Comparative Analysis of Fuzzy, Intuitionistic and Neutrosophic Assignment Problem Using Nonagonal Fuzzy Number

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Abstract: The assignment problem is a great topic and frequently used in engineering and management sciences. In this problem, $\tilde{c}_{ij}$ indicates the cost of assigning the ith person to a jth job. In this paper, $\tilde{c}_{ij}$ denotes fuzzy assignment problem, Intuitionistic assignment problem, and Neutrosophic assignment problem by using Nonagonal fuzzy numbers, which are more logical and standardly in nature. Here, we proposed three different algorithms for finding optimal solution to the assignment problem. Finally, comparing all the results with an illustrative example.

Keywords: Nonagonal fuzzy number, Fuzzy assignment problem, Intuitionistic assignment problem, Neutrosophic assignment problem.

1. INTRODUCTION

The assignment problem is one of the fundamental combinatorial optimization problems. The assignment problem is incredibly famous and frequently in engineering, management sciences, and industries. The problem is to find the optimum schedule so that the total cost is minimum, or the total profit is maximum. To find the solution for the assignment problem, we used so many methods such as the Hungarian method, linear programming, Neural network, genetic algorithm, etc.

In 1965, the concepts of fuzzy set theory and the degree of membership introduced by Prof. L.A. Zadeh. The fuzzy number defined as a fuzzy subset of the real line by D.Dubais and H.Prade. [3] A fuzzy number is a quantity whose values as precise, rather than exact, as in the case with single value. To deal imprecise in real-life situations, many researchers used triangular and trapezoidal fuzzy numbers. Most of the researchers have focused on hexagonal, octagonal, and decagonal fuzzy numbers. If the vagueness arises in nine different points, we restricted to use other fuzzy numbers.

In this situation, a Nonagonal fuzzy number can be used to clear the uncertainty and get a clear solution. Atanassov first introduced intuitionistic fuzzy sets as a generalization of fuzzy sets. Florentin Smarandache first introduced the Neutrosophic set in the year 1995. The idea of Neutrosophic set characterized by three independent membership degrees, namely truth (T), indeterminacy (I), and falsity (F), which are lie between nonstandard unit interval $]0, 1^+[$. We proposed a new ranking for Nonagonal fuzzy number, and it has been applied in the illustrative example.
2. PRELIMINARIES

Fuzzy Set [8]

A fuzzy set $A$ characterized by a membership function mapping element of a domain, space, or the universe of discourse $X$ to the unit interval $[0,1]$, i.e., $A = \{(x, \mu_A(x))/ x \in X\}$. Here $\mu_A(x): X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set $A$ and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set $A$. These membership grades often represented by real numbers ranging $[0,1]$.

Fuzzy Number [3]

A fuzzy set $A$ defined on the universal set of real line $R$ with membership function $\mu_A(x): R \rightarrow [0,1]$ is called a fuzzy number if

(i) $A$ is normal and convexity.
(ii) $A$ must be bounded.
(iii) $\mu_A(x)$ is piecewise continuous.

Nonagonal Fuzzy Number [6]

A Nonagonal fuzzy number $\tilde{A}_N$ denoted as $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$ and the membership function is defined as

$$
\mu_{\tilde{A}_N}(x) = \begin{cases}
\frac{1}{4} (x - a_1) & ; a_1 \leq x \leq a_2 \\
\frac{1}{4} (a_2 - a_1) & ; a_2 \leq x \leq a_3 \\
\frac{1}{4} (a_3 - a_2) & ; a_3 \leq x \leq a_4 \\
\frac{1}{3} (x - a_3) & ; a_4 \leq x \leq a_5 \\
\frac{1}{3} (a_4 - a_3) & ; a_5 \leq x \leq a_6 \\
\frac{1}{4} (x - a_5) & ; a_6 \leq x \leq a_7 \\
\frac{1}{4} (a_6 - a_5) & ; a_7 \leq x \leq a_8 \\
\frac{1}{2} (x - a_7) & ; a_8 \leq x \leq a_9 \\
\frac{1}{4} (a_9 - a_8) & ; a_9 \leq x \leq a_9 \\
0 & ; otherwise
\end{cases}
$$

Intuitionistic Fuzzy Set [1]

Let $X$ denote a universe of discourse, then the intuitionistic fuzzy set $A$ in $X$ is defined as an object of the form $\tilde{A}^I = \{(x, \mu_A^I(x), \vartheta_A^I(x)): x \in X\}$ where the functions $\mu_A^I: X \rightarrow [0,1]$, $\vartheta_A^I: X \rightarrow [0,1]$ define the degrees of membership and the degrees of non-membership of the element $x \in X$ to the set $\tilde{A}^I$ respectively and for every $x \in X$ in $\tilde{A}^I$, $0 \leq \mu_A^I(x) + \vartheta_A^I(x) \leq 1$.

Neutrosophic Set [2]

Let $X$ be a non-empty set. Then a Neutrosophic set $\tilde{A}^N$ of $X$ is defined as $\tilde{A}^N = \{(x: T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x), F_{\tilde{A}^N}(x)): x \in X, T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x), F_{\tilde{A}^N}(x) \in [0^-, 1^+]\}$, where $T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x), F_{\tilde{A}^N}(x)$ are truth, indeterminacy and falsity membership function and there is
no restriction on the sum of $T_{aN}(x), I_{aN}(x), F_{aN}(x)$, so $0^{-} \leq T_{aN}(x) + I_{aN}(x) + F_{aN}(x) \leq 3^*$ and $]0^-, 1^+[$ is a nonstandard unit interval.

3. THE MATHEMATICAL FORMULATION FOR ASSIGNMENT PROBLEM

Suppose there are $n$ jobs to be worked and $n$ persons need to be doing the tasks. It is also assumed that each person can perform each job at a time and thus depending on their performance to do the job. Let $\tilde{C}_{ij}$ be the cost of the $i^{th}$ person assigned to the $j^{th}$ job. The objective is to minimize the total cost of mapping all the tasks to the available persons (one job assigned to one person).

The assignment problem can be stated in the form of an $n \times n$ cost matrix $[\tilde{C}_{ij}]$ as given in the following table:

<table>
<thead>
<tr>
<th>Persons</th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>...</th>
<th>$J_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>$\tilde{c}_{11}$</td>
<td>$\tilde{c}_{12}$</td>
<td>$\tilde{c}_{13}$</td>
<td>...</td>
<td>$\tilde{c}_{1n}$</td>
</tr>
<tr>
<td>$P_2$</td>
<td>$\tilde{c}_{21}$</td>
<td>$\tilde{c}_{22}$</td>
<td>$\tilde{c}_{23}$</td>
<td>...</td>
<td>$\tilde{c}_{2n}$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>...</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$P_n$</td>
<td>$\tilde{c}_{n1}$</td>
<td>$\tilde{c}_{n2}$</td>
<td>$\tilde{c}_{n3}$</td>
<td>...</td>
<td>$\tilde{c}_{1n}$</td>
</tr>
</tbody>
</table>

Mathematically, Assignment problem can be stated as,

Minimize $\tilde{Z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{C}_{ij} x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = 1, \quad 1 \leq i \leq n$

$\sum_{i=1}^{n} x_{ij} = 1, \quad 1 \leq j \leq n$

Where $x_{ij} = \begin{cases} 1, & \text{if the } i^{th} \text{ person assigned with } j^{th} \text{ job} \\ 0, & \text{otherwise} \end{cases}$

It is the decision variable denoting the assignment of the $i^{th}$ person to the $j^{th}$ job. $\tilde{C}_{ij}$ is the cost of assigning the $j^{th}$ task to the $i^{th}$ person.

4. RANKING OF NONAGONAL FUZZY NUMBER [7]

The new ranking method for Nonagonal fuzzy number is given below:

$$R(\tilde{A}_N) = \left( \frac{f_1 + 3f_2 + 3f_3 + 5f_4 + 3f_5 + 5f_6 + 3f_7 + 3f_8 + f_9}{27} \right)$$

5. ALGORITHMS AND EXAMPLES FOR FUZZY, INTUITIONISTIC AND NEUTROSOPHIC ASSIGNMENT PROBLEM

An Algorithm for Solving Fuzzy Assignment Problem

Step 1: Test whether the given fuzzy cost matrix is a balanced one or not. Suppose it is a balanced one (i.e., no. of persons = no. of jobs); then go to step 2. Suppose it is not a balanced one (i.e., no. of persons ≠ no. of jobs), then add dummy rows or columns with zero costs to be balanced. Go to step 2.

Step 2: Case (i) Select the maximum fuzzy costs value for the $\tilde{C}_{ij}$ and then solve it by Hungarian method and allocate the optimal schedule.

Case (ii) Select the minimum fuzzy cost value for the $\tilde{C}_{ij}$ and then solve it by Hungarian method and allocate the optimal schedule.
Case (iii) Find the average fuzzy costs value for the $\tilde{C}_{ij}$ and then solve it by Hungarian method and allocate the optimal schedule.

- Numerical Example

Let us consider a fuzzy assignment problem having four projects to the four researchers. The cost matrix is a Nonagonal fuzzy number denoting time for completing the $i^{th}$ projects to the $j^{th}$ researchers. It is required to find the optimal assignment of projects allotted to the researchers.

The elements of the fuzzy cost matrix are as follows:

\[
\begin{align*}
    a_{11} &= (3,5,7,11,13,15,17,19,21) \\
    a_{12} &= (11,13,15,17,19,21,23,25,27) \\
    a_{13} &= (11,13,15,17,19,21,23,25,27) \\
    a_{14} &= (4,8,12,16,20,24,28,32,36) \\
    a_{21} &= (15,16,17,18,19,20,21,22,23) \\
    a_{22} &= (15,17,19,21,23,27,31,35,37) \\
    a_{23} &= (9,10,11,12,13,14,15,16,17) \\
    a_{24} &= (17,18,19,20,21,22,23,24,25)
\end{align*}
\]

Case I: Maximization Method

Select the Maximum fuzzy costs for the above matrix are:

<table>
<thead>
<tr>
<th>Projects</th>
<th>Researchers</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>21</td>
<td>27</td>
<td>d27</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>23</td>
<td>37</td>
<td>17</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>R3</td>
<td>23</td>
<td>19</td>
<td>20</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>R4</td>
<td>36</td>
<td>17</td>
<td>23</td>
<td>21</td>
<td></td>
</tr>
</tbody>
</table>

The fuzzy optimum schedule is

<table>
<thead>
<tr>
<th>Projects</th>
<th>Researchers</th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>[0]</td>
<td>6</td>
<td>8</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>6</td>
<td>20</td>
<td>[0]</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>R3</td>
<td>4</td>
<td>[0]</td>
<td>1</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>R4</td>
<td>19</td>
<td>0</td>
<td>6</td>
<td>[0]</td>
<td></td>
</tr>
</tbody>
</table>

The minimum optimal fuzzy costs are $C_{11} + C_{23} + C_{32} + C_{44} = 21+17+19+21 = 78$

Case II: Minimization Method

Select the Minimum fuzzy costs for the above matrix are:

<table>
<thead>
<tr>
<th>Projects</th>
<th>Researchers</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>3</td>
<td>11</td>
<td>11</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>15</td>
<td>15</td>
<td>9</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>R3</td>
<td>7</td>
<td>11</td>
<td>12</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>R4</td>
<td>4</td>
<td>9</td>
<td>7</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
The fuzzy optimum schedule is

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Projects</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>[0]</td>
<td>4</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>R2</td>
<td>6</td>
<td>2</td>
<td>[0]</td>
<td>8</td>
</tr>
<tr>
<td>R3</td>
<td>0</td>
<td>[0]</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>R4</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>[0]</td>
</tr>
</tbody>
</table>

The minimum optimal fuzzy costs are $C_{11} + C_{23} + C_{32} + C_{44} = 3+9+11+3 = 26$

Case III: Average Method

To find the average of the Fuzzy cost matrix:

<table>
<thead>
<tr>
<th>Projects</th>
<th>Researchers</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>12.33</td>
<td>19</td>
<td>19</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>19</td>
<td>25</td>
<td>13</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>R3</td>
<td>15</td>
<td>15</td>
<td>16</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>R4</td>
<td>20</td>
<td>13</td>
<td>15</td>
<td>12.33</td>
<td></td>
</tr>
</tbody>
</table>

The fuzzy optimum schedule is

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Projects</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>[0]</td>
<td>6.67</td>
<td>6.67</td>
<td>7.67</td>
</tr>
<tr>
<td>R2</td>
<td>6</td>
<td>12</td>
<td>[0]</td>
<td>8</td>
</tr>
<tr>
<td>R3</td>
<td>0</td>
<td>[0]</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>R4</td>
<td>7.67</td>
<td>0.67</td>
<td>2.67</td>
<td>[0]</td>
</tr>
</tbody>
</table>

The minimum optimal fuzzy costs are $C_{11} + C_{23} + C_{32} + C_{44} = 12.33+13+15+12.33 = 52.63$

An Algorithm for Solving the Intuitionistic Assignment Problem

**Step 1:** Test whether the given fuzzy Intuitionistic cost matrix is a balanced one or not. Suppose it is a balanced one (i.e., no. of persons = no. of jobs); then go to step 2. Suppose it is not a balanced one (i.e., no. of persons ≠ no. of jobs), then add dummy rows or columns with zero Intuitionistic costs to be a balanced one. Go to step 2.

**Step 2:** Find the rank of each cell $\tilde{C}_{ij}$ of the chosen Intuitionistic fuzzy cost matrix by using the given ranking formula whose elements are Nonagonal fuzzy numbers. After that, we get the Intuitionistic fuzzy cost matrix (i.e., each element consists of membership function and non-membership function)

**Step 3:** **Case (i)** Select the maximum intuitionistic costs value for the $\tilde{C}_{ij}$ and then solve it by Hungarian method and allocate the optimal schedule.

**Case (ii)** Select the minimum intuitionistic costs to value for the $\tilde{C}_{ij}$ and then solve it by Hungarian method and allocate the optimal schedule.
Case (iii) Find the average intuitionistic cost value for the $\tilde{c}_{ij}$ and then solve it by Hungarian method and allocate the optimal schedule.

- Numerical Example

Let us consider an Intuitionistic assignment problem having four Banks to the four Employees. The cost matrix is a Nonagonal fuzzy number denoting time for completing the $i^{th}$ Banks to the $j^{th}$ Employees. It is required to find the optimal assignment of Banks is assigned to the Employees.

The elements of the fuzzy cost matrix are as follows:

\[
\begin{align*}
    a_{11} &= ((3,4,5,6,7,8,9,10,11); (3,5,7,11,13,15,17,19,21)) \\
    a_{12} &= ((2,4,6,8,10,12,14,16),(18;2,3,4,5,6,7,8,9,10)) \\
    a_{13} &= ((11,12,13,14,15,16,17,18,19); (11,13,15,17,19,21,23,25,27)) \\
    a_{14} &= ((4,8,12,16,20,24,28,32,36); (12,13,14,15,16,17,18,19,20)) \\
    a_{21} &= ((15,16,17,18,19,20,21,22,23); (6,7,8,9,10,11,12,13,14)) \\
    a_{22} &= ((15,17,19,21,23,27,31,35,37); (9,10,11,12,13,14,15,16,17)) \\
    a_{23} &= ((6,7,8,9,10,11,12,13,14); (9,10,11,12,13,14,15,16,17)) \\
    a_{24} &= ((17,18,19,20,21,22,23,24,25); (12,13,14,15,16,17,18,19,20)) \\
    a_{31} &= ((5,6,7,8,9,10,11,12,13); (7,9,11,13,15,17,19,21,23)) \\
    a_{32} &= ((2,4,6,8,10,12,14,16,18); (2,3,4,5,6,7,8,9,10)) \\
    a_{33} &= ((7,8,9,10,11,12,13,14,15); (12,13,14,15,16,17,18,19,20)) \\
    a_{34} &= ((2,5,7,9,11,13,15,17,19); (15,17,19,21,23,27,31,35,57)) \\
    a_{41} &= ((4,8,12,16,20,24,28,32,36); (12,13,14,15,16,17,18,19,20)) \\
    a_{42} &= ((6,7,8,9,10,11,12,13,14); (9,10,11,12,13,14,15,16,17)) \\
    a_{43} &= ((5,6,7,8,9,10,11,12,13); (7,9,11,13,15,17,19,21,23)) \\
    a_{44} &= ((3,4,5,6,7,8,9,10,11); (3,5,7,11,13,15,17,19,21)) 
\end{align*}
\]

The ranking of the Nonagonal fuzzy number is given below:

<table>
<thead>
<tr>
<th>Employees</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>(7,12.48)</td>
<td>(10,6)</td>
<td>(15,19)</td>
<td>(20,16)</td>
</tr>
<tr>
<td>E2</td>
<td>(19,10)</td>
<td>(24.7,13)</td>
<td>(10,13)</td>
<td>(21,16)</td>
</tr>
<tr>
<td>E3</td>
<td>(9,15)</td>
<td>(10,6)</td>
<td>(11,13)</td>
<td>(10.96,24.7)</td>
</tr>
<tr>
<td>E4</td>
<td>(20,16)</td>
<td>(10,13)</td>
<td>(9,15)</td>
<td>(7,12.48)</td>
</tr>
</tbody>
</table>

Case I: Maximization Method

Select the Maximum Intuitionistic costs for the above matrix are:

<table>
<thead>
<tr>
<th>Employees</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>12.48</td>
<td>10</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>E2</td>
<td>19</td>
<td>24.7</td>
<td>13</td>
<td>21</td>
</tr>
<tr>
<td>E3</td>
<td>15</td>
<td>10</td>
<td>13</td>
<td>24.7</td>
</tr>
<tr>
<td>E4</td>
<td>20</td>
<td>13</td>
<td>15</td>
<td>12.48</td>
</tr>
</tbody>
</table>

The Intuitionistic optimum schedule is

<table>
<thead>
<tr>
<th>Employees</th>
<th>B1</th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>[0]</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

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The minimum optimal fuzzy costs are $C_{11} + C_{23} + C_{32} + C_{44} = 12.48 + 13 + 10 + 12.48 = 47.96$

Case II: Minimization Method
Select the Minimum Intuitionistic costs matrix are:

<table>
<thead>
<tr>
<th></th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>7</td>
<td>6</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>B2</td>
<td>10</td>
<td>13</td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>B3</td>
<td>9</td>
<td>6</td>
<td>11</td>
<td>10.96</td>
</tr>
<tr>
<td>B4</td>
<td>16</td>
<td>10</td>
<td>9</td>
<td>7</td>
</tr>
</tbody>
</table>

The Intuitionistic optimum schedule is

<table>
<thead>
<tr>
<th></th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>[0]</td>
<td>0</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>B2</td>
<td>0</td>
<td>4</td>
<td>[0]</td>
<td>6</td>
</tr>
<tr>
<td>B3</td>
<td>2</td>
<td>[0]</td>
<td>4</td>
<td>3.96</td>
</tr>
<tr>
<td>B4</td>
<td>9</td>
<td>4</td>
<td>2</td>
<td>[0]</td>
</tr>
</tbody>
</table>

The minimum optimal fuzzy costs are $C_{11} + C_{23} + C_{32} + C_{44} = 7 + 10 + 6 + 7 = 30$

Case III: Average Method
To find the average of the Intuitionistic cost matrix:

<table>
<thead>
<tr>
<th></th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>9.74</td>
<td>8</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>B2</td>
<td>14.5</td>
<td>18.85</td>
<td>11.5</td>
<td>18.5</td>
</tr>
<tr>
<td>B3</td>
<td>12</td>
<td>8</td>
<td>12</td>
<td>17.83</td>
</tr>
<tr>
<td>B4</td>
<td>18</td>
<td>11.5</td>
<td>12</td>
<td>9.74</td>
</tr>
</tbody>
</table>

The Intuitionistic optimum schedule is

<table>
<thead>
<tr>
<th></th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>[0]</td>
<td>0</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>B2</td>
<td>1.26</td>
<td>7.35</td>
<td>[0]</td>
<td>7</td>
</tr>
<tr>
<td>B3</td>
<td>2.26</td>
<td>[0]</td>
<td>4</td>
<td>9.83</td>
</tr>
<tr>
<td>B4</td>
<td>2.52</td>
<td>1.76</td>
<td>2.26</td>
<td>[0]</td>
</tr>
</tbody>
</table>

The minimum optimal fuzzy costs are $C_{11} + C_{23} + C_{32} + C_{44} = 9.74 + 11.5 + 8 + 9.74 = 38.98$

An Algorithm for Solving the Neutrosophic Assignment Problem
Step 1: Test whether the given fuzzy Neutrosophic cost matrix is a balanced one or not. Suppose it is a balanced one (i.e., no. of persons = no. of jobs); then go to step 2. Suppose it is not a balanced one (i.e., no. of persons ≠ no. of jobs), then add dummy rows or columns with zero Neutrosophic costs to be a balanced one. Go to step 2.

Step 2: Find the rank of each cell $\tilde{C}_{ij}$ of the chosen Neutrosophic fuzzy cost matrix by using the given ranking formula whose elements are Nonagonal fuzzy numbers. After that, we get the Neutrosophic fuzzy cost matrix (i.e., each element consists of truth, indeterminacy and falsity membership function).

Step 3: Case (i) Select the maximum Neutrosophic costs value for the $\tilde{C}_{ij}$ and then solve it by Hungarian method and allocate the optimal schedule.

Case (ii) Select the minimum Neutrosophic costs value for the $\tilde{C}_{ij}$ and then solve it by Hungarian method and allocate the optimal schedule.

Case (iii) Find the average of the Neutrosophic cost value for the $\tilde{C}_{ij}$ and then solve it by the Hungarian method and allocate the optimal schedule.

Numerical Example
Let us consider a Neutrosophic assignment problem having four Houses to the four Tenants. The cost matrix is a Nonagonal fuzzy number denoting time for completing the $i^{th}$ Houses to the $j^{th}$ Tenants. It is required to find the optimal assignment of Houses issued to the Tenants.

The elements of the fuzzy cost matrix are as follows:

<table>
<thead>
<tr>
<th>Value</th>
<th>Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{11} = ((0,1,2,3,4,5,6,7,8); (0,2,4,6,8,10,12,14,16); (0,3,5,7,9,11,13,15,17))$</td>
<td></td>
</tr>
<tr>
<td>$a_{12} = ((1,2,3,4,5,6,7,8,9); (1,3,5,7,9,11,13,15,17); (3,5,7,9,11,13,15,17,19,21))$</td>
<td></td>
</tr>
<tr>
<td>$a_{13} = ((2,4,6,8,10,12,14,16,18); (2,3,4,5,6,7,8,9,10); (2,5,7,9,11,13,15,17,19))$</td>
<td></td>
</tr>
<tr>
<td>$a_{14} = ((3,4,5,6,7,8,9,10,11); (3,5,7,9,11,13,15,17,19,21); (3,6,9,12,15,18,21,24,27))$</td>
<td></td>
</tr>
<tr>
<td>$a_{21} = ((4,5,6,7,8,9,10,11,12); (4,6,8,10,12,14,16,18,20); (4,8,12,16,20,24,28,32,36))$</td>
<td></td>
</tr>
<tr>
<td>$a_{22} = ((5,6,7,8,9,10,11,12,13); (5,7,9,11,13,15,17,19,21); (6,7,8,9,10,11,12,13,14))$</td>
<td></td>
</tr>
<tr>
<td>$a_{23} = ((0,1,2,3,4,5,6,7,8); (1,2,3,4,5,6,7,8,9); (2,4,6,8,10,12,14,16,18))$</td>
<td></td>
</tr>
<tr>
<td>$a_{24} = ((6,7,8,9,10,11,12,13,14); (6,8,10,12,14,16,18,20,22); (7,8,9,10,11,12,13,14,15))$</td>
<td></td>
</tr>
<tr>
<td>$a_{31} = ((2,4,6,8,10,12,14,16,18); (2,3,4,5,6,7,8,9,10); (2,5,7,9,11,13,15,17,19))$</td>
<td></td>
</tr>
<tr>
<td>$a_{32} = ((7,8,9,10,11,12,13,14,15); (7,9,11,13,15,17,19,21,23); (8,9,10,11,12,13,14,15,16))$</td>
<td></td>
</tr>
<tr>
<td>$a_{33} = ((1,2,3,4,5,6,7,8,9); (1,3,5,7,9,11,13,15,17,19); (3,5,7,9,11,13,15,17,19,21))$</td>
<td></td>
</tr>
<tr>
<td>$a_{34} = ((8,9,10,11,12,13,14,15,16); (8,10,12,14,16,18,20,22,24); (9,10,11,12,13,14,15,16,17))$</td>
<td></td>
</tr>
<tr>
<td>$a_{41} = ((3,4,5,6,7,8,9,10,11); (3,5,7,9,11,13,15,17,19,21); (3,6,9,12,15,18,21,24,27))$</td>
<td></td>
</tr>
<tr>
<td>$a_{42} = ((0,1,2,3,4,5,6,7,8); (1,2,3,4,5,6,7,8,9); (2,4,6,8,10,12,14,16,18))$</td>
<td></td>
</tr>
<tr>
<td>$a_{43} = ((0,1,2,3,4,5,6,7,8); (0,2,4,6,8,10,12,14,16); (0,3,5,7,9,11,13,15,17))$</td>
<td></td>
</tr>
<tr>
<td>$a_{44} = ((4,5,6,7,8,9,10,11,12); (4,6,8,10,12,14,16,18,20); (4,8,12,16,20,24,28,32,36))$</td>
<td></td>
</tr>
</tbody>
</table>

The ranking of the Nonagonal fuzzy number is given below:

<table>
<thead>
<tr>
<th>Banks</th>
<th>Employees</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E1</td>
</tr>
<tr>
<td>B1</td>
<td>(4,8,8.96)</td>
</tr>
<tr>
<td>B2</td>
<td>(8,12,20)</td>
</tr>
<tr>
<td>B3</td>
<td>(10,6,10.96)</td>
</tr>
<tr>
<td>B4</td>
<td>(7,12,48,17.51)</td>
</tr>
</tbody>
</table>

Case I: Maximization Method
Select the Maximum Neutrosophic costs for the above matrix are:
The Neutrosophic optimum schedule is

<table>
<thead>
<tr>
<th>Employees</th>
<th>Banks</th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>8.96</td>
<td>12.48</td>
<td>10.96</td>
<td>17.51</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>20</td>
<td>13</td>
<td>10</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>10.96</td>
<td>15</td>
<td>12.48</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>B4</td>
<td>17.51</td>
<td>10</td>
<td>8.96</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

The minimum optimal fuzzy costs are $C_{11} + C_{23} + C_{34} + C_{42} = 8.96 + 10 + 16 + 10 = 44.96$

Case II: Minimization Method

Select the Minimum Neutrosophic costs matrix is:

<table>
<thead>
<tr>
<th>Employees</th>
<th>Banks</th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>[0]</td>
<td>1.44</td>
<td>0.96</td>
<td>3.51</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>11.04</td>
<td>1.96</td>
<td>[0]</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>0</td>
<td>1.96</td>
<td>0.48</td>
<td>[0]</td>
<td></td>
</tr>
<tr>
<td>B4</td>
<td>9.59</td>
<td>[0]</td>
<td>0</td>
<td>7.04</td>
<td></td>
</tr>
</tbody>
</table>

The Neutrosophic optimum schedule is

<table>
<thead>
<tr>
<th>Employees</th>
<th>Banks</th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>[0]</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>3</td>
<td>4</td>
<td>[0]</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>[0]</td>
<td>5</td>
<td>0</td>
<td>[0]</td>
<td></td>
</tr>
<tr>
<td>B4</td>
<td>3</td>
<td>[0]</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

The minimum optimal fuzzy costs are $C_{14} + C_{23} + C_{31} + C_{42} = 7 + 4 + 6 + 4 = 21$

Case III: Average Method

To find the average of the Neutrosophic cost matrix:

<table>
<thead>
<tr>
<th>Employees</th>
<th>Banks</th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>6.99</td>
<td>8.83</td>
<td>8.99</td>
<td>12.33</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>13.33</td>
<td>10.66</td>
<td>6.33</td>
<td>11.66</td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>8.99</td>
<td>12.66</td>
<td>8.83</td>
<td>13.66</td>
<td></td>
</tr>
<tr>
<td>B4</td>
<td>12.33</td>
<td>6.33</td>
<td>6.99</td>
<td>13.33</td>
<td></td>
</tr>
</tbody>
</table>

The Intuitionistic optimum schedule is
6. ANALYSIS OF ALL THE THREE ASSIGNMENT PROBLEM

<table>
<thead>
<tr>
<th>Employees</th>
<th>Cases</th>
<th>Optimum Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>Case I</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td>Case II</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Case III</td>
<td>52.33</td>
</tr>
<tr>
<td>B2</td>
<td>Case I</td>
<td>47.96</td>
</tr>
<tr>
<td></td>
<td>Case II</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Case III</td>
<td>38.98</td>
</tr>
<tr>
<td>B3</td>
<td>Case I</td>
<td>44.96</td>
</tr>
<tr>
<td></td>
<td>Case II</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>Case III</td>
<td>33.31</td>
</tr>
<tr>
<td>B4</td>
<td>Case I</td>
<td>33.31</td>
</tr>
</tbody>
</table>

7. CONCLUSION

In this paper, three different algorithms (Fuzzy, Intuitionistic, and Neutrosophic Assignment Problem) are given to solving an assignment problem with costs as a Nonagonal fuzzy number by using the proposed ranking method. Comparing all the three algorithms, Neutrosophic Assignment problem gives minimum cost than the Intuitionistic and Fuzzy Assignment problem.

8. REFERENCES


