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Stationary Flow Of A Viscous Fluid In A Flat Channel With Permeable Walls (In The Example Of Blood Circulation)

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Abstract. This article is devoted to the study of the stationary flow of a viscous fluid in a flat channel with permeable walls. As is known, in a pulsating flow in pipelines, a pressure wave propagates through its walls, which is called a pulse wave. This wave, as it moves away from the initial cross section, gradually weakens and practically attenuates at the ends. In the present work, stationary flows of a viscous fluid in a plane channel with permeable walls are considered to be sufficiently long for the channel. The problems are solved and calculation formulas for determining the longitudinal and transverse speeds are obtained. Formulas are also found and numerical calculations are made that determine the change in pressure and fluid flow in the direction of the main stream for various values of the permeability coefficient. It is shown that at lower values of the coefficient of permeability, the pressure is distributed along the flow according to linear laws, and the fluid flow rate remains almost constant along the channel length. Other values of the permeability coefficient, it is characteristic that with an increase in the permeability coefficient of the pressure distribution differs significantly from the linear distribution, with the maximum deviation in the middle of the channel. The fluid flow rate with an increase in the permeability coefficient increases several times in the initial section compared to the fluid flow rate, in the same pressure drop in the flow of a viscous fluid in a flat channel with impermeable walls.
Keywords: viscosity , stationarity , flat, permeability, hydraulic resistance, longitudinal speed , lateral speed, pressure, density , pressure.

1. INTRODUCTION.

Most works [1–7] focus on determining the propagation of a pulse pressure wave taking into account the elasticity of the pipe wall, and its permeability has not been taken into account anywhere. However, the permeability of the wall significantly affects the propagation of the pulse pressure wave and its attenuation [8-15]. In biological mechanics, this can be applied to the pulsating blood flow in the arterial bed. As you know, branching of the arterial tree is modeled by the introduction of “permeability” of the wall with volumetric outflows of lateral velocity to the walls. In fact, this speed is a discontinuous coordinate function; when using the model of a permeable tube, this function is always smoothed by the wall surface.

Apparently, the idea of modeling an artery in the form of a permeable tube was first expressed in [12] and further developed in [8,9]. The most complete study of such a model is contained in [10], where the dog's circulatory system is modeled. Unlike other works [7–9], to determine the hydraulic resistance in the central arterial vessel, in [10, 11] the formulation of the problem of a pulsating blood flow in the arterial bed was considered, where the blood outflow is mathematically modeled as a permeable vessel wall. It should be noted that in addition to wave flows, it is advisable to study the stationary flow of blood in a vessel with permeable walls. However, this case does not follow from the solution for wave flows by the limiting transition when the frequency tends to zero, therefore the stationary problem should be considered in a separate formulation.

Problem statement and solution methods.

Let us state the solution of the problem for the case of the flow of a viscous incompressible fluid in a flat channel with permeable walls**.** Denote the distance between the walls through $2h$, the length of the walls through L, and $h/L << 1$. The x axis extends in the middle of the channel along the stream. The y axis is directed perpendicular to the x axis. We believe that in both walls the permeability of the liquid has the same values, which preserves the symmetry of the flow. We estimate the terms of the Navier - Stokes equations in the Cartesian coordinate system for a symmetric flow in them:

$$x = Lx_1,\ y = hy_1, u = Uu_1,\ \upsilon = V\upsilon_1,\ p = \frac{\mu UL}{h^2}p_1, \mathrm{Re} = \frac{Uh}{\nu}$$

where $x_1, y_1, \upsilon_1,, p_1, u_1, -$is the dimensionless quantity. U, V - characteristic longitudinal and transverse speeds, with $\frac{V}{U} = \frac{h}{L} = \delta$; and $\mathrm{Re}-$ Reynolds numbers are small quantities. Then the system of Navier - Stokes equations will take the form

$$\begin{cases} \mathrm{Re}\,\delta\left(u_1\frac{\partial u_1}{\partial x_1} + \upsilon_1\frac{\partial u_1}{\partial y_1}\right) = -\frac{\partial p_1}{\partial x_1} + \delta^2\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial y_1^2} \\ \mathrm{Re}\,\delta^3\left(\upsilon_1\frac{\partial \upsilon_1}{\partial x_1} + \upsilon_1\frac{\partial \upsilon_1}{\partial y_1}\right) = -\frac{\partial p_1}{\partial y_1} + \delta^2\left(\delta^2\frac{\partial^2 \upsilon_1}{\partial x_1^2} + \frac{\partial^2 \upsilon_1}{\partial y_1^2}\right) \end{cases} \quad (1)$$

Neglecting the terms containing $\delta^2, \delta^3, \delta^4$ and the Reynolds number, then returning to dimensional variables, we obtain

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \nu\frac{\partial^2 u}{\partial y^2},\quad \frac{\partial p}{\partial y} = 0,\ \frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \quad (2)$$

In this case, the boundary conditions will be as follows:

$$\vartheta = \frac{\dot{\gamma} h}{\mu}(p - p_c),\quad \mathrm{u} = 0 \ \text{ at } y = h,\quad \frac{\partial u}{\partial y} = 0,\ \upsilon = 0 \ \text{ at } y = 0, \quad (3)$$

$$p = p_0 \ \text{ at } x = 0,\quad p = p_L \ \text{ at } x = L,$$

where $\overline{p} = p - p_c$, u, υ - longitudinal and transverse speeds; ρ - fluid density; ν - kinematic viscosity of the liquid; p – internal pressure; p_c – environmental pressure; p_0, p_L - fluid pressure of the initial and final sections; μ – dynamic viscosity; γ^* – permeability coefficient.

The solution is the first equation of system (2) taking into account the boundary conditions (3) and provided that $\frac{\partial p}{\partial y} = 0$ has the form

$$u(x, y) = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)\left(h^2 - y^2\right), \qquad (4)$$

Using the third equation of system (2), we find

$$\upsilon(x, y) = \frac{1}{2\mu}\left(-\frac{\partial^2 p}{\partial x^2}\right)\left(h^2 y - \frac{y^3}{3}\right), \qquad (5)$$

Now, we find the volumetric flow rate of the fluid.

$$Q = \int_{-h}^{h} u(x, y), \quad Q = \frac{2}{3\mu}\left(-\frac{\partial p}{\partial x}\right)h^3. \qquad (6)$$

Denoting by $z = \frac{3\mu}{2h^3}$, we have

$$-zQ = \frac{\partial \overline{p}}{\partial x} \qquad (7)$$

in the third equation of system (2), we integrate both sides from $-h$ to h with respect to variable y, and taking into account the boundary conditions (3), we obtain

$$\frac{dQ}{dx} = -\frac{2h\gamma^*}{\mu}\left(p - p_c\right) \qquad (8)$$

From (7) and (8) we can write the following equations for determining the pressure and flow rate of a liquid

$$\frac{d^2 \overline{p}}{dx^2} - z^* k^* \overline{p} = 0, \quad \frac{d^2 Q}{dx^2} - z^* k^* Q = 0, \qquad (9)$$

Solving the first equation (9) first, we find formulas for determining the fluid pressure

$$\overline{p}(x) = A_1 e^{\sqrt{kz}x} + A_2 e^{-\sqrt{kz}x}, \qquad (10)$$

Where $k = \frac{2h\gamma^*}{\mu}, z = \frac{3\mu}{2h^3}$

Then, solving the second equation (9), we find the change in the volumetric flow rate of the liquid

$$Q(x) = B_1 e^{\sqrt{kz}x} + B_2 e^{-\sqrt{kz}x} \qquad (11)$$

where A_1, A_2, B_1, B_2, -are constant integration coefficients. After applying equation (7) in (10) and (11), we can determine the relationship between these coefficients.

$$-zB_1 e^{\sqrt{kz}x} - zB_2 e^{-\sqrt{kz}x} = \sqrt{kz} A_1 e^{\sqrt{kz}x} - \sqrt{kz} A_2 e^{-\sqrt{kz}x} \qquad (12)$$

The following flows from here

$$-zB_1 = A_1\sqrt{kz}, \quad -zB_2 = -A_2\sqrt{kz}, \quad B_1 = -\sqrt{\frac{k}{z}} A_1, \; B_2 = \sqrt{\frac{k}{z}} A_2$$

Then

$$\bar{p}(x) = A_1 e^{\sqrt{kz}x} + A_2 e^{-\sqrt{kz}x} \qquad (13)$$

$$Q(x) = -\sqrt{\frac{k}{z}}\left(A_1 e^{\sqrt{kz}x} - A_2 e^{-\sqrt{kz}x}\right) \qquad (14)$$

Thus, the pressure and flow rate are interconnected with constant coefficients A_1 and A_2, these coefficients, we determine from the boundary conditions (3), when

$\bar{p} = p_0$ at $x = 0$, $\bar{p} = p_L$ at $x = L$,

Then

$$A_1 = -\frac{p_0 e^{-\sqrt{kz}L} - p_L}{e^{\sqrt{kz}L} - e^{-\sqrt{kz}L}}, \quad A_2 = -\frac{p_0 e^{\sqrt{kz}L} - p_L}{e^{\sqrt{kz}L} - e^{-\sqrt{kz}L}} \qquad (15)$$

The pressure distribution of the longitudinal coordinate is determined from formula (13), substituting their value instead of A_1 and A_2, we find

$$\bar{p}(x) = \frac{p_0 e^{-\sqrt{kz}L} - p_L}{e^{\sqrt{kz}L} - e^{-\sqrt{kz}L}} e^{\sqrt{kz}x} + \frac{p_0 e^{\sqrt{kz}L} - p_L}{e^{\sqrt{kz}L} - e^{-\sqrt{kz}L}} e^{-\sqrt{kz}x} \qquad (16)$$

The fluid flow rate is determined by the formula

$$Q(x) = -\sqrt{\frac{k}{z}}\left(\frac{p_0 e^{-\sqrt{kz}L} - p_L}{e^{\sqrt{kz}L} - e^{-\sqrt{kz}L}} e^{\sqrt{kz}x} - \frac{p_0 e^{\sqrt{kz}L} - p_L}{e^{\sqrt{kz}L} - e^{-\sqrt{kz}L}} e^{-\sqrt{kz}x}\right) \qquad (17)$$

In the limiting case, when $L \to \infty$ flows of a viscous fluid pass to the flow of a semi-infinite flat channel. In this case, the flow rate and pressure distribution along the flow decreases exponentially

$$Q(x) = \sqrt{\frac{k}{z}} p_0 e^{-\sqrt{kz}x}, \; p(x) = p_0 e^{-\sqrt{kz}x} \qquad (18)$$

Now, using formulas (14), we can find expressions for other parameters, such as the velocity of the longitudinal and pepper directions

$$u(x,y) = -\frac{\sqrt{kz}}{2\mu}\left(\frac{p_0 e^{-\sqrt{kz}L} - p_L}{e^{\sqrt{kz}L} - e^{-\sqrt{kz}L}} e^{\sqrt{kz}x} - \frac{p_0 e^{\sqrt{kz}L} - p_L}{e^{\sqrt{kz}L} - e^{-\sqrt{kz}L}} e^{-\sqrt{kz}x}\right)\left(h^2 - y^2\right),$$

$$\upsilon(x,y) = -\frac{kz}{2\mu}\left(\frac{p_0 e^{-\sqrt{kz}L} - p_L}{e^{\sqrt{kz}L} - e^{-\sqrt{kz}L}} e^{\sqrt{kz}x} + \frac{p_0 e^{\sqrt{kz}L} - p_L}{e^{\sqrt{kz}L} - e^{-\sqrt{kz}L}} e^{-\sqrt{kz}x}\right)\left(h^2 y - \frac{y^3}{3}\right) \quad (19)$$

In order to carry out numerical calculations and for convenience, we transform the exponential functions containing formulas (16) and (17) to a hyperbolic function. Then formulas (16) and (17) have the following form

$$\bar{p}(x) = \frac{p_0 sh\left(\sqrt{kz}(L-x)\right)}{sh\left(\sqrt{kz}L\right)} + \frac{p_L sh\left(\sqrt{kz}x\right)}{sh\left(\sqrt{kz}L\right)} \quad (20)$$

$$Q(x) = \sqrt{\frac{k}{z}}\left(\frac{p_0 ch\left(\sqrt{kz}(L-x)\right)}{sh\left(\sqrt{kz}L\right)} - \frac{p_L ch\left(\sqrt{kz}x\right)}{sh\left(\sqrt{kz}L\right)}\right) \quad (21)$$

Instead of z and k, setting their values in formulas (20) and (21), we obtain calculation formulas for the distribution of pressure and fluid flow along the flow in the form

$$\frac{\bar{p}(x)}{p_0} = \frac{sh\left(\sqrt{3\gamma^*}\frac{(L-x)}{h}\right)}{sh\left(\sqrt{3\gamma^*}\frac{L}{h}\right)} + \frac{p_L}{p_0}\frac{sh\left(\sqrt{3\gamma^*}\frac{x}{h}\right)}{sh\left(\sqrt{3\gamma^*}\frac{L}{h}\right)} \quad (22)$$

$$\frac{Q(x)}{Q_0} = \frac{1}{1-\frac{p_L}{p_0}}\frac{L}{h}\sqrt{3\gamma^*}\left(\frac{ch\left(\sqrt{3\gamma^*}\frac{(L-x)}{h}\right)}{sh\left(\sqrt{3\gamma^*}\frac{L}{h}\right)} + \frac{p_L}{p_0}\frac{ch\left(\sqrt{3\gamma^*}\frac{x}{h}\right)}{sh\left(\sqrt{3\gamma^*}\frac{L}{h}\right)}\right) \quad (23)$$

$$Q_0 = \frac{2h^3}{3\mu}\left(\frac{p_0 - p_L}{L}\right)$$

Where 1 is the given pressure in the initial section, 2 is the flow rate of the Poiseuille fluid.

2. CALCULATION RESULTS

Based on formulas (22) and (23), we will carry out numerical calculations that determine the change in pressure and fluid flow in the direction of the main flow for various values of the permeability coefficient.

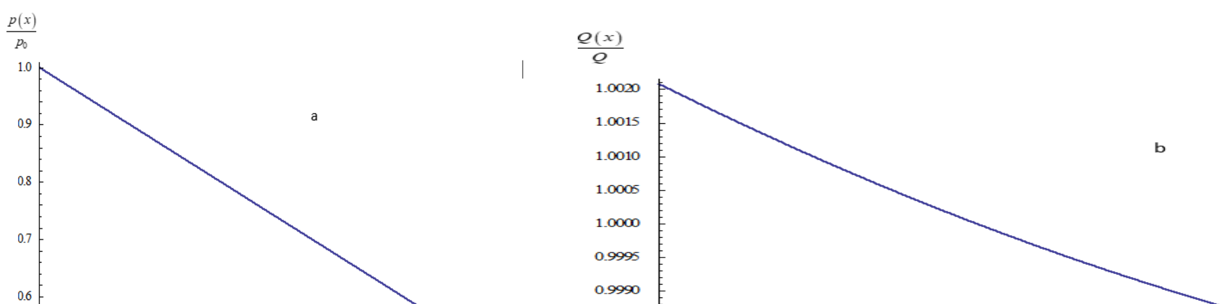

Fig. 1. Change in pressure (a) and fluid flow (b) by the direction of the main stream, depending on lower values of the permeability coefficient

$$\left(\frac{L}{h}=5\ ,\ \frac{p_L}{p_0}=0{,}5\ ,\ \gamma^*=0{,}0001\right)$$

In fig. 1, the change in pressure (a) and fluid flow (b) is shown by the direction of the main flow at lower values of the permeability coefficient. From this figure it can be seen that the pressure is distributed along the flow according to linear laws, and the fluid flow rate remains almost constant along the channel length

This proves that by passing to the limit, when $\gamma^* \to 0$ from formulas (22) and (23), we obtain formulas for the pressure and flow rate of the fluid for the Poiseuille flow

$$\lim_{\gamma^* \to 0} \frac{p(x)}{p_0} = 1 - \left(\frac{1 - \dfrac{p_L}{p_0}}{L} \right) x, \quad \text{or} \quad \lim_{\gamma^* \to 0} p(x) = p_0 - \frac{p_0 - p_L}{L} x.$$

$$\lim_{\gamma^* \to 0} \frac{Q(x)}{Q_0} = 1, \quad \text{or} \quad \lim_{\gamma^* \to 0} Q(x) = Q_0. \quad \text{where} \quad Q_0 = \frac{3h^3}{2\mu}\left(\frac{p_0 - p_L}{L}\right)$$

Thus, when $\gamma^* \to 0$, the stationary flow of a viscous fluid in a flat channel with permeable walls, goes to the flow in the same channel with impermeable walls. This proves the reliability of the obtained solutions for pressure and fluid flow.

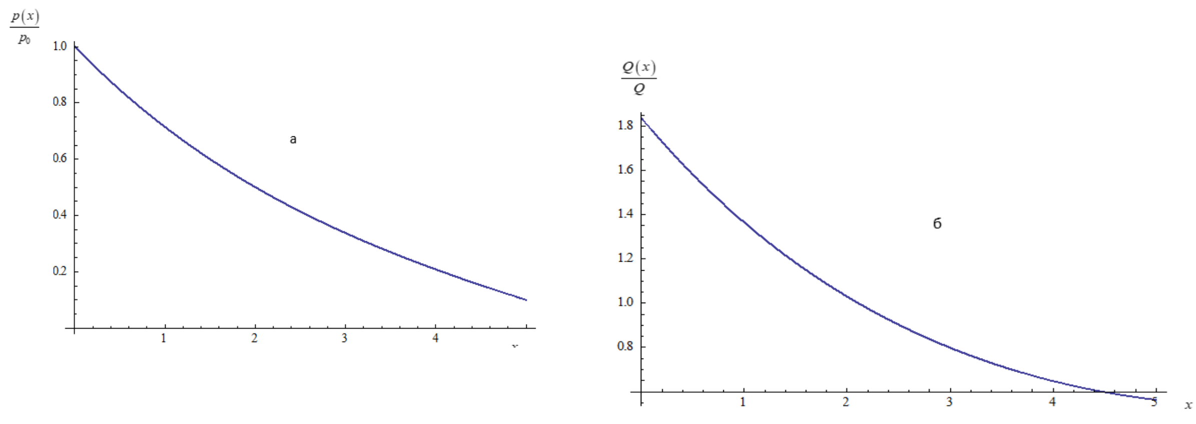

Fig. 2. Change in pressure (a) and fluid flow (b) by the direction of the main flow depending on the permeability coefficient ($\frac{L}{h}=5,\ \frac{p_L}{p_0}=0{,}1\ ,\gamma^*=0{,}1$)

Fig. 3. Change in pressure (a) and fluid flow (b) by the direction of the main flow depending on the permeability coefficient ($\frac{L}{h}=5$, $\frac{p_L}{p_0}=0{,}1$, $\gamma^*=0{,}3$)

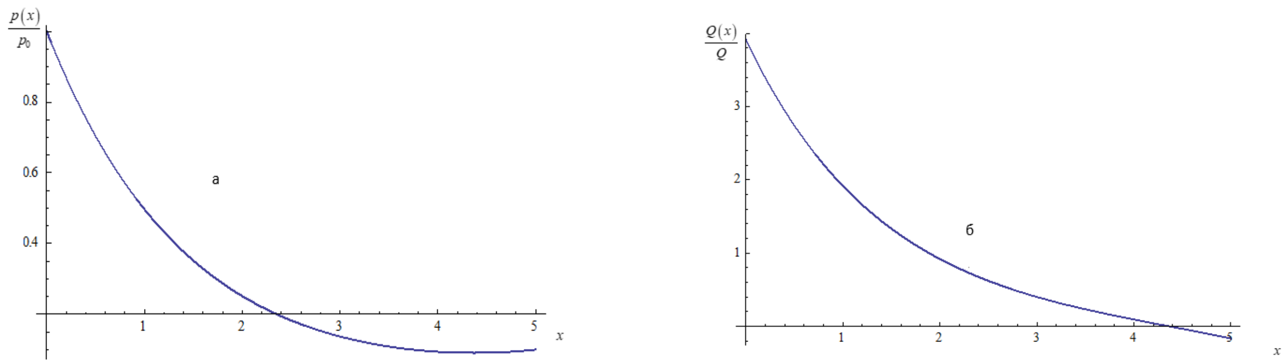

Fig. 4. Change in pressure (a) and fluid flow (b) by the direction of the main flow depending on the permeability coefficient ($\frac{L}{h}=5$, $\frac{p_L}{p_0}=0{,}1$, $\gamma^*=0{,}5$)

At considerable other values of the permeability coefficient, the change in pressure and fluid flow along the length of the flat channel is illustrated in Fig. 2., fig. 3., fig. 4.

3. CONCLUSION

When solving the problem of viscous fluid flow in a flat channel with permeable walls, calculation formulas were obtained and numerical calculations were made that determine the change in pressure and fluid flow in the direction of the main flow for various values of the permeability coefficient. It is shown that at lower values of the coefficient of permeability, the pressure is distributed along the flow according to linear laws, and the fluid flow rate remains almost constant along the channel length. Since, in this case, the flow of a viscous fluid passes to a stationary flow in a flat channel with impermeable walls. Other values of the permeability coefficient, it is characteristic that with an increase in the permeability coefficient of the pressure distribution differs significantly from the linear distribution, with the maximum deviation in the middle of the channel. The fluid flow rate with an increase in the permeability coefficient increases several times in the initial section compared to the fluid flow rate, in the same pressure drop in the flow of a viscous fluid in a flat channel with impermeable walls. In this case, when $\gamma^*=0{,}01;0{,}1;0{,}2;$ is accompanied by an increase in flow rate in the initial section about 3-4 times. The hydraulic resistance in this one-dimensional theory of the motion of a viscous fluid in a flat channel with permeable walls does not depend on the permeability coefficient. In order to determine the change in hydraulic resistance depending on the permeability coefficient, it is necessary to solve a more complex problem in a two-dimensional formulation.

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