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U]\rangle\right), \lambda_{ij}\right]$$

$= \widetilde{A}_s^c$.

iv) $\widetilde{A}_s^c \wedge_R \widetilde{A}_s^c = \left[\left(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle\left[a'_{ij}{}^L, a'_{ij}{}^U\right]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U]\rangle\right), \lambda_{ij}\right] \wedge_R \left[\left(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle\left[a'_{ij}{}^L, a'_{ij}{}^U\right]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U]\rangle\right), \lambda_{ij}\right]$

$$= \begin{Bmatrix} \left(\min\{[a_{ij}{}^L, a_{ij}{}^U], [a_{ij}{}^L, a_{ij}{}^U]\}, \min\left\{\left[a'_{ij}{}^L, a'_{ij}{}^U\right], \left[a'_{ij}{}^L, a'_{ij}{}^U\right]\right\}, \min\{[a^*_{ij}{}^L, a^*_{ij}{}^U], [a^*_{ij}{}^L, a^*_{ij}{}^U]\}\right), \\ \left(\max[\lambda_{ij}, \lambda_{ij}]\right) \end{Bmatrix}$$

$$= \left[\left(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle\left[a'_{ij}{}^L, a'_{ij}{}^U\right]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U]\rangle\right), \lambda_{ij}\right]$$

$= \widetilde{A}_s^c$.

4. DETERMINANT AND ADJOINT OF P – ORDERED (R- ORDERED) SPHERICAL CUBIC SOFT MATRICES

In this section we define determinant and adjoint of P – Ordered (R- Ordered) Spherical cubic soft matrices and related property

4.1 COMPONENT WISE ADDITION, MULTIPLICATION AND PRODUCT OF P – ORDERED SPHERICAL CUBIC SOFT MATRICES

Let $\widetilde{A}_s^c$ and $\widetilde{B}_s^c$ are spherical cubic soft matrices of order $m \times n$, Thus $\widetilde{A}_s^c = \left[\left(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle\left[a'_{ij}{}^L, a'_{ij}{}^U\right]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U]\rangle\right), \lambda_{ij}\right]_{(m\times n)}$, and $\widetilde{B}_s^c = \left[\left(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle\left[a'_{ij}{}^L, a'_{ij}{}^U\right]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U]\rangle\right), \lambda_{ij}\right]_{(m\times n)}$ then their Component wise addition and Component wise multiplication are defined as

i) $\widetilde{A}_s^c \oplus_P \widetilde{B}_s^c =$

$$\begin{Bmatrix} \max\{[a_{ij}{}^L, a_{ij}{}^U], [b_{ij}{}^L, b_{ij}{}^U]\}, \max\left\{\left[a'_{ij}{}^L, a'_{ij}{}^U\right], \left[b'_{ij}{}^L, b'_{ij}{}^U\right]\right\}, \max\{[a^*_{ij}{}^L, a^*_{ij}{}^U], [b^*_{ij}{}^L, b^*_{ij}{}^U]\}, \\ \left(\max[\lambda_{ij}, \mu_{ij}]\right) \end{Bmatrix}$$

ii) $\widetilde{A}_S^c \odot_P \widetilde{B}_S^c = \begin{Bmatrix} \min\{[a_{ij}{}^L, a_{ij}{}^U], [b_{ij}{}^L, b_{ij}{}^U]\}, \min\left\{\left[a'_{ij}{}^L, a'_{ij}{}^U\right], \left[b'_{ij}{}^L, b'_{ij}{}^U\right]\right\}, \min\{[a^*_{ij}{}^L, a^*_{ij}{}^U], [b^*_{ij}{}^L, b^*_{ij}{}^U]\}, \\ (\min[\lambda_{ij}, \mu_{ij}]) \end{Bmatrix}$

iii) $\widetilde{A}_S^c *_P \widetilde{B}_S^c = \left\{\left[\sum_{j=1}^n \left\{\widetilde{A}_S^{c\,aL} \wedge \widetilde{B}_S^{c\,bL}\right\}, \sum_{j=1}^n \left\{\widetilde{A}_S^{c\,aU} \wedge \widetilde{B}_S^{c\,bU}\right\}\right], \sum_{j=1}^n \{\lambda_{ij}{}^a \wedge \mu_{ij}{}^b\}\right\}$ for all i,j. The Product $\widetilde{A}_S^c *_P \widetilde{B}_S^c$ is defined if and only if $\widetilde{A}_S^c$ and $\widetilde{B}_S^c$ are square matrices of the same order.

4.2. DETERMINANT OF P- ORDEREDSPHERICAL CUBIC SOFT MATRICES OF ORDER n × n

The determinant $|\widetilde{A}_S^c|_P$ of an n × n SCSM

$\widetilde{A}_S^c = \left[\left(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle\left[a'_{ij}{}^L, a'_{ij}{}^U\right]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U], \lambda_{ij}\rangle\right)\right]_{(n\times n)}$ is defined as

$|\widetilde{A}_S^c|_P =$
$\langle \vee_{\sigma\in S_n}\left\{\widetilde{A}_S^{c\,aL}{}_{1\sigma(1)} \ldots \wedge \widetilde{A}_S^{c\,aL}{}_{n\sigma(n)}\right\}, \vee_{\sigma\in S_n}\left\{\widetilde{A}_S^{c\,aU}{}_{1\sigma(1)} \ldots \wedge \widetilde{A}_S^{c\,aU}{}_{n\sigma(n)}\right\}, \vee_{\sigma\in S_n}\left\{\lambda_{ij}{}^a{}_{1\sigma(1)} \ldots \wedge \lambda_{ij}{}^a{}_{n\sigma(n)}\right\}, \rangle$ where S_n denotes the symmetric group of all permutations of the indices 1,2,3,….n.

4.2 ADJOINT OF P- ORDERED SPHERICAL CUBIC SOFT MATRICES

The adjoint of an $n \times n$ SCSM $\tilde{A}_S^c$ denoted by $adj_P \tilde{A}_S^c$ as $adj_P \tilde{A}_S^c = [b_{ij}] = \sum_{\varphi\in S_{n_{jn_i}}} \prod_{t\in n_j} \langle\left[\tilde{A}_S^{c\,aL}{}_{t\varphi(t)}, \tilde{A}_S^{c\,aU}{}_{t\varphi(t)}\right], \lambda_{ij}{}^a{}_{t\varphi(t)}\rangle$, where $n_j = \{1,2,3, \ldots . n\}\backslash\{j\}$ and $S_{n_{jn_i}}$ is the set of all permutation of set n_j over the set n_i.

4.3 COMPONENT WISE ADDITION, MULTIPLICATION AND PRODUCT OF R – ORDERED SPHERICAL CUBIC SOFT MATRICES

Let $\tilde{A}_S^c$ and $\tilde{B}_S^c$ be two R-ordered spherical cubic soft matrices of order $m \times n$, Thus $\tilde{A}_S^c = [(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle[a'_{ij}{}^L, a'_{ij}{}^U]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U], \lambda_{ij}\rangle)]_{(m\times n)}$, and $\tilde{B}_S^c = [(\langle[b_{ij}{}^L, b_{ij}{}^U]\rangle, \langle[b'_{ij}{}^L, b'_{ij}{}^U]\rangle, \langle[b^*_{ij}{}^L, b^*_{ij}{}^U], \mu_{ij}\rangle)]_{(m\times n)}$ then their Component wise addition and Component wise multiplication are defined as

i) $\tilde{A}_S^c \oplus_R \tilde{B}_S^c = \begin{Bmatrix} max\{[a_{ij}{}^L, a_{ij}{}^U], [b_{ij}{}^L, b_{ij}{}^U]\}, max\{[a'_{ij}{}^L, a'_{ij}{}^U], [b'_{ij}{}^L, b'_{ij}{}^U]\}, max\{[a^*_{ij}{}^L, a^*_{ij}{}^U], [b^*_{ij}{}^L, b^*_{ij}{}^U]\}, \\ (min[\lambda_{ij},, \mu_{ij}]) \end{Bmatrix}$

ii) $\tilde{A}_S^c \odot_R \tilde{B}_S^c = \begin{Bmatrix} min\{[a_{ij}{}^L, a_{ij}{}^U], [b_{ij}{}^L, b_{ij}{}^U]\}, min\{[a'_{ij}{}^L, a'_{ij}{}^U], [b'_{ij}{}^L, b'_{ij}{}^U]\}, min\{[a^*_{ij}{}^L, a^*_{ij}{}^U], [b^*_{ij}{}^L, b^*_{ij}{}^U]\}, \\ (max[\lambda_{ij},, \mu_{ij}]) \end{Bmatrix}$

iii) $\tilde{A}_S^c *_R \tilde{B}_S^c = \left\{\left[\sum_{j=1}^n \left\{\tilde{A}_S^{c\,aL} \wedge \tilde{B}_S^{c\,bL}\right\}, \sum_{j=1}^n \left\{\tilde{A}_S^{c\,aU} \wedge \tilde{B}_S^{c\,bU}\right\}\right], \sum_{j=1}^n \{\lambda_{ij}{}^a \vee \mu_{ij}{}^b\}\right\}$ for all i,j. The Product $\tilde{A}_S^c *_P \tilde{B}_S^c$ is defined if and only if $\tilde{A}_S^c$ and $\tilde{B}_S^c$ are square matrices of the same order.

4.3.1. PROPOSITION

Consider $\tilde{A}_S^c, \tilde{B}_S^c$ and $\tilde{C}_S^c$ are spherical cubic soft matrices of order $m \times n$,then the following are holds.

i) $\tilde{A}_S^c \oplus_R \tilde{B}_S^c = \tilde{B}_S^c \oplus_R \tilde{A}_S^c$

ii) $(\tilde{A}_S^c \oplus_R \tilde{B}_S^c) \oplus_R \tilde{C}_S^c = \tilde{A}_S^c \oplus_R (\tilde{B}_S^c \oplus_R \tilde{C}_S^c)$

iii) $\tilde{A}_s^c \odot_R \tilde{B}_s^c = \tilde{B}_s^c \odot_R \tilde{A}_s^c$

iv) $\left(\tilde{A}_s^c \odot_R \tilde{B}_s^c\right) \odot_R \tilde{C}_s^c = \tilde{A}_s^c \odot_R \left(\tilde{B}_s^c \odot_R \tilde{C}_s^c\right)$

v) $\left(\tilde{A}_s^c *_R \tilde{B}_s^c\right) = \left(\tilde{B}_s^c *_R \tilde{A}_s^c\right)$

vi) $\left(\tilde{A}_s^c *_R \tilde{B}_s^c\right) *_R \tilde{C}_s^c = \tilde{A}_s^c *_R \left(\tilde{B}_s^c *_R \tilde{C}_s^c\right)$

vii) $\left(\tilde{A}_s^c \oplus_R \tilde{B}_s^c\right) \odot_R \tilde{C}_s^c = \left(\tilde{A}_s^c \odot_R \tilde{C}_s^c\right) \oplus_R \left(\tilde{B}_s^c \odot_R \tilde{C}_s^c\right)$

viii) $\left(\tilde{A}_s^c \odot_R \tilde{B}_s^c\right) \oplus_R \tilde{C}_s^c = \left(\tilde{A}_s^c \oplus_R \tilde{C}_s^c\right) \odot_R \left(\tilde{B}_s^c \oplus_R \tilde{C}_s^c\right)$

PROOF

$\tilde{A}_s^c = \left[\left(\langle\left[{a_{ij}}^L, {a_{ij}}^U\right]\rangle, \langle\left[{a'_{ij}}^L, {a'_{ij}}^U\right]\rangle, \langle\left[{a^*_{ij}}^L, {a^*_{ij}}^U\right], \lambda_{ij}\rangle\right)\right]_{(m\times n)}$, $\tilde{B}_s^c = \left[\left(\langle\left[{b_{ij}}^L, {b_{ij}}^U\right]\rangle, \langle\left[{b'_{ij}}^L, {b'_{ij}}^U\right]\rangle, \langle\left[{b^*_{ij}}^L, {b^*_{ij}}^U\right], \mu_{ij}\rangle\right)\right]_{(m\times n)}$ and

$\tilde{C}_s^c = \left[\left(\langle\left[{c_{ij}}^L, {c_{ij}}^U\right]\rangle, \langle\left[{c'_{ij}}^L, {c'_{ij}}^U\right]\rangle, \langle\left[{c^*_{ij}}^L, {c^*_{ij}}^U\right], \gamma_{ij}\rangle\right)\right]_{(m\times n)}$

i) $\tilde{A}_s^c \oplus_R \tilde{B}_s^c$

$$= \begin{Bmatrix} max\left\{\left[{a_{ij}}^L, {a_{ij}}^U\right], \left[{b_{ij}}^L, {b_{ij}}^U\right]\right\}, max\left\{\left[{a'_{ij}}^L, {a'_{ij}}^U\right], \left[{b'_{ij}}^L, {b'_{ij}}^U\right]\right\}, max\left\{\left[{a^*_{ij}}^L, {a^*_{ij}}^U\right], \left[{b^*_{ij}}^L, {b^*_{ij}}^U\right]\right\}, \\ \left(min\left[\lambda_{ij}, \mu_{ij}\right]\right) \end{Bmatrix}$$

$$= \begin{Bmatrix} max\left\{\left[{b_{ij}}^L, {b_{ij}}^U\right], \left[{a_{ij}}^L, {a_{ij}}^U\right]\right\}, max\left\{\left[{b'_{ij}}^L, {b'_{ij}}^U\right], \left[{a'_{ij}}^L, {a'_{ij}}^U\right]\right\}, max\left\{\left[{b^*_{ij}}^L, {b^*_{ij}}^U\right], \left[{a^*_{ij}}^L, {a^*_{ij}}^U\right]\right\}, \\ \left(min\left[\mu_{ij}, \lambda_{ij}\right]\right) \end{Bmatrix}$$

$= \tilde{B}_s^c \oplus_R \tilde{A}_s^c$

ii) $\left(\tilde{A}_s^c \oplus_R \tilde{B}_s^c\right) \oplus_R \tilde{C}_s^c =$

$$\begin{Bmatrix} max\left\{\left[{a_{ij}}^L, {a_{ij}}^U\right], \left[{b_{ij}}^L, {b_{ij}}^U\right]\right\}, max\left\{\left[{a'_{ij}}^L, {a'_{ij}}^U\right], \left[{b'_{ij}}^L, {b'_{ij}}^U\right]\right\}, max\left\{\left[{a^*_{ij}}^L, {a^*_{ij}}^U\right], \left[{b^*_{ij}}^L, {b^*_{ij}}^U\right]\right\}, \\ \left(min\left[\lambda_{ij}, \mu_{ij}\right]\right) \end{Bmatrix}$$

$$\oplus_P \left[\left(\langle\left[{c_{ij}}^L, {c_{ij}}^U\right]\rangle, \langle\left[{c'_{ij}}^L, {c'_{ij}}^U\right]\rangle, \langle\left[{c^*_{ij}}^L, {c^*_{ij}}^U\right], \gamma_{ij}\rangle\right)\right]_{(m\times n)}$$

$$= \begin{Bmatrix} max\left(max\left\{\left[{a_{ij}}^L, {a_{ij}}^U\right], \left[{b_{ij}}^L, {b_{ij}}^U\right]\right\}, \left[{c_{ij}}^L, {c_{ij}}^U\right]\right), max\left(max\left\{\left[{a'_{ij}}^L, {a'_{ij}}^U\right], \left[{b'_{ij}}^L, {b'_{ij}}^U\right]\right\}, \left[{c'_{ij}}^L, {c'_{ij}}^U\right]\right), \\ max\left(max\left\{\left[{a^*_{ij}}^L, {a^*_{ij}}^U\right], \left[{b^*_{ij}}^L, {b^*_{ij}}^U\right]\right\}, \left[{c^*_{ij}}^L, {c^*_{ij}}^U\right]\right), \\ \left(min\left(min\left[\lambda_{ij}, \mu_{ij}\right], \gamma_{ij}\right)\right) \end{Bmatrix}$$

$$= \begin{Bmatrix} max\left\{\left[{a_{ij}}^L, {a_{ij}}^U\right], max\left(\left[{b_{ij}}^L, {b_{ij}}^U\right], \left[{c_{ij}}^L, {c_{ij}}^U\right]\right)\right\}, max\left\{\left[{a'_{ij}}^L, {a'_{ij}}^U\right], max\left(\left[{b'_{ij}}^L, {b'_{ij}}^U\right], \left[{c'_{ij}}^L, {c'_{ij}}^U\right]\right)\right\}, \\ max\left\{\left[{a^*_{ij}}^L, {a^*_{ij}}^U\right], max\left(\left[{b^*_{ij}}^L, {b^*_{ij}}^U\right], \left[{c^*_{ij}}^L, {c^*_{ij}}^U\right]\right)\right\}, \\ \left(min\left[\lambda_{ij}, min\left(\mu_{ij}, \gamma_{ij}\right)\right]\right) \end{Bmatrix}$$

$= \tilde{A}_s^c \oplus_R \left(\tilde{B}_s^c \oplus_R \tilde{C}_s^c\right)$

iii) $\tilde{A}_s^c \odot_R \tilde{B}_s^c$

$$= \begin{Bmatrix} max\left\{\left[{a_{ij}}^L, {a_{ij}}^U\right], \left[{b_{ij}}^L, {b_{ij}}^U\right]\right\}, max\left\{\left[{a'_{ij}}^L, {a'_{ij}}^U\right], \left[{b'_{ij}}^L, {b'_{ij}}^U\right]\right\}, max\left\{\left[{a^*_{ij}}^L, {a^*_{ij}}^U\right], \left[{b^*_{ij}}^L, {b^*_{ij}}^U\right]\right\}, \\ \left(min\left[\lambda_{ij}, \mu_{ij}\right]\right) \end{Bmatrix}$$

$$= \begin{Bmatrix} max\{[b_{ij}{}^{L}, b_{ij}{}^{U}], [a_{ij}{}^{L}, a_{ij}{}^{U}]\}, max\{[b'_{ij}{}^{L}, b'_{ij}{}^{U}], [a'_{ij}{}^{L}, a'_{ij}{}^{U}]\}, max\{[b^{*}_{ij}{}^{L}, b^{*}_{ij}{}^{U}], [a^{*}_{ij}{}^{L}, a^{*}_{ij}{}^{U}]\}, \\ (min[\mu_{ij}, \lambda_{ij}]) \end{Bmatrix}$$

$= \tilde{B}_s^c \odot_R \tilde{A}_s^c$

iv) $(\tilde{A}_s^c \odot_R \tilde{B}_s^c) \odot_R \tilde{C}_s^c$

$$= \begin{Bmatrix} max\{[a_{ij}{}^{L}, a_{ij}{}^{U}], [b_{ij}{}^{L}, b_{ij}{}^{U}]\}, max\{[a'_{ij}{}^{L}, a'_{ij}{}^{U}], [b'_{ij}{}^{L}, b'_{ij}{}^{U}]\}, max\{[a^{*}_{ij}{}^{L}, a^{*}_{ij}{}^{U}], [b^{*}_{ij}{}^{L}, b^{*}_{ij}{}^{U}]\}, \\ (min[\lambda_{ij}, \mu_{ij}]) \end{Bmatrix}$$

$$\oplus_R [(\langle [c_{ij}{}^{L}, c_{ij}{}^{U}] \rangle, \langle [c'_{ij}{}^{L}, c'_{ij}{}^{U}] \rangle, \langle [c^{*}_{ij}{}^{L}, c^{*}_{ij}{}^{U}], \gamma_{ij} \rangle)]_{(m \times n)}$$

$$= \begin{Bmatrix} max(max\{[a_{ij}{}^{L}, a_{ij}{}^{U}], [b_{ij}{}^{L}, b_{ij}{}^{U}]\}, [c_{ij}{}^{L}, c_{ij}{}^{U}]), max(max\{[a'_{ij}{}^{L}, a'_{ij}{}^{U}], [b'_{ij}{}^{L}, b'_{ij}{}^{U}]\}, [c'_{ij}{}^{L}, c'_{ij}{}^{U}]), \\ max(max\{[a^{*}_{ij}{}^{L}, a^{*}_{ij}{}^{U}], [b^{*}_{ij}{}^{L}, b^{*}_{ij}{}^{U}]\}, [c^{*}_{ij}{}^{L}, c^{*}_{ij}{}^{U}]), \\ (min(min[\lambda_{ij}, \mu_{ij}], \gamma_{ij})) \end{Bmatrix}$$

$$= \begin{Bmatrix} max\{[a_{ij}{}^{L}, a_{ij}{}^{U}], max([b_{ij}{}^{L}, b_{ij}{}^{U}], [c_{ij}{}^{L}, c_{ij}{}^{U}])\}, max\{[a'_{ij}{}^{L}, a'_{ij}{}^{U}], max([b'_{ij}{}^{L}, b'_{ij}{}^{U}], [c'_{ij}{}^{L}, c'_{ij}{}^{U}])\}, \\ max\{[a^{*}_{ij}{}^{L}, a^{*}_{ij}{}^{U}], max([b^{*}_{ij}{}^{L}, b^{*}_{ij}{}^{U}], [c^{*}_{ij}{}^{L}, c^{*}_{ij}{}^{U}])\}, \\ (min[\lambda_{ij}, min(\mu_{ij},, \gamma_{ij})]) \end{Bmatrix}$$

$$= \tilde{A}_s^c \odot_R (\tilde{B}_s^c \odot_R \tilde{C}_s^c)$$

v) $\widetilde{A}_s^c *_R \tilde{B}_s^c = \left\{ \left[\sum_{j=1}^{n} \left\{ \tilde{A}_s^{c\,aL} \wedge \tilde{B}_s^{c\,bL} \right\}, \sum_{j=1}^{n} \left\{ \tilde{A}_s^{c\,aU} \wedge \tilde{B}_s^{c\,bU} \right\} \right], \sum_{j=1}^{n} \{ \lambda_{ij}{}^{a} \vee \mu_{ij}{}^{b} \} \right\}$

$$= \left\{ \left[\sum_{j=1}^{n} \left\{ \tilde{B}_s^{c\,bL} \wedge \tilde{A}_s^{c\,aL} \right\}, \sum_{j=1}^{n} \left\{ \tilde{B}_s^{c\,bU} \wedge \tilde{A}_s^{c\,aU} \right\} \right], \sum_{j=1}^{n} \{ \mu_{ij}{}^{b} \vee \lambda_{ij}{}^{a} \} \right\}$$

$$= \tilde{B}_s^c *_R \tilde{A}_s^c$$

vi) $(\tilde{A}_s^c *_P \tilde{B}_s^c) *_P \tilde{C}_s^c = \left\{ \left[\sum_{j=1}^{n} \left\{ \tilde{A}_s^{c\,aL} \wedge \tilde{B}_s^{c\,bL} \right\}, \sum_{j=1}^{n} \left\{ \tilde{A}_s^{c\,aU} \wedge \tilde{B}_s^{c\,bU} \right\} \right], \sum_{j=1}^{n} \{ \lambda_{ij}{}^{a} \vee \mu_{ij}{}^{b} \} \right\} *_P \tilde{C}_s^c$

$$= \left\{ \left[\sum_{j=1}^{n} \left\{ \tilde{A}_s^{c\,aL} \wedge \tilde{B}_s^{c\,bL} \right\} \wedge \tilde{C}_s^{c\,cL}, \sum_{j=1}^{n} \left\{ \tilde{A}_s^{c\,aU} \wedge \tilde{B}_s^{c\,bU} \right\} \wedge \tilde{C}_s^{c\,cU} \right], \sum_{j=1}^{n} \{ \lambda_{ij}{}^{a} \vee \mu_{ij}{}^{b} \} \vee \gamma_{ij}{}^{c} \right\}$$

$$= \left\{ \left[\sum_{j=1}^{n} \left\{ \tilde{A}_s^{c\,aL} \wedge \left(\tilde{B}_s^{c\,bL} \wedge \tilde{C}_s^{c\,cL} \right) \right\}, \sum_{j=1}^{n} \left\{ \tilde{A}_s^{c\,aU} \wedge \left(\tilde{B}_s^{c\,bU} \wedge \tilde{C}_s^{c\,cU} \right) \right\} \right], \sum_{j=1}^{n} \{ \lambda_{ij}{}^{a} \vee (\mu_{ij}{}^{b} \vee \gamma_{ij}{}^{c}) \} \right\}$$

$$= \tilde{A}_s^c *_R (\tilde{B}_s^c *_R \tilde{C}_s^c)$$

vii) $(\tilde{A}_s^c \oplus_R \tilde{B}_s^c) \odot_R \tilde{C}_s^c =$

$$\begin{Bmatrix} max\{[a_{ij}{}^{L}, a_{ij}{}^{U}], [b_{ij}{}^{L}, b_{ij}{}^{U}]\}, max\{[a'_{ij}{}^{L}, a'_{ij}{}^{U}], [b'_{ij}{}^{L}, b'_{ij}{}^{U}]\}, max\{[a^{*}_{ij}{}^{L}, a^{*}_{ij}{}^{U}], [b^{*}_{ij}{}^{L}, b^{*}_{ij}{}^{U}]\}, \\ (min[\lambda_{ij}, \mu_{ij}]) \end{Bmatrix} \odot_R \tilde{C}_s^c$$

$$= \left\{\begin{matrix} min(max\{[a_{ij}{}^L, a_{ij}{}^U], [b_{ij}{}^L, b_{ij}{}^U]\}, [c_{ij}{}^L, c_{ij}{}^U]), min(max\{[a'_{ij}{}^L, a'_{ij}{}^U], [b'_{ij}{}^L, b'_{ij}{}^U]\}, [c'_{ij}{}^L, c'_{ij}{}^U]), \\ min(max\{[a^*_{ij}{}^L, a^*_{ij}{}^U], [b^*_{ij}{}^L, b^*_{ij}{}^U]\}, [c^*_{ij}{}^L, c^*_{ij}{}^U]), \\ (max(min[\lambda_{ij}, \mu_{ij}], \gamma_{ij})) \end{matrix}\right\}$$

$$= \left\{\begin{matrix} max(min\{[a_{ij}{}^L, a_{ij}{}^U], [c_{ij}{}^L, c_{ij}{}^U]\}, min\{[b_{ij}{}^L, b_{ij}{}^U], [c_{ij}{}^L, c_{ij}{}^U]\}), \\ max(min\{[a'_{ij}{}^L, a'_{ij}{}^U], [c'_{ij}{}^L, c'_{ij}{}^U]\}, min\{[b'_{ij}{}^L, b'_{ij}{}^U], [c'_{ij}{}^L, c'_{ij}{}^U]\}), \\ max(min\{[a^*_{ij}{}^L, a^*_{ij}{}^U], [b^*_{ij}{}^L, b^*_{ij}{}^U]\}, min\{[b^*_{ij}{}^L, b^*_{ij}{}^U], [c^*_{ij}{}^L, c^*_{ij}{}^U]\}), \\ (min(max[\lambda_{ij}, \gamma_{ij}], max[\mu_{ij}, \gamma_{ij}], max[\gamma_{ij}, \lambda_{ij}])) \end{matrix}\right\}$$

$$= (\tilde{A}_s^c \odot_R \tilde{C}_s^c) \oplus_R (\tilde{B}_s^c \odot_R \tilde{C}_s^c)$$

viii) $(\tilde{A}_s^c \odot_R \tilde{B}_s^c) \oplus_R \tilde{C}_s^c$

$$= \left\{\begin{matrix} min\{[a_{ij}{}^L, a_{ij}{}^U], [b_{ij}{}^L, b_{ij}{}^U]\}, min\{[a'_{ij}{}^L, a'_{ij}{}^U], [b'_{ij}{}^L, b'_{ij}{}^U]\}, min\{[a^*_{ij}{}^L, a^*_{ij}{}^U], [b^*_{ij}{}^L, b^*_{ij}{}^U]\}, \\ (max[\lambda_{ij}{}^a, \mu_{ij}{}^b], max\left[\lambda_{ij}{}^{a'}, \mu_{ij}{}^{b'}\right], max[\lambda_{ij}{}^{a^*}, \mu_{ij}{}^{b^*}]) \end{matrix}\right\} \oplus_R \tilde{C}_s^c$$

$$= \left\{\begin{matrix} max(min\{[a_{ij}{}^L, a_{ij}{}^U], [b_{ij}{}^L, b_{ij}{}^U]\}, [c_{ij}{}^L, c_{ij}{}^U]), max(min\{[a'_{ij}{}^L, a'_{ij}{}^U], [b'_{ij}{}^L, b'_{ij}{}^U]\}, [c'_{ij}{}^L, c'_{ij}{}^U]), \\ max(min\{[a^*_{ij}{}^L, a^*_{ij}{}^U], [b^*_{ij}{}^L, b^*_{ij}{}^U]\}, [c^*_{ij}{}^L, c^*_{ij}{}^U]), \\ (min(max[\lambda_{ij}, \mu_{ij}], \gamma_{ij})) \end{matrix}\right\}$$

$$= \left\{\begin{matrix} min(max\{[a_{ij}{}^L, a_{ij}{}^U], [c_{ij}{}^L, c_{ij}{}^U]\}, max\{[b_{ij}{}^L, b_{ij}{}^U], [c_{ij}{}^L, c_{ij}{}^U]\}), \\ min(max\{[a'_{ij}{}^L, a'_{ij}{}^U], [c'_{ij}{}^L, c'_{ij}{}^U]\}, max\{[b'_{ij}{}^L, b'_{ij}{}^U], [c'_{ij}{}^L, c'_{ij}{}^U]\}), \\ min(max\{[a^*_{ij}{}^L, a^*_{ij}{}^U], [b^*_{ij}{}^L, b^*_{ij}{}^U]\}, max\{[b^*_{ij}{}^L, b^*_{ij}{}^U], [c^*_{ij}{}^L, c^*_{ij}{}^U]\}), \\ (max(min[\lambda_{ij}, \gamma_{ij}], min[\mu_{ij}, \gamma_{ij}], min[\gamma_{ij}, \lambda_{ij}])) \end{matrix}\right\}$$

$$= (\tilde{A}_s^c \oplus_R \tilde{C}_s^c) \odot_R (\tilde{B}_s^c \oplus_R \tilde{C}_s^c)$$

4.4 DETERMINANT OF R- ORDERED SPHERICAL CUBIC SOFT MATRICES OF ORDER n × n

The determinant $|\tilde{A}_s^c|_R$ of an $n \times n$ SCSM

$\tilde{A}_s^c = [(\langle[a_{ij}{}^L, a_{ij}{}^U]\rangle, \langle[a'_{ij}{}^L, a'_{ij}{}^U]\rangle, \langle[a^*_{ij}{}^L, a^*_{ij}{}^U]\rangle), \lambda_{ij}]_{(n\times n)}$ is defined as

$|\tilde{A}_s^c|_R =$

$\langle \vee_{\sigma\in S_n}\left\{\tilde{A}_s^c{}^{aL}{}_{1\sigma(1)} \ldots \wedge \tilde{A}_s^c{}^{aL}{}_{n\sigma(n)}\right\}, \vee_{\sigma\in S_n}\left\{\tilde{A}_s^c{}^{aU}{}_{1\sigma(1)} \wedge \ldots \wedge \tilde{A}_s^c{}^{aU}{}_{n\sigma(n)}\right\}, \vee_{\sigma\in S_n}\left\{\lambda_{ij}{}^a{}_{1\sigma(1)} \vee \ldots \vee \lambda_{ij}{}^a{}_{n\sigma(n)}\right\}, \rangle$
where S_n denotes the symmetric group of all permutations of the indices 1,2,3,….n.

4.5 ADJOINT OF R- ORDERED SPHERICAL CUBIC SOFT MATRICES

The adjoint of an $n \times n$ SCSM $\tilde{A}_s^c$ denoted by $adj_R\tilde{A}_s^c$ as $adj_R\tilde{A}_s^c = [b_{ij}] =$
$\sum_{\varphi\in S_{n_{jn_i}}} \prod_{t\in n_j} \langle\left[\tilde{A}_s^c{}^{aL}{}_{t\varphi(t)}, \tilde{A}_s^c{}^{aU}{}_{t\varphi(t)}\right], \lambda_{ij}{}^a{}_{t\varphi(t)}\rangle$, where $n_j = \{1,2,3, \ldots .n\}\backslash\{j\}$ and $S_{n_{jn_i}}$ is the set of all permutation of set n_j over the set n_i.

5. APPLICATION OF SPHERICAL CUBIC SOFT MATRICES IN MEDICAL FIELD

5.1 SCSMS IN DECISION MAKING

In this section, we define the concepts of the value function, score function and total score for a SCSM. Later these notions are used in decision making process.

(1) DEFINITION : VALUE FUNCTION

The value function of a SCSM helps to transform the elements of a spherical cubic matrix into real numbers to bring out the significance or importance of these fuzzy membership values. Each element of a spherical cubic matrix is a combination of three closed sub interval of [0,1] and a real number from [0,1]. As such they do not convey any useful meaning about the object. It is necessary to integrate the three interval-valued membership value and the fuzzy membership value into a single real number to arrive at a numerical value that can be viewed as a measure of that property.

Let $A = [a_{ij}] = \langle [{a_{ij}}^L, {a_{ij}}^U], [{a'_{ij}}^L, {a'_{ij}}^U], [{a^*_{ij}}^L, {a^*_{ij}}^U], \lambda_{ij} \rangle$

Then the value function for the element a_{ij} is defined as

$$V(a_{ij}) = v_{ij} = \frac{\left[{a_{ij}}^L + {a_{ij}}^U + {a'_{ij}}^L + {a'_{ij}}^U + {a^*_{ij}}^L + {a^*_{ij}}^U\right]}{6} + \lambda_{ij} \text{ for all i, j}$$

Thus the value function for the SCSM, $A = [a_{ij}]$ is given by

$$V = \left[\frac{\left[{a_{ij}}^L + {a_{ij}}^U + {a'_{ij}}^L + {a'_{ij}}^U + {a^*_{ij}}^L + {a^*_{ij}}^U\right]}{6} + \lambda_{ij}\right] = [v_{ij}]$$

V is also an$m \times n$ matrix, having the same dimension as A and has non-negative entries.

(2) DEFINITION: SCORE FUNCTION AND TOTAL SCORE

The score function is a tool to consolidate and compare the outputs generated by the value function. Each element of a row of the value function matrix of a SCSM repressed the unified numerical value associated with that object in respect of a particular parameter assessed by the experts. Thus the sum of all the value of a row give the total score of the object, which can be considered as a measure to facilitate the choice of that object based on all the parameter. Thus the total score of (T_{ij}) of the ith object is defined as the sum of all score values of theith row of the value matrix.

ie$(T_{ij}) = \sum_{j=1}^{n} v_{ij}$, i = 1,2,3,……m.

APPLICATION OF SCSMS IN MEDICAL FIELD

In this section an application of SCSM in multi-criteria decision making (MCDM) is provided. An algorithm is developed for this purpose. The working of the algorithm is illustrated with suitable example.

STATEMENT OF THE PROBLEM

Let us consider that the government officials are in the process of checking the standards of the hospitals in a particular region during COVID-19. Let U = { $h_1, h_2, h_3, \dots \dots \dots h_m$ }

be the list of hospitals under consideration. Let E = $\{e_1, e_2, e_3, \dots \dots \dots e_n\}$ be the set of criteria based on which the selection is to be finalized. Assume that a group of experts assess the hospitals individually and independently with respect to the criteria set E. Let G = $\{g_1, g_2, g_3, \dots \dots \dots g_k\}$ be the set of experts. Each of the government officials verify the hospitals credentials based on the set E and present their results in the form of a SCSMs for each element of the set E. The respective SCSM's are denoted by $s_1, s_2, , \dots \dots \dots s_k$.

The problem is to convert the SCSM's into meaningful matrices to help ordering the hospitals and to select the best hospital .

THE METHOD

Let $s_1, s_2, , \dots \dots \dots s_k$be the SCSMs obtained from the government officials .Using definition (1) convert each entryinto corresponding value function v_{ij}. Then using the Definition (2) construct the value function matrices $V^r = [v_{ij}{}^r]$; r = 1,2,….,k. Calculate the total value $T_i{}^r = \sum_{j=1}^{n} v_{ij}{}^r$ from each of the V^r matrices. Now the overall total score Ts_i for each hospital is obtained as $Ts_i = \sum_{r=1}^{k} T_i{}^r$; i=1,2,….m. Arrange the Ts_i values in increasing order . The hospital with highest Ts_i value is the best hospital .

ALGORITHM

The algorithm for ranking the alternative of MCDM problem based on SCSM is given below :

Step 1: Identify the list of hospitals and the list of parameter .

Step 2: Government officials assign spherical cubic values for each hospital based on the parameter .

Step 3: Form the SCSMs $s_1, s_2, , \ldots \ldots \ldots s_k$ for each of the government officials .

Step 4: Calculate the value function matrices $V^r = [v_{ij}{}^r]$ using Definition (1) and (2).

Step 5: Calculate the total value $T_i{}^r$ from each of the V^r matrices .

Step 6 : Evaluate the Ts_i for the each hospital .

Step 7: Order the Ts_i values and select the hospital with highest Ts_i value as the best hospital .

CASE STUDY

The government officials are in the process of checking the standards of the hospitals in a particular region during COVID – 19.

(1) Let U = $\{h_1, h_2, h_3, h_4\}$ be the list of hospitals .

(2) Let E = $\{e_1, e_2, e_3, e_4\}$ be the set of parameter which form the criteria for selecting the best hospital .

(3) Form SCSMs S_1, S_2 as

$$S_1 = \begin{bmatrix} \langle [0.5,0.5],[0.5,0.5],[0,1],0.2\rangle & \langle [0,1],[0.5,0.5],[0.5,0.5],0.3\rangle & \langle [0.5,0.5],[0,1],[0.5,0.5],0.1\rangle & \langle [0.5,0.5],[0.5,0.5],[0,1],0.3\rangle \\ \langle [0.5,0.5],[0,1],[0.5,0.5],0.2\rangle & \langle [0.5,0.5],[0,1],[0.5,0.5],0.1\rangle & \langle [0.5,0.5],[0.5,0.5],[0,1],0.3\rangle & \langle [0,1],[0.5,0.5],[0.5,0.5],0.4\rangle \\ \langle [0.5,0.5],[0.5,0.5],[0,1],0.4\rangle & \langle [0.5,0.5],[0.5,0.5],[0,1],0.3\rangle & \langle [0,1],[0.5,0.5],[0.5,0.5],0.1\rangle & \langle [0.5,0.5],[0,1],[0.5,0.5],0.1\rangle \\ \langle [0.5,0.5],[0,1],[0.5,0.5],0.3\rangle & \langle [0.5,0.5],[0.5,0.5],[0,1],0.4\rangle & \langle [0.5,0.5],[0.5,0.5],[0,1],0.4\rangle & \langle [0,1],[0.5,0.5],[0.5,0.5],0.2\rangle \end{bmatrix}$$

and

$$S_2 = \begin{bmatrix} \langle [0.5,0.5],[0,1],[0.5,0.5],0.1\rangle & \langle [0.5,0.5],[0,1],[0.5,0.5],0.3\rangle & \langle [0,1],[0.5,0.5],[0.5,0.5],0.4\rangle & \langle [0.5,0.5],[0,1],[0.5,0.5],0.3\rangle \\ \langle [0,1],[0.5,0.5],[0.5,0.5],0.2\rangle & \langle [0.5,0.5],[0.5,0.5],[0,1],0.3\rangle & \langle [0.5,0.5],[0,1],[0.5,0.5],0.1\rangle & \langle [0.5,0.5],[0.5,0.5],[0,1],0.2\rangle \\ \langle [0.5,0.5],[0,1],[0.5,0.5],0.1\rangle & \langle [0,1],[0.5,0.5],[0.5,0.5],0.2\rangle & \langle [0.5,0.5],[0.5,0.5],[0,1],0.3\rangle & \langle [0,1],[0.5,0.5],[0.5,0.5],0.4\rangle \\ \langle [0.5,0.5],[0.5,0.5],[0,1],0.4\rangle & \langle [0.5,0.5],[0,1],[0.5,0.5],0.1\rangle & \langle [0,1],[0.5,0.5],[0.5,0.5],0.2\rangle & \langle [0.5,0.5],[0,1],[0.5,0.5],0.1\rangle \end{bmatrix}$$

(4) Using Definition1 the value function matrices are obtained as

$$V_1 = \begin{bmatrix} 0.7 & 0.8 & 0.6 & 0.8 \\ 0.7 & 0.6 & 0.8 & 0.9 \\ 0.9 & 0.8 & 0.6 & 0.6 \\ 0.8 & 0.9 & 0.9 & 0.7 \end{bmatrix} \text{and} V_2 = \begin{bmatrix} 0.6 & 0.8 & 0.9 & 0.8 \\ 0.7 & 0.8 & 0.6 & 0.7 \\ 0.6 & 0.7 & 0.8 & 0.9 \\ 0.9 & 0.6 & 0.7 & 0.6 \end{bmatrix}$$

(5) Using Definition 2the total of the value function are calculated as

$$T^1 = \begin{bmatrix} 2.9 \\ 3 \\ 2.9 \\ 3.3 \end{bmatrix} \text{and} T^2 = \begin{bmatrix} 3.1 \\ 2.8 \\ 3 \\ 2.8 \end{bmatrix}$$

(6) The total value for each hospital is calculated and presented as

$$T_s = \begin{bmatrix} 6 \\ 5.8 \\ 5.9 \\ 6.1 \end{bmatrix}$$

Arranging the hospitals according to their total score we obtain the ranking of the hospitals as :

Hospital	Score	Rank
h_1	6	II
h_2	5.8	IV
h_3	5.9	III
h4	6.1	I

The hospital h4 ranks first and it is the best hospital in the region .

CONCLUSION

In this paper, we have introduced the notions of SCSM. We have also discussed properties of SCSM, determinant and adjoint of SCSM.Finally,we have applied the motto of spherical cubic soft matrices and complement of spherical cubic soft matrices in decision making problem under the medical field.

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