# Modified Fuzzy Resolving Number

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***Abstact: The concept of modified fuzzy resolving set, the modified fuzzy resolving number and the modified fuzzy super resolving number are introduced and their properties are discussed. In this article, we have proved that the modified fuzzy resolving number of a fuzzy graph $G$ with no fuzzy bridge is $n-1$ and it is also proved that the modified fuzzy resolving number and modified fuzzy super resolving number of a fuzzy labelling graph are the same.***

***Keywords: Connectedness of fuzzy graph, Fuzzy resolving number, Fuzzy super resolving number, Isomorphic fuzzy graph.***

## 1. INTRODUCTION

Fuzzy graph theory is a current research area in which a lot of research scholars do their research study. Fuzzy Mathematics is introduced by Lotfi Asker Zadeh in 1965 and Fuzzy Graph Theory is introduced by Rosenfield in the year 1975. R shanmugapriya and Mary Jiny D defined fuzzy resolving number in the year 2019 and this study is motivated by the concept of resolving number in graph theory by Slater in 1975. The difference between the fuzzy resolving number of a fuzzy graph and the resolving number of a graph is the environment in which the movement of the object happens. In graph theory, it is assumed that the robots are moving in a graph-structured frame-work and the resolving number is the minimum number of landmark needed to identify the robot's position uniquely in terms of distance. And in fuzzy graph theory, we assume that the robots are moving in a fuzzy graph-structured frame-work and the fuzzy resolving number is the minimum number of landmark required to identify the position of the robot uniquely in terms of safety level. In this article as an extension of the concept of resolving number we introduce the modified fuzzy resolving set and modified fuzzy resolving number by considering the weakness of connectedness between the nodes of the fuzzy graph.

## 2. PRELIMINARY

**Definition 2.1** Let $V$ be a set of vertices, $|V| \neq \phi$, $\sigma$ is the function from $V$ to $[0,1]$ and $\mu$ is a function from $V \times V$ to $[0,1]$ such that for all $r, s \in V$, $\mu(r,s) \leq \sigma(r) \wedge \sigma(s)$. The ordered triple $G(V,\sigma,\mu)$ is called the fuzzy graph.

**Definition 2.2** The fuzzy graph $G$ is said to be **isomorphic** to $G'$, if there exist a bijective map $h: V \to V'$ which satisfies (i) $\sigma(x) = \sigma'(h(x)) \forall x \in V$ (ii)$\mu(x,y) = \mu'(h(x),h(y)) \forall x,y \in V$ denoted as $G \cong G'$.

**Definition 2.3** The fuzzy graph $G$ is said to be **co-weak isomorphic** to $G'$ if there exist a bijective homomorphism $h: V \to V'$, such that $\mu(u,v) = \mu'(h(u),h(v)) \forall u,v \in V$.

**Definition 2.4 Strength (or weight) of the fuzzy path** $P = r_1, r_2, \ldots r_n$ is the weight of the edge with minimum membership value in the path $P$. The path $P$ is called as cycle if $r_1 = r_n$.

**Definition 2.5 The strength of connectedness** between any two vertices $r_1$ to $r_n$ is the maximum value of strength of all paths connecting $r_1$ to $r_n$, which is denoted as $\mu^\infty(r_1, r_n)$.

**Definition 2.6** An edge $uv$ of the fuzzy graph $G$ is called **fuzzy bridge** if in the graph $G'$ obtained by removal of $uv$, $\mu'^\infty(x,y) < \mu^\infty(x,y)$ for some $x, y \in V$. And $\mu^\infty(u,v) = \mu(uv)$.

**Definition 2.7** Let $G$ be a fuzzy graph. A subset $H$ of $\sigma$ is called **the fuzzy resolving set** of $G$, if the representations of all the element in $\sigma - H$ with respect to $H$ are all distinct. If $\sigma$ has $n$ element and $|H| = k$ then $\sigma - H$ has $n - k$ elements. And the representation of an element in $\sigma - H$ with respect to $H$ is an ordered $k$-tuple whose elements are the strength of connectedness between the element in $\sigma - H$ to all the $k$ elements in $H$. The cardinality of the minimum resolving set is named as **the resolving number of $G$**. And if the representations of every element in $\sigma$ with respect to $H$ are all distinct then $H$ is called the fuzzy super resolving set of $G$.

**Illustration 2.1** Consider the following fuzzy graph $G$:

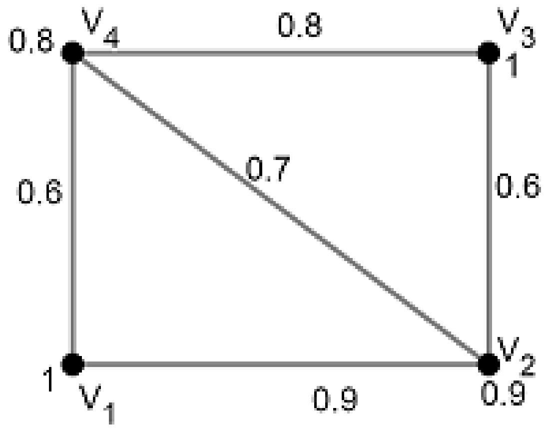


Fig 2.1 The Fuzzy Graph $G(V, \sigma, \mu)$

let $H = \{\sigma_1, \sigma_3\}$, $\sigma - H = \{\sigma_2, \sigma_4\}$ where $\sigma_1 = (v_1, \sigma(v_1))$
the representation of $\sigma - H$ with respect to $H$ are:

$$\sigma_2/H = (\mu^\infty(v_2, v_1), \mu^\infty(v_2, v_3)) = (.9, .7)$$
$$\sigma_4/H = (\mu^\infty(v_4, v_1), \mu^\infty(v_4, v_3)) = (.7, .8)$$

Since the representations are distinct $H$ is the resolving set of $G$. Similarly we can check all other subset of $\sigma$ for the resolving set. The minimum resolving set of $G$ has cardinality 2. Therefore the resolving number of $G$ is 2.

**Definition 2.8** Let $V$ be a set of vertices, $|V| \neq \phi$, $\sigma$ is the function from $V$ to $[0,1]$ and $\mu$ is a function from $V \times V$ to $[0,1]$ such that for all $r, s \in V$, $\mu(r,s) \leq \sigma(r) \vee \sigma(s)$. Then the ordered triple $G(V, \sigma, \mu)$ is called **modified fuzzy graph**.

**Definition 2.9** A fuzzy graph $G(V_n, \sigma, \mu)$ is said to be a **fuzzy labeling graph** if there exist an one-one onto function between the set of vertices and edges of $G$ to $[0,1]$.

## 3. THE MODIFIED FUZZY RESOLVING NUMBER

In a fuzzy graph, the strength of any path is the membership value of the weakest arc in the path and the connectedness between any two nodes is the maximum of the strength of all path connecting those nodes. However consider a situation, a robot is moving in a fuzzy graph-structured framework, each node is denoted a place or landmark and the membership value of the edges indicate the temperature level as a fuzzy number. If the robot needs to travel from one landmark to the other landmark safely (without exploring to high temperature). It has to find the maximum temperature in edges of all the path connecting the two landmark and then select the minimum value among that, the corresponding path is the safest path. This motivates us to find the weakness of connectedness between any two nodes of the fuzzy graph.

**Definition 3.1 The weakness of the fuzzy path** $P = v_1, v_2, \dots v_n$ connecting the two vertices $v_1$ and $v_2$ is the maximum membership value of all edges in the path.

**Definition 3.2** We define, **the weakness of connectedness** between any two vertices $v_1$ and $v_2$ of a modified fuzzy graph $G$ is the minimum of the weaknesses of all the path joining the two vertices and which is denoted as $\mu_\infty(v_1, v_2)$.

**Definition 3.3** Let $H = \{v_1, v_2, \dots v_k\}$ be a subset of $\sigma = \{v_1, v_2, \dots v_k, \dots v_n\}$. The modified representation of $\sigma - H = \{v_{k+1}, v_{k+2}, \dots v_n\}$ with respect to $H$ is an ordered $k$-tuple, $(\mu_\infty(v_{k+1}, v_1), \mu_\infty(v_{k+2}, v_2), \dots \mu_\infty(v_n, v_k))$, $\mu_\infty(v_i, v_i) = \sigma(v_i)$.

**Definition 3.4** Let $G(V_n, \sigma, \mu)$ be a fuzzy graph. A proper subset $H$ of $\sigma$, $2 \leq |H| \leq n - 1$ is called **the modified fuzzy resolving set** of $G$, if the modified representation of all the element in $\sigma - H$ with respect to $H$ are all distinct. The cardinality of the minimum resolving set is named as **the modified fuzzy resolving number** of $G$ denoted as $Fr_m(G)$. And if the modified representation of every element in $\sigma$ with respect to $H$ are all distinct then $H$ is called **the modified fuzzy super resolving set** of $G$. The cardinality of the minimum modified fuzzy super resolving set is called **the modified fuzzy super resolving set** of $G$ denoted as $Sr_m(G)$.

**Illustration 3.1** Find the modified fuzzy resolving number of the following modified fuzzy graph $G$.

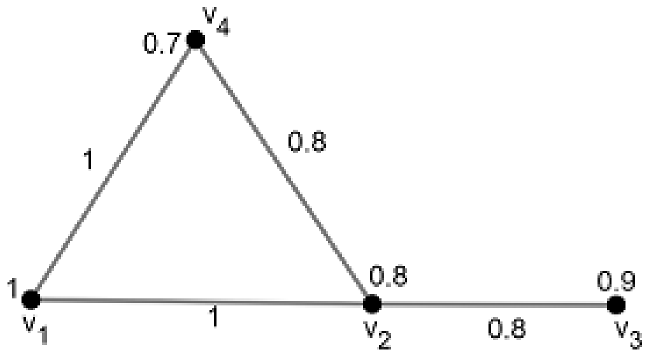


Fig. 3.1 The Fuzzy Graph $G(V, \sigma, \mu)$

$\sigma = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$, the two element subset of $\sigma$ are $H_1 = \{\sigma_1, \sigma_2\}, H_2 = \{\sigma_1, \sigma_3\}, H_3 = \{\sigma_1, \sigma_4\}$, $H_4 = \{\sigma_2, \sigma_3\}$, $H_5 = \{\sigma_2, \sigma_4\}$

The modified representation of $\sigma - H_1$ with respect to $H_1$ are as follows:

$$\sigma_3/H_1 = (\mu_\infty(u_3, u_1), \mu_\infty(u_3, u_2)) = (1, .8)$$
$$\sigma_4/H_1 = (\mu_\infty(u_4, u_1), \mu_\infty(u_4, u_2)) = (1, .8)$$

these representations are not distinct. Therefore $H_1$ is not a modified fuzzy resolving set.

Now, $\sigma_2/H_3 = (\mu_\infty(u_2, u_1), \mu_\infty(u_2, u_4)) = (.8, .8)$

$$\sigma_3/H_3 = (\mu_\infty(u_3, u_1), \mu_\infty(u_3, u_4)) = (1, .8)$$

These representations are distinct, hence $H_3$ is the modified fuzzy resolving set of $G$. Similary we can see that $H_4, H_5$ and $H_6$ are the modified fuzzy resolving set of $G$. Hence $Fr_m(G) = 2$.

**Theorem 3.1** For a fuzzy graph $G$ with no fuzzy bridge, there does not exist an end vertex.

**Proof:** Let $G$ be a fuzzy graph with no fuzzy bridge. If there exist a end vertex say $u$, let $v$ is the adjacent vertex of $u$ in $G$. Let $G'$ is a fuzzy graph obtained by the removal of the edge $uv$ then $\mu'^{\infty}(u,v) < \mu^{\infty}(u,v)$. Since $\mu'^{\infty}(u,v) = 0$ and $u$ is the isolated vertex of $G'$. Which will imply that $uv$ is a fuzzy bridge. Which is a contradiction to our assumption that $G$ has no fuzzy bridge. Hence $G$ does not contain an end vertex.

**Theorem 3.2** For a fuzzy graph $G$ with no fuzzy bridge, $\mu$ is a constant function.

**Proof:** Let $G$ be a fuzzy graph with no fuzzy bridge. Then $G$ does not contain an end vertex by theorem 3.1. Therefore every vertex $v$ of $G$ must lie in a cycle. Now let us assume that $\mu$ is not a constant function then there exist at least one edge $vw$ in a fuzzy cycle of $G$ with $\mu(vw)$ is the maximum edge membership value of that cycle. then the removal of the edge $vw$ will reduce the strength of connectedness between $v$ and $w$. that is $\mu'^{\infty}(v,w) < \mu^{\infty}(v,w)$, hence $vw$ is a fuzzy bridge. Which is a contradiction to our assumption that $G$ has no fuzzy bridge. Therefore $\mu$ is a constant function.

**Theorem 3.3** Let $G$ be a fuzzy graph with no fuzzy bridge, then the modified fuzzy resolving number $Fr_m(G) = n - 1$.

**Proof:** If $G$ is a fuzzy graph with no fuzzy bridge then $\mu$ is a constant function by theorem 3.1. Therefore for any subset $H$ of $\sigma$ with $|H| < n - 1$, the modified representation of $\sigma - H$ with respect to $H$ are the same. Now, if $|H| = n - 1$ there will be only one element in $\sigma - H$. Therefore the modified representation of $\sigma - H$ with respect to $H$ is unique, which will imply that $Fr_m(G) = n - 1$

**Theorem 3.4** If the fuzzy graph $G$ is isomorphic to $G'$ then $Fr_m(G) = Fr_m(G')$.

**Proof:** Let $G(V,\sigma,\mu)$ be isomorphic to $G'(V',\sigma',\mu')$. Then there exist a bijective map $h: V \to V'$ such that $\sigma(v) = \sigma'(h(v)) \forall v \in V$ and $\mu(u,v) = \mu'(h(u),h(v)) \forall u,v \in V$ (1)

Let $V = \{v_1, v_2, \dots v_n\}$, $\sigma = \{(v_1,\sigma(v_1)),(v_2,\sigma(v_2)),\dots(v_n,\sigma(v_n))\} = \{\sigma_1,\sigma_2,\dots\sigma_n\}$

Assume that the modified resolving number of $G$ is $m$. And let the minimum resolving set be $K = \{\sigma_1,\sigma_2,\dots\sigma_m\}$ then the representation of $\sigma - K$ with respect to $H$ are distinct.

That is, $\sigma_{m+i}/K = (\mu_\infty(v_{m+i},v_1),\mu_\infty(v_{m+i},v_2),\dots\mu_\infty(v_{m+i},v_m))$ are distinct for $i = 1,2,\dots n-m$. (2)

Now, take $K' = \{\sigma_{1'},\sigma_{2'},\dots\sigma_{m'}\}$

$$\begin{aligned}\sigma_{m+i}'/K' &= (\mu_{\infty'}(h(v_{m+i}),h(v_1)),\mu_{\infty'}(h(v_{m+i}),h(v_2)),\dots\mu_{\infty'}(h(v_{m+i}),h(v_m)))\\ &= (\mu_\infty(v_{m+i},v_1),\mu_\infty(v_{m+i},v_2),\dots\mu_\infty(v_{m+i},v_m)) \quad \text{[by equation (1)]}\end{aligned}$$

Which are distinct for $i = 1,2,\dots n-m$ by equation (2). Therefore $K'$ is the modified resolving set of $G'$. Now to prove it is minimum modified resolving set of $G'$.

If there exist a resolving set $K_{1'}$ of $G'$, $K_{1'} = \{\sigma'_1,\sigma'_2,\dots\sigma'_{m_1}\} \ni |K'_1| = m_1 < |K'| = m$.

Then $\sigma'_{m_1+i}/I$ are distinct for $i = 1,2,\dots n-m_1$

Now let $K_1 = \{\sigma_1,\sigma_2,\dots\sigma_{m_1}\}$

$$\begin{aligned}\sigma_{m_1+i}/K_1 &= (\mu_\infty(v_{m_1+i},v_1),\mu_\infty(v_{m_1+i},v_2),\dots\mu_\infty(v_{m_1+i},v_{m_1}))\\ &= \left(\mu_{\infty'}\left(h\left(v_{m_1+i}\right),h(v_1)\right),\mu_{\infty'}\left(h\left(v_{m_2+i}\right),h(v_2)\right),\dots\mu_{\infty'}\left(h\left(v_{m_1+i}\right),h\left(v_{m_1}\right)\right)\right)\end{aligned}$$

[by equation (1)]

which are all distinct since $K_{1'}$ is a modified resolving set. Therefore $K_1$ is a resolving set of $G$, which is a contradiction to our assumption that the modified resolving number of $G$ is $m$ $[m_1 < m]$. Which implies that $K'$ is the minimum modified fuzzy resolving set of $G'$.

Hence, $Fr_m(G) = Fr_m(G') = m$.

**Corollary 3.1** If the fuzzy graph $G$ is co-weak isomorphic to $G'$ then $Fr_m(G) = Fr_m(G')$.

**Corollary 3.2** If $G$ is a self complementary fuzzy graph then $Fr_m(G) = Fr_m(\bar{G})$, where $\bar{G}$ is the complement of $G$.

**Theorem 3.5** If $G$ be a fuzzy labelling graph, then $Fr_m(G) = Sr_m(G)$.

**Proof:** Let $G(V_n, \sigma, \mu)$ be a fuzzy labelling graph and let $Fr_m(G) = k,\ k < n$. Assume that, $H$ is the corresponding minimum modified resolving set of $G$ then the modified representations of $\sigma - H$ with respect to $H$ are distinct. Now, $G$ is a fuzzy labeling graph there exist a one-one onto map from the set of all vertices to [0,1]. Therefore the modified representations of $H$ with respect to $H$ are distinct, since $\mu_\infty(v_i, v_i) = \sigma(v_i)$. That is, the modified representation of $\sigma$ with respect to $H$ are all distinct. Which implies that $H$ is also a modified fuzzy super resolving set of $G$ with minimum cardinality. Hence $Fr_m(G) = Sr_m(G) = k$.

**Theorem 3.6** let $G$ be a fuzzy labeling cycle with $2k$ vertices then there exist a modified fuzzy resolving set of cardinality k.

**Proof:** let $G(V, \sigma, \mu)$ be a fuzzy labeling cycle $v_1, v_2, \ldots v_{2k}, v_1$. Without loss of generosity let us assume that $v_1v_2$ is the strongest edge of the cycle. Let us take the subset $A = \{v_2, v_4, \ldots v_{2k}\}$ of $\sigma$. Now to prove the modified representation of $\sigma - A$ with respect to $A$ is distinct. Assume the contrary that if any two modified representation $\sigma_i/A$ and $\sigma_j/A$ are same for $\sigma_i, \sigma_j \in \sigma - A, i \neq j$ Then $(\mu_\infty(v_i, v_2), \mu_\infty(v_i, v_4), \ldots \mu_\infty(v_i, v_{2k})) = (\mu_\infty(v_j, v_2),$ $\mu_\infty(v_j, v_4), \ldots \mu_\infty(v_j, v_{2k}))$ that is $\mu_\infty(v_i, v_2) = \mu_\infty\left(v_j, v_2\right), \mu_\infty(v_i, v_4) = \mu_\infty\left(v_j, v_4\right),$ $\ldots \mu_\infty(v_i, v_{2k})) = \mu_\infty(v_j, v_{2k})$. Which is not possible since $G$ is a fuzzy labeling graph. Hence our assumption that any two modified representation $\sigma_i/A$ and $\sigma_j/A$ are the same is wrong, that is no two modified representation of $\sigma - A$ with respect to $A$ are same. Which implies that $A$ is a fuzzy resolving set of cardinality $2k$.

**Theorem 3.7** Every modified fuzzy super resolving set of the fuzzy graph $G$ is a modified fuzzy resolving set of $G$ but the converse is need not to be true.

**Proof:** Let $G(V, \sigma, \mu)$ is the fuzzy graph and let $A$ be the modified super resolving set of $G$. Then the modified representation of $\sigma$ with respect to $A$ are all distinct. Therefore the modified representation of $\sigma - A$ with respect to $A$ is also distinct. But if the modified representation of $\sigma - A$ with respect to $A$ is distinct then it does not imply that the modified representation of $\sigma$ with respect to $A$ are distinct. Hence Every modified fuzzy super resolving set of the fuzzy graph $G$ is a modified fuzzy resolving set of $G$ but the converse is need not to be true.

**Corollary 3.3** If $G(V, \sigma, \mu)$ be a fuzzy labeling four cycle then the modified fuzzy resolving number of $G$ is 2.

**Proof:** Let $G$ be a fuzzy labeling four cycle then by theorem 3.6 there exist a modified fuzzy resolving set of cardinality 2. Hence by the definition of modified fuzzy resolving set the modified resolving number of $G$ is 2.

## 4. CONCLUSION

The strength of connectedness of fuzzy graph and the resolving set of a fuzzy graph is a current research area in the fuzzy graph. This article explains the weakness of connectedness of fuzzy graph and the modified resolving set of the fuzzy graph which is different from the resolving set of the fuzzy graph. We have proved some properties of modified fuzzy resolving set of the fuzzy graph and fuzzy labeling graph. We would like to study more properties of a modified fuzzy resolving set in our future work.

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