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APPLICATIONS OF CUBIC ROOT FUZZY SETS IN DECISION MAKING APPROACH

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ABSTRACT: The Cubic root fuzzy set is the extension of fuzzy set**.** In this paper, we investigate various algebraic structures of cubic root fuzzy set (CR-fuzzy set) which is an extension of fuzzy set. Various operations, score function, accuracy functions and some standard results to be proved based on CR-fuzzy set. Finally, we propose applications of CR-fuzzy set in decision making approach with suitable example.

Keywords: Intuitionistic fuzzy set, Pythagorean fuzy set, Fermatean fuzzy set, (3,2)- fuzzy set, score function, accuracy function, CR-fuzzy set, operations, Decision making.

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1.Introduction: In 1965, Zadeh [19] introduced uncertainty sets. After the introduction of the idea of fuzzy sets, several researches were conducted on the generalizations of fuzzy sets. The integration between fuzzy sets and some uncertainty approaches such as soft sets and rough sets have been analysed in [1, 4, 8]. The idea of intuitionistic fuzzy set has been introduced by Atanassov [3] as a generalization concept of uncertainty sets. The intuitionistic uncertainty set theory is applicable in various application areas, such as algebraic structures, control systems and various engineering fields. Many researchers have worked various applications of intuitionistic uncertainty set such as medical application, real life situations, education and networking [10–12]. Recently, Yager [18] launched a nonstandard uncertainty set referred to as Pythagorean uncertainty set which is the generalization of intuitionistic uncertainty sets. The construct of Pythagorean uncertainty sets can be used to characterize uncertain information more sufficiently and accurately than intuitionistic uncertainty set. Garg [9] presented a developed score function for the ranking order of interval-valued pythagorean fuzzy sets. Ibrahim et al. [13] defined a new generalized Pythagorean uncertainty set is called (3, 2)-Fuzzy sets. In 2020, fermatean uncertainty sets proposed by Senapati and Yager [17], can handle uncertain information more easily in the process of decision making. They also discussed basic operations over the fermatean uncertainty sets. The main advantage of fermatean fuzzy sets is that it can describe more uncertainties than pythagorean fuzzy sets, which can be applied in many decision-making problems. The relevant research can be referred to [15, 16]. K.Balamurugan and R.Nagarajan [6] defined some algebraic attributes of (3,2)- fuzzy set structures. Al-shami [2] introduced a new extensions of fuzzy sets called square-root fuzzy sets (briefly, SR-Fuzzy sets). Y.A.Salih et.al [14] introduced a new type of generalised fuzzy sets is called cubic root (briefly, CR-Fuzzy set). They also defined operations, score function and accuracy function of CR-fuzzy set with several properties. In this article, we investigate various algebraic structures of cubic root fuzzy set (CR-fuzzy set) which is an extension of fuzzy set. Various operations, score function , accuracy functions and some standard results to be proved based on CR-fuzzy set. Finally, we propose applications of CR-fuzzy set in decision making approach with suitable example.

2.PREMINARIES AND VARIUOS BASIC CONCEPTS

Before we present our main concepts and results, we recall the definitions of Intuitionistic fuzzy set (IFS) and Fermatean fuzzy set (FFS).

Definition 2.1: (Intuitionistic fuzzy set) Let$X$ be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$, where the functions $\mu_A(x), \nu_A(x): X \to [0,1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A , which is a subset of X , and for every element $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Furthermore, we have $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A .$\pi_A(x)$is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0,1]$ i.e., $\pi_A(x): X \to [0,1]$ and $0 \leq \pi_A(x) \leq 1$ for every $x \in X$.$\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS or not.

Definition 2.2: (Pythagorean Fuzzy set) A Pythagorean fuzzy set D on a set X is defined by $D = \{(x, (\alpha_D(x), \beta_D(x)) / x \in X\}$where $\alpha_D: X \to [0,1]$ is the degree of membership and $\beta_D: X \to [0,1]$ is the degree of non – membership of $x \in X$, respectively which fulfil the condition $0 \leq {\alpha_D}^2(x) + {\beta_D}^2(x) \leq 1$ for all $x \in X$.

Definition 2.3: (Fermatean fuzzy set) Let ‘X’ be a universe of discourse A. Fermatean fuzzy set ‘F’ in X is an object having the form $F = \{\langle x, m_F(x), n_F(x)\rangle / x \in X\}$ where $m_F(x): X \to [0,1]$and$n_F(x): X \to [0,1]$, including the condition $0 \leq (m_F(x))^3 + (n_F(x))^3 \leq 1$ for all $x \in X$. The numbers $m_F(x)$signifies the level (degree) of membership and $n_F(x)$indicate the non-membership of the element ‘x’ in the set F.

Definition 2.4: [(3,2)- fuzzy set]Let X be a universal set. Then the (3, 2)-fuzzy set (briefly, (3, 2)-FS) D is defined by $D = \{\langle x, \alpha_D(x), \beta_D(x)\rangle / x \in X\}$where $\alpha_D: X \to [0,1]$ is the degree of membership and $\beta_D: X \to [0,1]$is the degree of non – membership of $x \in X$ to D, with the condition$0 \leq (\alpha_D(x))^3 + (\beta_D(x))^2 \leq 1$the degree of indeterminacy of $x \in X$ to D is defined by $\pi_D(x) = 1 - \left[(\alpha_D(x))^3 + (\beta_D(x))^2\right]$.

In this section ,we discuss the notion of cube root fuzzy set (briefly, CR-FS) and study its factors in detail.

For computations, we use only five decimal places in the whole paper.

Let X be a universal set such that $\alpha_D: X \to [0,1]$ and $\beta_D: X \to [0,1]$ are mapping. Then, the CR-fuzzy set 'D' is defined by $D = \{\langle x, \alpha_D(x), \beta_D(x)\rangle / x \in X\}$……………...(1) where $\alpha_D(x)$is the degree of membership and $\beta_D(x)$is the degree of non – membership of $x \in X$, such that $0 \le (\alpha_D(x))^3 + \sqrt{\beta_D(x)} \le 1$……………………………………………(2).
Then, there is a degree of indeterminacy of $x \in X$ to D defined by
$\pi_D(x) = 1 - \left[(\alpha_D(x))^3 + \sqrt{\beta_D(x)}\right]$……………………………………………………..(3)
It is obvious that $(\alpha_D(x))^3 + \sqrt{\beta_D(x)} + \pi_D(x) = 1$………………………………………(4)
Otherwise, $\pi_D(x) = 0$ whenever $(\alpha_D(x))^3 + \sqrt{\beta_D(x)} = 1$. In the interest of simplicity, we Shall mention the symbol $D = (\alpha_D, \beta_D)$ for the CR- fuzzy set
$D = \{\langle x, \alpha_D(x), \beta_D(x)\rangle / x \in X\}$.The space of CR-fuzzy membership grades is displaced in figure-1.

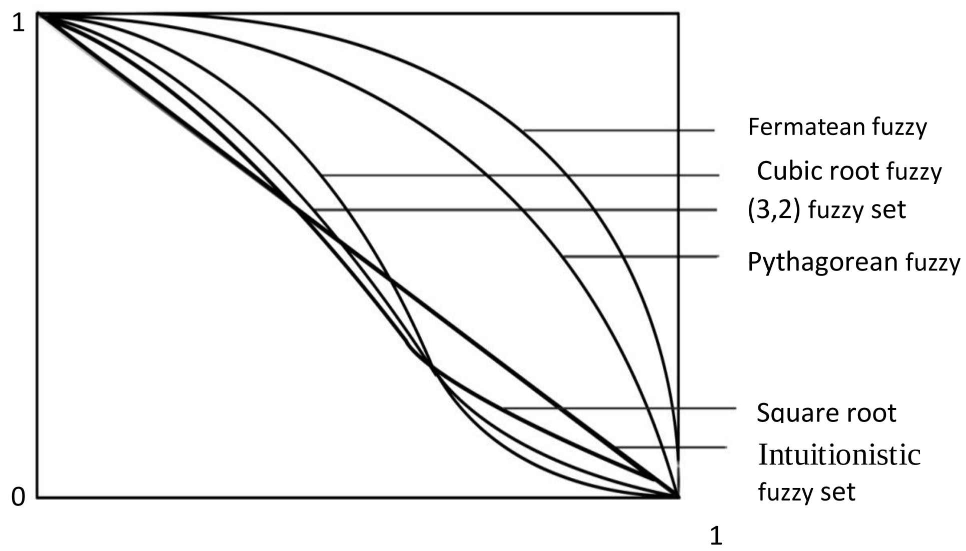

Figure-1

Comparison of grades space of IFSs, PFSs, FFSs, (3,2)-FSs and CR-FSs.

Author	Set domain	Membership function
L.A.Zadeh[1965]	Fuzzy set	$0 \le \mu(x) \le 1$
K.Atanassov[1983]	Intuitionistic fuzzy set	$0 \le \mu(x) + \nu(x) \le 1$
F. Smarandache [1998]	Neutrosophic fuzzy set	$0 \le T(x) + I(x) + F(x) \le 3$
R.R.Yager, [2013]	Pythagorean Fuzzy set	$0 \le (\mu(x))^2 + (\nu(x))^2 \le 1$
Senapati and Yager (2020)	Fermatean fuzzy set	$0 \le (\mu(x))^3 + (\nu(x))^3 \le 1$
H. Z. Ibrahim.etal [2022]	(3,2)- fuzzy set	$0 \le (\mu(x))^3 + (\nu(x))^2 \le 1$
H. Z. Ibrahim.etal [2022]	Square root fuzzy set	$0 \le (\mu(x))^2 + \sqrt{\nu(x)} \le 1$
H. Z. Ibrahim.etal [2022]	Cubic root fuzzy set	$0 \le (\mu(x))^3 + \sqrt{\nu(x)} \le 1$

Definition 2.5: Let $D_1 = (\alpha_{D_1}, \beta_{D_1})$ and $D_2 = (\alpha_{D_2}, \beta_{D_2})$ be two CR-Fuzzy sets. Then

(i) $D_1 = D_2$ if and only if $\alpha_{D_1} = \alpha_{D_2}$ and $\beta_{D_1} = \beta_{D_2}$.

(ii) $D_1 \geq D_2$ if and only if $\alpha_{D_1} \geq \alpha_{D_2}$ and $\beta_{D_1} \leq \beta_{D_2}$.

Example 2.6

(i) If $D_1 = (0.2,0.9)$ and $D_2 = (0.2,0.8)$for $S = \{x\}$then $D_1 = D_2$.

(ii)If $D_1 = (0.3,0.9)$ and $D_2 = (0.2,0.94\ )$for $S = \{x\}$ then $\alpha_{D_1} \geq \alpha_{D_2}$ and $\beta_{D_1} \leq \beta_{D_2}$.

Definition-2.7: Let $D_1 = (\alpha_{D_1}, \beta_{D_1})$ and $D_2 = \left(\alpha_{D_2}, \beta_{D_2}\right)$ be two CR-Fuzzy sets. Then

(i) $D_1 \cap D_2 = \left(\min\{\alpha_{D_1}, \alpha_{D_2}\},\ \max\{\beta_{D_1}, \beta_{D_2}\}\right)$

(ii) $D_1 \cup D_2 = \left(\max\{\alpha_{D_1}, \alpha_{D_2}\},\ \min\{\beta_{D_1}, \beta_{D_2}\}\right)$

(iii) $D_1{}^C = \left(\left(\beta_{D_1}\right)^6, \sqrt{\alpha_{D_1}}\right)$.

Note that $\left(\sqrt[2]{(\beta_D)^6} + \sqrt[4]{(\alpha_D)^2}\right) = (\beta_D)^3 + \sqrt{\alpha_{D_1}}$

$= (0.3)^3 + \sqrt{0.9} = 0.97568 < 1.$

So $D_1{}^C$ is a CR-fuzzy set.

It is obvious that $\left(D_1{}^C\right)^c = \left(\beta_{D_1}{}^6, \sqrt[4]{\alpha_{D_1}}\right)^C = (\alpha_{D_1}, \beta_{D_1}) = D_1$.

Remark2.8: It is noticed that $(D^C)^c = D$. This shows a validity of complementary law in CR-Fuzzy sets.

Example 2.9: Assume that $D_1 = (\alpha_{D_1} = 0.42, \beta_{D_1} = 0.54)$ and $D_2 = (\alpha_{D_2} = 0.45, \beta_{D_2} = 0.57)$are both CR-Fuzzy sets for $S = \{x\}$.Then

(i) $D_1 \cap D_2 = \left(\min\{\alpha_{D_1}, \alpha_{D_2}\},\ \max\{\beta_{D_1}, \beta_{D_2}\}\right)$

$= (\min\{0.42,0.45\},\ \max\{0.54,0.57\})$

$= (0.42\ ,\ 0.57).$

(ii) $D_1 \cup D_2 = \left(\max\{\alpha_{D_1}, \alpha_{D_2}\},\ \min\{\beta_{D_1}, \beta_{D_2}\}\right)$

$= (\max\{0.42,0.45\},\ \min\{0.54,0.57\})$

$= (0.45\ ,\ 0.54)$

(iii) $D_1{}^C = \left((0.54)^6, \sqrt[4]{0.42}\right)$

3.SOME STANDARD RESULTS IN CR- FUZZY SET STRUCTURES

Theorem3.1 :(Commutative law) Let $D_1 = (\alpha_{D_1}, \beta_{D_1})$ and $D_2 = (\alpha_{D_2}, \beta_{D_2})$ be two CR-Fuzzy sets. Then the following properties hold;

(i) $D_1 \cap D_2 = D_2 \cap D_1$

(ii) $D_1 \cup D_2 = D_2 \cup D_1$

Proof: From the definition-2, we can obtain the following

(i) $D_1 \cap D_2 = \left(\min\{\alpha_{D_1}, \alpha_{D_2}\},\ \max\{\beta_{D_1}, \beta_{D_2}\}\right)$

$= \left(\min\{\alpha_{D_2}, \alpha_{D_1}\},\ \max\{\beta_{D_2}, \beta_{D_1}\}\right)$

$= D_2 \cap D_1$

(ii) The proof is similar to (i).

Theorem-3.2: A Cube Root fuzzy subset $D = \{x,\ \alpha_D(x), \beta_D(x)\}$ of a group G is cube root fuzzy subgroup of G if and only if $\alpha_D{}^3(xy^{-1}) \geq \min\{\alpha_D{}^3(x), \alpha_D{}^3(y)\}$ and $\sqrt{\beta_D(xy^{-1})} \leq \max\left\{\sqrt{\beta_D(x)}, \sqrt{\beta_D(y)}\right\}$for all $x, y \in G$.

Proof: Let $D = \left\{x,\ \alpha_D(x), \beta_D(x) / x \in G, \alpha_D{}^3(x) + \sqrt{\beta_D(x)} \leq 1\right\}$ be a cube root fuzzy subgroup of G. Then for all $x, y \in G$,

$$\alpha_D{}^3(xy^{-1}) \geq \min\{\alpha_D{}^3(x), \alpha_D{}^3(y^{-1})\} = \min\{\alpha_D{}^3(x), \alpha_D{}^3(y)\} \text{ and}$$
$$\sqrt{\beta_D(xy^{-1})} \leq \max\left\{\sqrt{\beta_D(x)}, \sqrt{\beta_D(y^{-1})}\right\} = \max\left\{\sqrt{\beta_D(x)}, \sqrt{\beta_D(y)}\right\}.$$

Conversely, suppose that $\alpha_D{}^3(xy^{-1}) \geq \min\{\alpha_D{}^3(x), \alpha_D{}^3(y)\}$ and $\sqrt{\beta_D(xy^{-1})} \leq \max\left\{\sqrt{\beta_D(x)}, \sqrt{\beta_D(y)}\right\}$for all $x, y \in G$. Then

$$\alpha_D{}^3(xy) = \alpha_D{}^3(x(y^{-1})^{-1}) \geq \min\{\alpha_D{}^3(x), \alpha_D{}^3(y^{-1})\} = \min\{\alpha_D{}^3(x), \alpha_D{}^3(y)\}$$

Thus $\alpha_D{}^3(xy) \geq \min\{\alpha_D{}^3(x), \alpha_D{}^3(y)\}$……………. (1)

Similarly, $\sqrt{\beta_D(xy)} \leq \max\left\{\sqrt{\beta_D(x)}, \sqrt{\beta_D(y)}\right\}$ ………………(2)

Next, $\alpha_D{}^3(x^{-1}) = \alpha_D{}^3(ex^{-1}) \geq \min\{\alpha_D{}^3(e), \alpha_D{}^3(x)\} = \alpha_D{}^3(x)$

i.e. $\alpha_D{}^3(x^{-1}) \geq \alpha_D{}^3(x)$………………….(3)

Similarly, $\sqrt{\beta_D(x^{-1})} \leq \sqrt{\beta_D(x)}$…………..(4).

The inequalities (1) to (4), show that Dis cube root fuzzy subgroup of G.

Theorem-3.3: Let $D = \{x,\ \alpha_D(x), \beta_D(x)\}$ be a cube root fuzzy subgroup of G. Then $\alpha_D{}^3(x^m) \geq \alpha_D{}^3(x)and\sqrt{\beta_D(x^m)} \leq \sqrt{\beta_D(x)}$for all $x \in G$ and $m \in N$.

Proof: We prove this theorem by mathematical induction method. Suppose $x \in G$, then

$\sqrt{\beta_D(x^2)} = \sqrt{\beta_D(xx)} \leq \max\left\{\sqrt{\beta_D(x)}, \sqrt{\beta_D(x)}\right\} = \sqrt{\beta_D(x)}$.

Therefore, the inequality is valid for $m = 2$. Assume that the theorem is true for $m = n - 1$. That is, $\sqrt{\beta_D(x^{n-1})} \leq \sqrt{\beta_D(x)}$.Then ,

$$\sqrt{\beta_D(x^n)} = \sqrt{\beta_D(xx^{n-1})} \leq \max\left\{\sqrt{\beta_D(x)}, \sqrt{\beta_D(x^{n-1})}\right\}$$
$$\leq \max\left\{\sqrt{\beta_D(x)}, \sqrt{\beta_D(x^{n-1})}\right\} \leq \sqrt{\beta_D(x)}$$

Thus, by induction principle, we have $\sqrt{\beta_D(x^n)} \leq \sqrt{\beta_D(x)}$, for all $m \in N$.

Similarly, we can show $\alpha_D{}^3(x^m) \geq \alpha_D{}^3(x)$, for all $m \in N$.

Theorem-3.4: Let $D = \{x,\ \alpha_D(x), \beta_D(x)\}$ be a cube root fuzzy subgroup of G. If $\alpha_D(x_1) \neq \alpha_D(x_2)and\beta_D(x_1) \neq \beta_D(x_2)$, for some x_1,$x_2 \in G$, then $\alpha_D{}^3(x_1x_2) = \min\{\alpha_D{}^3(x_1), \alpha_D{}^3(x_2)\}$and $\sqrt{\beta_D(x_1x_2)} \leq \max\left\{\sqrt{\beta_D(x_1)}, \sqrt{\beta_D(x_2)}\right\}$.

Proof: Suppose that for some x_1,$x_2 \in G$. We have $\alpha_D(x_1) < \alpha_D(x_2)$; then obviously

${\alpha_D}^3(x_1) < {\alpha_D}^3(x_2)$. Consider ,

$${\alpha_D}^3(x_1) = {\alpha_D}^3({x_2}^{-1}x_2x_1) \geq \min\{{\alpha_D}^3({x_2}^{-1}), {\alpha_D}^3(x_2x_1)\}$$
$$\geq \min\{{\alpha_D}^3(x_2), {\alpha_D}^3(x_1x_2)\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

Since ${\alpha_D}^3(x_1) < {\alpha_D}^3(x_2)$, therefore from relation (1) we obtain ${\alpha_D}^3(x_1) \leq {\alpha_D}^3(x_1x_2)\ldots(2)$

Also, ${\alpha_D}^3(x_1x_2) \geq \min\{{\alpha_D}^3(x_1), {\alpha_D}^3(x_2)\} = {\alpha_D}^3(x_1)$. That is ${\alpha_D}^3(x_1x_2) \geq {\alpha_D}^3(x_1)\ldots.(3)$

From (2) and (3), we have ${\alpha_D}^3(x_1x_2) = {\alpha_D}^3(x_1) = \min\{{\alpha_D}^3(x_2), {\alpha_D}^3(x_1)\}\ldots\ldots.(4)$

Similarly, ${\alpha_D}^3(x_1x_2) = \min\{{\alpha_D}^3(x_2), {\alpha_D}^3(x_1)\}$ if ${\alpha_D}^3(x_1) > {\alpha_D}^3(x_2)$.

Next, assume that $\beta_D(x_1) < \beta_D(x_2)$; then clearly $\sqrt{\beta_D(x_1)} < \sqrt{\beta_D(x_2)}$.

Consider, $\sqrt{\beta_D(x_2)} = \sqrt{\beta_D({x_1}^{-1}x_1x_2)} \leq \max\left\{\sqrt{\beta_D({x_1}^{-1})}, \sqrt{\beta_D(x_1x_2)}\right\}$

$$= \max\left\{\sqrt{\beta_D(x_1)}, \sqrt{\beta_D(x_1x_2)}\right\}\ldots\ldots\ldots\ldots (5)$$

Since $\sqrt{\beta_D(x_1)} < \sqrt{\beta_D(x_2)}$, therefore from relation (1) ,

we obtain $\sqrt{\beta_D(x_2)} \leq \sqrt{\beta_D(x_1x_2)}\ldots\ldots\ldots\ldots..(6)$

Also, $\sqrt{\beta_D(x_1x_2)} \leq \max\left\{\sqrt{\beta_D(x_1)}, \sqrt{\beta_D(x_2)}\right\} = \sqrt{\beta_D(x_2)}$, that is $\sqrt{\beta_D(x_1x_2)} \leq \sqrt{\beta_D(x_2)}$..(7)

From (6) and (7), we have $\sqrt{\beta_D(x_1x_2)} = \sqrt{\beta_D(x_2)} = \max\left\{\sqrt{\beta_D(x_1)}, \sqrt{\beta_D(x_2)}\right\}\ldots..(8)$

Similarly, the result can be proved if $\sqrt{\beta_D(x_1)} > \sqrt{\beta_D(x_2)}$.

Theorem-3.5: Let 'e' denote the identity element of G and $D = \{x,\ \alpha_D(x), \beta_D(x)\}$ be a cube root fuzzy subgroup of G. Then

(i) if ${\alpha_D}^3(x_1) = {\alpha_D}^3(e)$ for some $x_1 \in G$, then ${\alpha_D}^3(x_1x_2) = {\alpha_D}^3(x_2)$ for all $x_2 \in G$.

(ii) if $\sqrt{\beta_D(x_1)} = \sqrt{\beta_D(e)}$ for some $x_1 \in G$, then $\sqrt{\beta_D(x_1x_2)} = \sqrt{\beta_D(x_2)}$ for all $x_2 \in G$.

Proof: Suppose that $D = \{x,\ \alpha_D(x), \beta_D(x)\}$ is a Cube Root fuzzy subgroup of G.

(i) Let ${\alpha_D}^3(x_1) = {\alpha_D}^3(e)$ for some $x_1 \in G$. Then

$${\alpha_D}^3(x_2) = {\alpha_D}^3({x_1}^{-1}x_1x_2)$$
$$\geq \min\{{\alpha_D}^3({x_1}^{-1}), {\alpha_D}^3(x_1x_2)\}$$
$$= \min\{{\alpha_D}^3(x_1), {\alpha_D}^3(x_1x_2)\}$$
$$= \min\{{\alpha_D}^3(e), {\alpha_D}^3(x_1x_2)\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Since ${\alpha_D}^3(e) = {\alpha_D}^3(x_1)$, therefore from relation (1), we obtain ${\alpha_D}^3(x_2) \leq {\alpha_D}^3(x_1x_2)\ldots(2)$

Also, ${\alpha_D}^3(x_1x_2) \geq \min\{{\alpha_D}^3(x_1), {\alpha_D}^3(x_2)\} = {\alpha_D}^3(x_2)$, that is ${\alpha_D}^3(x_1x_2) \geq {\alpha_D}^3(x_2)\ldots(3)$

From (2) and (3), we get ${\alpha_D}^3(x_1x_2) = {\alpha_D}^3(x_2)\ldots\ldots\ldots\ldots\ldots..(4)$

(ii)The proof is similar to that of (i).

Theorem-3.6: Let 'e' denote the identity element of G and $D = \{x,\ \alpha_D(x), \beta_D(x)\}$ be a cuberoot fuzzy subgroup of G. Then $H = \left\{x \in G\ /\ {\alpha_D}^3(x) = {\alpha_D}^3(e) \text{and} \sqrt{\beta_D(x)} = \sqrt{\beta_D(e)}\right\}$ is a subgroup of G.

Proof: By definition of H, we have $e \in H$. Therefore H is non-empty subset of G. Let $x_1, x_2 \in H$, then ${\alpha_D}^3(x_1) = {\alpha_D}^3(e) = {\alpha_D}^3(x_2)$ and $\sqrt{\beta_D(x_1)} = \sqrt{\beta_D(e)} = \sqrt{\beta_D(x_2)}$.

Now, $\sqrt{\beta_D(x_1{x_2}^{-1})} \leq \max\left\{\sqrt{\beta_D(x_1)}, \sqrt{\beta_D({x_2}^{-1})}\right\}$

$$= \max\left\{\sqrt{\beta_D(x_1)}, \sqrt{\beta_D(x_2)}\right\}$$

$$= \max\left\{\sqrt{\beta_D(e)}, \sqrt{\beta_D(e)}\right\}$$

$\sqrt{\beta_D(x_1x_2^{-1})} \leq \sqrt{\beta_D(e)}$.

Also, by definition, we have $\sqrt{\beta_D(e)} \leq \sqrt{\beta_D(x_1x_2^{-1})}$.

Therefore, $\sqrt{\beta_D(x_1x_2^{-1})} = \sqrt{\beta_D(e)}$. Similarly, we can show that $\alpha_D{}^3(x_1x_2^{-1}) = \alpha_D{}^3(e)$.

Thus, $x_1x_2^{-1} \in H$, which completes the proof.

Theorem-3.7: (Involution Law) Let $D_1 = (\alpha_{D_1}, \beta_{D_1})$ and $D_2 = (\alpha_{D_2}, \beta_{D_2})$ be two CR-Fuzzy sets. Then,

(i) $(D_1 \cap D_2) \cup D_2 = D_2$,

(ii) $(D_1 \cup D_2) \cap D_2 = D_2$.

Proof: From the definition-2, we can obtain the following

(i)$(D_1 \cap D_2) \cup D_2 = \left(\min\{\alpha_{D_1}, \alpha_{D_2}\},\ \max\{\beta_{D_1}, \beta_{D_2}\}\right) \cup (\alpha_{D_2}, \beta_{D_2})$

$$= \left(\max\{\min\{\alpha_{D_2}, \alpha_{D_1}\}, \alpha_{D_2}\}, \min\{\max\{\beta_{D_2}, \beta_{D_1}\}, \beta_{D_2}\}\right)$$

$$= \left(\alpha_{D_2}, \beta_{D_2}\right) = D_2$$

(ii) The proof is similar to (i).

Theorem-3.8: (Demorgon's Law) Let $D_1 = (\alpha_{D_1}, \beta_{D_1})$ and $D_2 = (\alpha_{D_2}, \beta_{D_2})$ be two CR-Fuzzy sets. Then,

(i) $(D_1 \cap D_2)^C = D_1{}^C \cup D_2{}^C$

(ii) $(D_1 \cup D_2)^C = D_1{}^C \cap D_2{}^C$

Proof: For the two CR-Fuzzy sets $D_1$and D_2, according to definition (2), we obtain the following

(i)$(D_1 \cap D_2)^C = \left(\min\{\alpha_{D_1}, \alpha_{D_2}\},\ \max\{\beta_{D_1}, \beta_{D_2}\}\right)^C \left(\beta_{D_1}\right)^6, \sqrt{\alpha_{D_1}}$

$$= \left(\max\left\{\left(\beta_{D_1}\right)^6, \left(\beta_{D_2}\right)^6\right\},\ \min\left\{\sqrt{\alpha_{D_1}}, \sqrt{\alpha_{D_2}}\right\}\right)$$

$$= \left(\left(\beta_{D_1}\right)^6, \sqrt[4]{\alpha_{D_1}}\right) \cup \left(\left(\beta_{D_2}\right)^6, \sqrt[4]{\alpha_{D_2}}\right)$$

$$= D_1{}^C \cup D_2{}^C$$

(ii) The proof is similar to (i).

Theorem-3.9: (Associative Law) Let $D_1 = (\alpha_{D_1}, \beta_{D_1})$,$D_2 = (\alpha_{D_2}, \beta_{D_2})$ and $D_3 = (\alpha_{D_3}, \beta_{D_3})$ be three CR-Fuzzy sets. Then,

(i) $D_1 \cap (D_2 \cap D_3) = (D_1 \cap D_2) \cap D_3$,

(ii) $D_1 \cup (D_2 \cup D_3) = (D_1 \cup D_2) \cup D_3$.

Proof: For the three CR- Fuzzy sets D_1,$D_2$and D_3, according to definition (2), we obtain the following

(i)$D_1 \cap (D_2 \cap D_3) = (\alpha_{D_1}, \beta_{D_1}) \cap \left(\min\{\alpha_{D_2}, \alpha_{D_3}\},\ \max\{\beta_{D_2}, \beta_{D_3}\}\right)$

$$= \left(\min\left\{\alpha_{D_1}, \min\{\alpha_{D_2}, \alpha_{D_3}\}\right\},\ \max\left\{\beta_{D_1}, \max\{\beta_{D_2}, \beta_{D_3}\}\right\}\right)$$

$$= \left(\min\{\min\{\alpha_{D_1}, \alpha_{D_2}\}, \alpha_{D_3}\},\ \max\{\max\{\beta_{D_1}, \beta_{D_2}\}, \beta_{D_3}\}\right)$$

$= \left(\min\{\alpha_{D_1}, \alpha_{D_2}\},\ \max\{\beta_{D_1}, \beta_{D_2}\}\right) \cap (\alpha_{D_3}, \beta_{D_3})$
$= (D_1 \cap D_2) \cap D_3$
(ii) The proof is similar to (i).

Example of CR-Fuzzy set3.10: Assume that $\alpha_D(x) = 0.3, \beta_D(x) = 0.9$for $S = \{x\}$. Then $D = (0.3,0.9)$is not an intuitionistic fuzzy set because $0.3 + 0.9 = 1.2 > 1$. But it is pythagorean fuzzy set, fermatean fuzzy set and (3,2)-fuzzy set. In contrast,
$(0.3)^3 + \sqrt{0.9} = 0.9759 < 1$.
Note that $\pi_D(x) = 0.0243$ and hence $\left(\alpha_D(x)\right)^3 + \sqrt{\beta_D(x)} + \pi_D(x) = 1$.

Remark3.11 :From the figure-1, we get that
(i) The space of pythagorean membership grades is larger than the space of CR-fuzzy membership grades.
(ii) The CR-fuzzy set and IFS's intersect at the point $D = (0.3,0.9)$.
(iii) For $\alpha_D \in (0,0.3)$ and $\beta_D = (0.9,1)$ the space of CR-fuzzy membership grades starts to be larger than the space of intuitionistic membership grades.
(iv) For$\alpha_D \in (0.3,1)$ and $\beta_D = (0,0.9)$ the space of CR-fuzzy membership grades starts to be smaller than the space of intuitionistic membership grades.

Definition-3.12: (i) The score function of a CR-fuzzy set $D = (\alpha_D, \beta_D)$ can be represented as $\text{score}(D) = s(D) = \alpha_D{}^3 - \sqrt{\beta_D}$.
(ii)The accuracy function of a CR-fuzzy set $D = (\alpha_D, \beta_D)$ can be represented as $\text{accuracy}(D) = a(D) = \alpha_D{}^3 + \sqrt{\beta_D}$.

Example 3.13: For an CR-fuzzy set $D = (0.3,0.9)$, we find that

$$\text{score}(D) = s(D) = (0.3)^3 - \sqrt{0.9} = 0.027 - 0.948 = -0.921.$$
$$\text{accuracy}(D) = a(D) = (0.3)^3 + \sqrt{0.9} = 0.027 + 0.948 = 0.975.$$

In particular if $D = (0,1)$, then

$$\text{score}(D) = s(D) = (0)^3 - \sqrt{1} = 0 - 1 = -1.$$
$$\text{accuracy}(D) = a(D) = (0)^3 + \sqrt{1} = 0 + 1 = 1.$$

Theorem 3.14: The suggested score function of any CR-fuzzy set $D = (\alpha_D, \beta_D)$ denoted by $\text{score}(D)$lies in $[-1, 1]$.
Proof: Since for any CR-fuzzy set D we have $\alpha_D{}^3 + \sqrt{\beta_D} \le 1$.
Hence $\alpha_D{}^3 - \sqrt{\beta_D} \le \alpha_D{}^3 \le 1$ and
$\alpha_D{}^3 - \sqrt{\beta_D} \ge \sqrt{\beta_D} \ge 1$.
Therefore, $-1 \le \alpha_D{}^3 - \sqrt{\beta_D} \le 1$.
Hence $\text{score}(D) \in [-1,1]$.

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Remark 3.15: The suggested accuracy function of any CR-fuzzy set $D = (\alpha_D, \beta_D)$, denoted by accuracy(D)lies in [0, 1].

Example 2.25: Let $D = (\alpha_D, \beta_D)$ be a CR-fuzzy set. If $s(D) = -0.5$and $a(D) = 0.7$, then it follows $\alpha_D(x) = (0.1)^3$, $\beta_D(x)=\sqrt{0.6}$ and $\pi_D(x) = \sqrt{0.5}$.

Theorem 3.16: Let $D = (\alpha_D, \beta_D)$ be a CR-fuzzy set. Then $s(D) = 0$ if and only if $\alpha_D(x) = \beta_D(x)$for all $x \in X$.

Proof: Let $s(D) = 0$.Then$(\alpha_D(x))^3 - \sqrt{\beta_D(x)}$ implies $\alpha_D(x) = \beta_D(x)$ for allx $\in X$.

Conversely, suppose that $\alpha_D(x) = \beta_D(x)$ for all $x \in X$. It follows immediately that, for allx $\in X$, $\left(\alpha_D(x)\right)^3 = \sqrt{\beta_D(x)}$. Therefore $\left(\alpha_D(x)\right)^3 - \sqrt{\beta_D(x)} = 0$.

Thus $s(D) = 0$.

4. APPLICATIONS OF CR- FUZZY SET IN DECISION MAKING PROCESS.

Decision making approach for CR- fuzzy set

Algorithm:

Step-1 Construct D_1complement and D_2 complement for the decision matrix

Step-2 Calculate CR-fuzzy complement of ${D_1}^C$,${D_2}^C$ of D_1,D_2.

Step -3 Calculate score function using the structure (c+d)/10.

Step -4 Calculate the correlation coefficient between diseases and symptoms

Step -5 Choose maximum valuefrom step-4.

Step -6 Choose rank the order from step-5.

By Numerical example, we construct the following problem based on the above algorithm

Step-1 **Table-1 CR- fuzzy set D_1**

D_1=

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[0.2, 0.5]	[0.1, 0.6]	[0.3, 0.7]	[0.2, 0.6]
α_2	[0.1, 0.4]	[0.2, 0.5]	[0.2, 0.6]	[0.1, 0.5]
α_3	[0.3, 0.4]	[0.1, 0.4]	[0.1, 0.5]	[0.1, 0.4]

Table-2 CR- fuzzy set D_2

D_2=

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[0.1, 0.3]	[0.3, 0.4]	[0.1, 0.3]	[0.2, 0.5]
α_2	[0.2, 0.4]	[0.1, 0.5]	[0.3, 0.4]	[0.1, 0.4]
α_3	[0.3, 0.5]	[0.2, 0.3]	[0.1, 0.2]	[0.3, 0.2]

Table-3CR- fuzzy set ${D_1}^C$

${D_1}^C$ =

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[0.8, 0.5]	[0.9, 0.4]	[0.7, 0.3]	[0.8, 0.4]

α_2	[0.9, 0.6]	[0.8, 0.5]	[0.8, 0.4]	[0.9, 0.5]
α_3	[0.7, 0.6]	[0.9, 0.6]	[0.9, 0.5]	[0.9, 0.6]

Table-4 (3,2)- fuzzy set D_2^C

D_2^C =

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[0.9, 0.7]	[0.7, 0.6]	[0.9, 0.7]	[0.8, 0.5]
α_2	[0.8, 0.6]	[0.9, 0.5]	[0.7, 0.6]	[0.9, 0.6]
α_3	[0.7, 0.5]	[0.8, 0.7]	[0.9, 0.8]	[0.7, 0.8]

Step -2

Table-5 Calculate (3,2)- fuzzy (D_1^C, D_2^C)

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
$\alpha_1\alpha_2$	[0.8, 0.5]	[0.9, 0.4]	[0.7, 0.3]	[0.8, 0.4]
$\alpha_2\alpha_3$	[0.7, 0.5]	[0.8, 0.5]	[0.8, 0.4]	[0.7, 0.5]
$\alpha_3\alpha_1$	[0.7, 0.6]	[0.7, 0.6]	[0.9, 0.5]	[0.8, 0.5]

Table-6 Calculate CR- fuzzy (D_2^C, D_1^C)

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
$\alpha_1\alpha_2$	[0.9, 0.6]	[0.7, 0.5]	[0.8, 0.4]	[0.8, 0.5]
$\alpha_2\alpha_3$	[0.7, 0.6]	[0.9, 0.5]	[0.7, 0.5]	[0.9, 0.6]
$\alpha_3\alpha_1$	[0.7, 0.6]	[0.8, 0.4]	[0.7, 0.3]	[0.7, 0.4]

Step -3 Calculate the score function left and right value of CR- fuzzy set L = (a+b) /3 where i = 1,2 ….n and R = (a+b) /3 where j = 1,2 ….n

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[0.567, 0.367]	[0.533, 0.300]	[0.500, 0.233]	[0.533, 0.300]
α_2	[0.467, 0.367]	[0.567, 0.333]	[0.500, 0.300]	[0.533, 0.367]
α_3	[0.467, 0.367]	[0.500, 0.333]	[0.533, 0.267]	[0.500, 0.300]

Score function S = (L + R) /10 = (6.200 + 3.834) /10 = 10.03/10=1.003
Where L is the sum of left value of the bracket and R is the sum of right value of the bracket.

Step -4 Calculate correlation coefficient between diseases and symptoms by using the formula is given by

$$\rho = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left(d_{ij}\right)^2}{\sqrt{\sum_{i=1}^{n}(d_i)^2}\sqrt{\sum_{j=1}^{n}\left(d_j\right)^2}}$$

Using step-3 we form a new calculation table as follows

Y \ X	1	2	3	4
α_1	0.200	0.233	0.267	0.233
α_2	0.100	0.234	0.200	0.166
α_3	0.100	0.167	0.266	0.200

$$\rho = 0.301 < 1$$

Step -5 Score function = 1.003
Correlation function = 0.301
∴ Maximum value = 0.267

Step -6 From the table the order preference is given from step (4) as given below
Rank the order
$r_1 < r_2 = r_4 < r_3$
$r_1 < r_4 < r_3 < r_2$
$r_1 < r_2 < r_4 < r_3$

For α_1 symptoms fever<Headache= Cancer<Typhoid
For α_2 symptoms fever<Cancer<Typhoid<Headache
For α_3 symptoms fever<Headache<Cancer<Typhoid

Result:
Finally we conclude that symptoms (1)(ie) α_1 which is decide to choose the nearest ranking order for the given problem.

Conclusion: Several mathematicians defined various algebraic structures in various fuzzy Environment. We propose a new algebraic structures based on cubic root fuzzy sets. One can obtain the similar idea in the field of square root fuzzy sets and ortho pair fuzzy sets.

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