An Analysis of Flow Shop Scheduling Problem under Interval-Valued Fuzzy and Interval – Valued Intuitionistic Fuzzy Environment

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Abstract: In this paper scheduling problems are solved in the Interval – valued Fuzzy and Interval – valued Intuitionistic Fuzzy Criteria. Generally, scheduling plays a major part in manufacturing and in service industries to schedule the jobs in order. Here scheduling jobs are done under different algorithm where the parameters are being defined under Fuzzy and Intuitionistic Interval -valued numbers. Numerical examples are illustrated to calculate Mean Flow Time and Total Elapsed Time and compared with Johnson's algorithm.

Keywords: Processing time, Mean flow time, Total elapsed time, Interval – valued Fuzzy and Interval – valued Intuitionistic Fuzzy Numbers.

1. INTRODUCTION

The concept of Fuzzy set to handle the uncertainity was initialised by Zadeh [9] in 1965.As a generalisation of fuzzy set, Gorzalczany [3] and Turksen [6] suggested the idea of interval valued fuzzy set. Further Wang and Li [7] defined interval-valued fuzzy numbers and their extended operations.

Atanassov [2] developed the idea of intuitionistic fuzzy set to overcome the vague situations conveniently with membership and non-membership values. An interval-valued intuitionistic fuzzy set gives the clear idea to solve the difficulty by giving the membership and non-membership values in terms of intervals with lower and upper boundaries.

In general, flow shop scheduling problems are applied in many manufacturing sectors and in small scale industries to arrange the jobs in a sequencial order to increase production and to minimize the total time taken to complete the jobs.

G. Ambiga and G. Uthra[1] studied the analysis of flow shop scheduling problem with transportation time and weightage. Shiny Jose, Sunny Kuriakose [4] developed an algorithm to solve assignment problem with intuitionistic fuzzy interval. G. Uthra, K. Thangavelu, S. Shunmugapriya [8] discussed the optimal solution of FSS problems in intuitionistic fuzzy environment and compared with fuzzy environment.

In this paper, in problem I, the problem is solved by assuming the processing time in interval – valued fuzzy numbers, which are further defuzzified by taking the width of the interval and formed the sequence by the proposed method and compared the results with the Johnson's algorithm.

Problem II is based under the idea of scheduling jobs with processing time, transportation time, and corresponding weightage with specified break-down interval to calculate the mean flow time, where each parameters are defined under the intuitionistic interval-valued fuzzy number. Further with the idea of Geometric Aggregation Operator, the intervals are defuzzified and sequence are framed with the ranking of Score and Accuracy function.

2. PRELIMINARIES

2.1 Let X be a nonempty set, a **fuzzy set** $\tilde{A}$ is defined by $\tilde{A}$ = {(x, $\mu_{\tilde{A}}$(x)): x∈A}. In the pair (x, $\mu_{\tilde{A}}$(x)), the first element belongs to the classical set A, the second element $\mu_{\tilde{A}}$(x), belong to the interval [0, 1] is called the membership function.

2.2 Fuzzy number $\tilde{A}$ is a fuzzy set on the real line$\Re$, must satisfy the following conditions.

(i) $\mu_{\tilde{A}}(x_0)$ is piecewise continuous

(ii) There exist at least one $x_0 \in \Re$ with $\mu_{\tilde{A}}(x_0)$ =1

(iii)$\widetilde{A}$ must be normal & convex

2.3 Intuitionistic Fuzzy number An Intuitionistic fuzzy subset $A^I = \{(x_i, \mu_{A^I}(x), \gamma_{A^I}(x) / x_i \in X)$ of the real line R is named as an intuitionistic fuzzy number if the following holds.

(i) There exist θ ∈R, $\mu_{A^I}(\theta)$ = 1 and $\gamma_{A^I}(\theta)$ = 0. Where θ is the mean value of A^I.

(ii) μ_{A^I} is continuous mapping from R to [0,1] for all x∈R, the relation $0 \leq \mu_{A^I}(x) + \gamma_{A^I}(x) \leq 1$ holds. The membership and non-membership function of A^I is of the following form,

$$\mu_{A^I}(x) = \begin{cases} 0, & if -\alpha < x < \theta - \alpha \\ f_1(x), & if\ x \in [\theta - \alpha, \theta] \\ 1, & if\ \ x = \theta \\ g_1(x), & if\ x \in [\theta, \theta + \beta] \\ 0, & if \theta + \beta \leq x < \alpha \end{cases}$$

$$\gamma_{A^I}(x) = \begin{cases} 1, & if -\alpha < x < \theta - \alpha' \\ f_2(x), & if\ x \in [\theta - \alpha', \theta]; 0 \leq f_1(x) + f_2(x) \leq 1 \\ 0, & if\ x = \theta \\ g_2(x), & if\ x \in [\theta, \theta + \beta']; 0 \leq g_1(x) + g_2(x) \leq 1 \\ 1, & if \theta + \beta' \leq x \leq \alpha \end{cases}$$

Where $f_i(x)$ and, $g_i(x)$; i =1,2 which are strictly increasing and decreasing functions in $[\theta - \alpha, \theta]$, $[\theta, \theta + \beta]$, $[\theta - \alpha', \theta]$ and $[\theta, \theta + \beta']$ respectively. α, β, α' and β' are left and right spreads of $\mu_{A^I}(x)$ and $\gamma_{A^I}(x)$.

3. INTERVAL-VALUED FUZZY NUMBER(IVFN)

Let I = [0,1] be a closed interval and [I] = $\{\bar{a} = [a^-, a^+], a^- \leq a^+, a^+, a^- \in I\}$

Let X be a ordinary non-empty set. Then the mapping A: X → [I] is called an interval valued fuzzy set on X. All Interval – valued Fuzzy Sets on X are denoted by IF(X).

An interval -Valued fuzzy set A defined an X is given by $A = \{x, [A^L(x), A^U(x): x \in X\}$, where $0 \leq A^L(x) \leq A^U(x) \leq 1$.The Interval – valued fuzzy set A can be represented by an interval $A(x) = [A^L(x), A^U(x)]$ and the ordinary fuzzy set $A^L: X \to I$ and $A^U: X \to I$ are called lower and an upper fuzzy set of A respectively.

The centre of the interval can be taken as $\frac{A^L+A^U}{2}$ and the width of the interval is $\frac{A^L-A^U}{2}$

3.1 Arithmetic Operations on Interval Valued Fuzzy Number

Let A = $[{a_1}^L(x), {a_2}^U(x)]$ and B =$[{b_1}^L(x), {b_2}^U(x)]$ be two interval – valued fuzzy set then

(i) A+B = $[{a_1}^L + {b_1}^L, {a_2}^U + {b_2}^U]$

(ii) A-B = $[{a_1}^L - {b_1}^L, {a_2}^U - {b_2}^U]$

(iii)Ka = k$[{a_1}^L, {a_2}^U]$ = $[k{a_1}^L, k{a_2}^U]$

3.2 Score and Accuracy Function of Interval Valued Fuzzy Number

Let $\theta = \{[a,b],[c,d]\}$ be the Interval valued fuzzy number, then the

Score of θ is defined as $S(\theta) = \frac{1}{2}(a - c + b - d)$ and

Accuracy of θ is defined as $A(\theta) = \frac{1}{2}(a + b - dc)$.

4. INTERVAL-VALUED INTUITIONISTIC FUZZY SET (IVIFS)

Let $X = \{x_1, x_2, x_3 \dots . x_n\}$ be a non- empty set of the universe. An IVIFS

$$\bar{A} = \{x_i, [\mu_{\bar{A}}^L(x_i), \mu_{\bar{A}}^U(x_i)][\vartheta_{\bar{A}}^L(x_i), \vartheta_{\bar{A}}^U(x_i)]/x_i \epsilon X\}$$

Where $[\mu_A^L(x_i), \mu_A^U(x_i)]$ and $[\vartheta_A^L(x_i), \vartheta_A^L(x_i)]$ denotes the interval of Membership and Non-Membership degree of the element $x_i \in \bar{A}$ satisfying the following conditions

1. $\mu_{\bar{A}}^U(x_i) + \vartheta_{\bar{A}}^U(x_i) \leq 1$
2. 0≤ $\vartheta_{\bar{A}}^L(x_i) + \vartheta_{\bar{A}}^U(x_i) \leq 1 \; \forall x_i \epsilon X$
3. If $\mu_{\bar{A}}^L(x_i) \leq \mu_{\bar{A}}^U(x_i)$ and $\vartheta_{\bar{A}}^L(x_i) \leq \vartheta_{\bar{A}}^U(x_i)$ then $\bar{A}$ is reduced to IFS

4.1Definition: The weighted Geometric Average Operator is defined by

$$G_w(J_1, J_2, J_3, \dots . J_n) = \prod {J_i}^{w_i}$$
$$= [\prod {\mu_{AL}}^{w_i}(x), \prod {\mu_{AU}}^{w_i}(x)],$$
$$[1 - \prod(1 - \vartheta_{AL}(x)))^{w_i}, 1 - \prod(1 - \vartheta_{AU}(x))^{w_i}],$$

where w_i is the weight of the corresponding job.

5. MATHEMATICAL FORMULATION OF INTERVAL – VALUED FUZZY FLOW SHOP SCHEDULING PROBLEM

jobs	Machine A	Machine B	Machine C
	p_i	q_i	r_i
1	$[a_1, b_1]$	$[c_1, d_1]$	$[e_1, f_1]$
2	$[a_2, b_2]$	$[c_2, d_2]$	$[e_2, f_2]$
3	$[a_3, b_3]$	$[c_3, d_3]$	$[e_3, f_3]$
….	……..	…….	…..
….	……..	…….	…...
N	$[a_n, b_n]$	$[c_n, d_n]$	$[e_n, f_n]$

5.1Notations

$p_i -$ Processing time of i^{th} job on Machine A

$q_i -$ Processing time of i^{th} job onMachine B

$r_i -$ Processing time of i^{th} job on Machine C

(All the processing times are being taken in the form of interval – valued fuzzy numbers $[a_{l,}a_u]$)

5.2 Proposed Algorithm for Interval – Valued Flow Shop Scheduling Problem

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Step 1: Consider the processing time as an interval – valued fuzzy number.

Step 2: Calculate the expected processing time as

$$A_i = p_i * \ l_i, B_i = q_i * \ m_i, C_i = r_i * \ n_i$$

Step 3: Defuzzify the interval – valued number by taking the width of the interval.

Step 4: Calculate $\frac{Max-Min}{Min(i,j)}$ for each row.

Step 5: Arrange the calculation in ascending order, in case of tie, calculate the difference of minimum and next minimum of the processing time.

Step 6: The arrangement will provide the job sequence.

Step 7: Calculate the minimum total elapsed time by framing the in-out table.

6. NUMERICAL EXAMPLE

In a most reputed hospital, during Sunday the hospital will work only for half a day.

On a particular Sunday, 5 patients are arriving in random, an hour before opening the hospital. After when the hospital are getting open, the patients are getting token and undergoing 3 stages (i.e.,) taking scan(S), consulting doctor (D) and getting medicine (M) in the hospital. Calculate the total hour the hospital open on that particular day.

Here the time taken for the patients for each stage is given in the interval valued fuzzy number.

Stages Patients	S	D	M
P_1	[0.1,0.7]	[0.2,0.7]	[0.4,0.6]
P_2	[0.2,0.6]	[0.3,0.6]	[0.2,0.7]
P_3	[0.1,0.7]	[0.4,0.5]	[0.2,0.6]
P_4	[0.2,0.8]	[0.3,0.6]	[0.4,0.5]
P_5	[0.1,0.7]	[0.2,0.4]	[0.1,0.7]

Solution

Interval – valued fuzzy numbers are defuzzified by the width of the interval and calculating the proposed method for each row.

Stages Patients	S	D	M	$\frac{Max-Min}{Min(i,j)}$
P_1	0.3	0.25	0.1	0.067
P_2	0.2	0.15	0.25	0.033
P_3	0.3	0.05	0.2	0.083
P_4	03	0.15	0.05	0.083
P_5	0.3	0.10	0.3	0.067

The optimal sequence obtained from the above table by arranging in ascending order is

$$S_2 - (S_1 - S_5) - (S_3 - S_4)$$

Here the sequence with in the brackets are having same calculated values. Therefore calculating the minimum and the next minimum value in the corresponding row and selecting the maximum among the values we get the sequence as

$$S_2 - S_5 - S_1 - S_3 - S_4$$

Stages	S		D		M	
Patients	In	Out	In	Out	In	Out

2	--	0.2	0.2	0.35	0.35	0.60
5	0.2	0.5	0.5	0.60	0.60	0.90
1	0.5	0.8	0.8	1.05	1.05	1.15
3	0.8	1.1	1.1	1.15	1.15	1.35
4	1.1	1.4	1.4	1.55	1.55	1.60

Comparison between Proposed and Existing Method

Proposed Method	Existing Method
Minimum Total Elapsed Time =1.60hrs	Minimum Total Elapsed Time = 1.60hrs
Waiting time for Scan = 0.2 hrs	Waiting time for Scan = 0.2 hrs
Waiting time for Consulting Doctor = 1.35hrs	Waiting time for Consulting Doctor =1.35hrs
Waiting time for Getting medicine = 0.70hrs	Waiting time for Getting medicine = 0.70hrs

7. MATHEMATICAL FORMULATION OF INTERVAL – VALUED INTUITIONISTIC FLOW SHOP SCHEDULING PROBLEM

Here 5 jobs assigned to 3 workers, the processing time of each job by the workers are represented as f_{ij} and transportation time is represented as t_j .The membership and non – membership values of the parameters are given as

$$f_{ij}{}^{[\mu_{jl}(f_{ij})\mu_{ju}(f_{ij})],[\vartheta_{jl}(f_{ij})\vartheta_{ju}(f_{ij})]}$$

$$t_j{}^{[\mu_{jl}(t_j)\mu_{ju}(t_j)],[\vartheta_{jl}(t_j)\vartheta_{ju}(t_j)]}$$

7.1 Assumption

i) The tasks to be processed are independent of each others.
ii) Pre-emption of employment are not allowed
iii) An appointment is not available to the next machine until and unless processing in current device is completed.
iv) Each job must be completed once it is started.

7.2 Notations

f_{ij} - Processing time of i^{th} job on a j^{th} machine
w_i - weight of the job
S_K – Sequence formed for i jobs (k = 1,2,…5)
M – Mean weight flow time
t_j - transportation time of jobs
$(b-a) =$ length of the break – down interval

8. PROPOSED ALGORITHM FOR INTERVAL – VALUED INTUITIONISTIC FLOW SHOP SCHEDULING PROBLEM

Step 1: Find the weight $w_i = \frac{f_{ij}}{\Sigma f_{ij}}$ for each job.

Step 2: Find the weighted geometric average operator for the interval – valued intuitionistic fuzzy number representing the parameters for each job.

Step 3: Calculate the score value for each job, if the values are same find the accuracy for that particular job.

Step 4: Arrange the score function in ascending order which gives the job sequence.

Step 5: Calculate the mean flow time for the respective break down interval.

9. NUMERICAL EXAMPLE

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A building contractor had a contract to build a house, in certain stage he has to complete the work of painting, carpenting and tiles covering in dining room, drawing room, kitchen, balcony and bed room. Calculate the minimum total completion time.

Workers /Jobs	Painters(P)	t_1	Carpenters (C)	t_2	Tiles Worker(T)
J_1	$7^{[0.2,0.4][0.3,0.5]}$	$2^{[0.1,0.3][0.2,0.4]}$	$4^{[0.2,0.4][0.3,0.5]}$	$1^{[0.1,0.3][0.2,0.5]}$	$8^{[0.2,0.4][0.3,0.5]}$
J_2	$9^{[0.3,0.4][0.4,0.5]}$	$1^{[0.2,0.4][0.3,0.5]}$	$3^{[0.1,0.3][0.2,0.4]}$	$2^{[0.1,0.2][0.3,0.4]}$	$6^{[0.2,0.4][0.3,0.5]}$
J_3	$8^{[0.1,0.3][0.2,0.5]}$	$3^{[0.1,0.4][0.2,0.5]}$	$5^{[0.2,0.4][0.3,0.5]}$	$3^{[0.2,0.4][0.3,0.5]}$	$7^{[0.1,0.4][0.2,0.5]}$
J_4	$6^{[0.2,0.4][0.3,0.5]}$	$2^{[0.1,0.3][0.2,0.4]}$	$4^{[0.2,0.4][0.3,0.5]}$	$2^{[0.1,0.3][0.2,0.4]}$	$10^{[0.1,0.4][0.2,0.5]}$
J_5	$8^{[0.1,0.3][0.2,0.4]}$	$1^{[0.2,0.4][0.3,0.5]}$	$3^{[0.1,0.3][0.2,0.4]}$	$4^{[0.2,0.4][0.3,0.5]}$	$6^{[0.2,0.3][0.4,0.5]}$

Solution

Calculating the weights of each job

Workers Jobs	**P**	**t$_i$**	**C**	**e$_i$**	**T**
J_1	$7^{[0.2,0.4][0.3,0.5]}$ $w = 0.318$	$2^{[0.1,0.3][0.2,0.4]}$ $w = 0.091$	$4^{[0.2,0.4][0.3,0.5]}$ $w = 0.182$	$1^{[0.1,0.3][0.2,0.5]}$ $w = 0.045$	$8^{[0.2,0.4][0.3,0.5]}$ $w = 0.364$
J_2	$9^{[0.3,0.4][0.4,0.5]}$ $w = 0.429$	$1^{[0.2,0.4][0.3,0.5]}$ $w = 0.048$	$3^{[0.1,0.3][0.2,0.4]}$ $w = 0.143$	$2^{[0.1,0.2][0.3,0.4]}$ $w = 0.095$	$6^{[0.2,0.4][0.3,0.5]}$ $w = 0.286$
J_3	$8^{[0.1,0.3][0.2,0.5]}$ $w = 0.308$	$3^{[0.1,0.4][0.2,0.5]}$ $w = 0.115$	$5^{[0.2,0.4][0.3,0.5]}$ $w = 0.192$	$3^{[0.2,0.4][0.3,0.5]}$ $w = 0.115$	$7^{[0.1,0.4][0.2,0.5]}$ $w = 0.269$
J_4	$6^{[0.2,0.4][0.3,0.5]}$ $w = 0.250$	$2^{[0.1,0.3][0.2,0.4]}$ $w = 0.083$	$4^{[0.2,0.4][0.3,0.5]}$ $w = 0.167$	$2^{[0.1,0.3][0.2,0.4]}$ $w = 0.083$	$10^{[0.1,0.4][0.2,0.5]}$ $w = 0.417$
J_5	$8^{[0.1,0.3][0.2,0.4]}$ $w = 0.364$	$1^{[0.2,0.4][0.3,0.5]}$ $w = 0.045$	$3^{[0.1,0.3][0.2,0.4]}$ $w = 0.136$	$4^{[0.2,0.4][0.3,0.5]}$ $w = 0.182$	$6^{[0.2,0.3][0.4,0.5]}$ $w = 0.273$

The weighted geometric average operator for each job is given

$$G(J_1) = (0.189,0.392,0.283,0.481)$$
$$G(J_2) = (0.212,0.385,0.324,0.468)$$
$$G(J_3) = (0.132,0.377,0.226,0.489)$$
$$G(J_4) = (0.139,0.389,0.269,0.478)$$
$$G(J_5) = (0.147,0.354,0.278,0.446)$$

The corresponding score and accuracy values are

$$S(J_1) = -0.092, A(J_1) = 0.222$$
$$S(J_2) = -0.098, A(J_2) = 0.223$$
$$S(J_3) = -0.103, A(J_3) = 0.199$$
$$S(J_4) = -0.110, A(J_4) = 0.200$$
$$S(J_5) = -0.112, A(J_5) = 0.189$$

By arranging the score functions in ascending order we get the sequence as

$$S_5 - S_4 - S_3 - S_2 - S_1$$

The In-out table for the sequence is

Workers	P		t_i	C		e_i	T	
Jobs	In	Out		In	Out		In	Out
5	--	8	1	9	12	4	16	22

4	8	14	2	16	20	2	22	32
3	14	22	3	25	30	3	32	39
2	22	31	1	32	35	2	39	45
1	31	38	2	40	44	1	45	53

The effect of break – down interval is (34 – 40) and the weights are considered with the minimum weight calculated with each row

Workers	p		t_i	C		e_i	T		w_i
JOBS	In	Out		In	Out		In	Out	
5	--	8	1	9	12	4	16	22	0.045
4	8	14	2	16	20	2	22	32	0.083
3	14	22	3	25	30	3	32	45	0.115
2	22	31	1	32	41	2	45	51	0.048
1	31	44	2	46	50	1	51	59	0.045

Mean Flow Time for Completing the Wholework

$$= \frac{(22-0)*0.045+(32-8)*0.083+(45-14)*0.115+(51-22)*0.048+(59-31)*0.045}{0.115+0.045+0.083+0.048+0.045}$$

$$= \frac{9.199}{0.336}$$

= 27.37rs

Makespan = 59 hrs

Comparison with the Job – block criteria

Considering the processing time and the transportation time for the jobs as below and taking (2,5) jobs equivalent with the breakdown effect (34 – 40)

Machines Jobs	P	t_i	C	e_i	T	w_i
J_1	7	2	4	1	8	0.045
J_2	9	1	3	2	6	0.048
J_3	8	3	5	3	7	0.115
J_4	6	2	4	2	10	0.083
J_5	8	1	3	4	6	0.045

Makespan = 62 hrs

Mean Flow Time = 28.5 hrs

10. CONCLUSION

Scheduling the jobs in an orderly manner increase the profit and consumes the labour work. Solving Flow Shop Scheduling problem under inter valued environment is very efficient and gives better comparison with Johnson's algorithm and complete the work in minimum time. Our Future work deals with the study of scheduling process under different environment to yield better results.

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