MATHEMATIC MODELING OF PULSATION MOVEMENT OF BLOOD IN LARGE ARTERIES

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Abstract: Pulsation motion of Newton liquid in flat channels has begun to be investigated as a result of the development of the liquid mechanics. The function of the pressure gradient due to time is given on mathematic modeling of pulsation motion of liquid and as a result, the rest of dimension mill be defined in relating to time. While investigating the process of Navier-Stokes, it is made lazier incessant equation according to the usage of their replacement of the tasks. For this, the terms, which are given in the systems of the equations are transformed into new dimensions and the value of the terms, which are contrasted with each other is defined in the system of equation. The terms, which have acquired boundless small quantity is omitted without counting. This article discusses the pulsating fluid flows in the flat tube. Here it is determined the dependency of the speed of the pulsating movement of fluid on the vibration parameter.
Keywords: newton liquids, continuity, vibration parameter, pulsing movements.

INTRODUCTION

The problems associated with the movement of blood in large arteries have long attracted the attention of researchers. However, only in the last decades the theory in this area has been able to achieve significant progress, since it became possible to take into account the complex mechanical properties of the walls in the formulation of the problem, and when solving it, use modern means of mathematics and computers.

The crudest approach to the theoretical description of the pulsating blood flow in the arteries, which relies on the assumption that the blood is an incompressible viscous liquid with a constant property and moves laminar in an infinitely long cylindrical tube of circular cross-section with rigid walls under the action of a pressure gradient harmoniously varying with time.

Our mention one more possible approach to the theoretical analysis of pulsating flows in tubes with deforming walls. This approach is based on equations in which all inertial and some viscous terms are discarded (equations such as the theory of the lubricating layer). In this form, it can be directly applied, of course, only for small blood vessels, but its simplest refinement - introduction of averaged inertial terms - makes it possible to take into account all the main features of the pulsating flow with no less success than the other methods listed above.

The propagation of pulse pressure waves along elastic vessels is analyzed. Of great interest is the study of the pulsating flow of viscous fluid, in particular blood, in pipes of viscoelastic material. Pulsating fluid flows in pipes with various mechanical properties of the wall (rigid, elastic, elastically permeable) are considered. Some hydrodynamic regularities in the pulsating flow (the nature of the pressure change, the fluid flow) are given.

The task in the system of the difficult equation is simplified. Based on these, we look at pulsation motion of Newton fluid in the flat pipe, which has enough length.

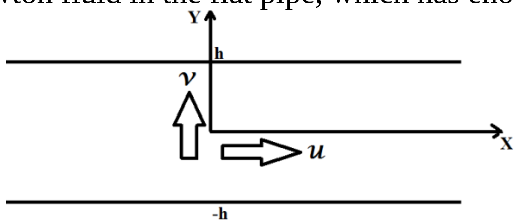

Figure 1. The motion of Newton liquid in a flat pipe

The plates of the flat pipe are located in the $2h$ distance, and the arrow Ox is lined from the middle part and the arrow Oy is crossed with it in a perpendicular. The flow of the liquid is considered as symmetrical to the arrow Oy and as a result, the flow is focused directly towards Ox and Oy arrows, but the flow towards the arrow Oz is not taken into consideration.

MATERIALS AND METHODS

Now the equation of motion of the liquid towards the investigated flow is expressed Descartes coordination system by the equations of Navier-Stokes and the incessant equation of the liquid. The dimensions are replaced as following:

$$\begin{cases} t = Tt_1, \quad x = Lx_1, \quad h = h_0 h_1 \\ u = U_0 u_1, \quad \upsilon = V_0 \upsilon_1, \quad p = \dfrac{\mu L U_0}{h_0^2} p_1 \\ \dfrac{V}{U_0} = \dfrac{h_0}{L} = \varepsilon \end{cases} \tag{1}$$

As a result, the following equation system, which is expressed by new equation system is formed:

$$\begin{cases} \alpha^2 \left[\dfrac{\partial u_1}{\partial t} + \varepsilon \left(u_1 \dfrac{\partial u_1}{\partial x_1} + \upsilon_1 \dfrac{\partial u_1}{\partial y_1} \right) \right] = -\dfrac{\partial p_1}{\partial x_1} + \left(\varepsilon^2 \dfrac{\partial^2 u_1}{\partial x^2} + \dfrac{\partial^2 u_1}{\partial y^2} \right) \\ \varepsilon^2 \alpha^2 \left[\dfrac{\partial \upsilon_1}{\partial t} + \varepsilon \left(u_1 \dfrac{\partial \upsilon_1}{\partial x_1} + \upsilon_1 \dfrac{\partial \upsilon_1}{\partial y_1} \right) \right] = -\dfrac{\partial p_1}{\partial y_1} + \varepsilon^2 \left(\dfrac{\partial^2 \upsilon_1}{\partial x^2} + \varepsilon^2 \dfrac{\partial^2 \upsilon_1}{\partial y^2} \right) \\ \dfrac{\partial u_1}{\partial x_1} + \dfrac{\partial \upsilon_1}{\partial y_1} = 0 \end{cases} \tag{2}$$

Here t – time, x and y – horizontal and vertical coordination, u_1 and v_1 – vertical and cross velocities, p_1 – pressure, $\alpha = \sqrt{\dfrac{\omega}{\nu}} R$ – the appropriate number of the frequency, given by Reynold $\varepsilon = \frac{h_0}{L} = \frac{V_0}{U_0} << 1$ – small quantities. Subsequently, we don't need to take into consideration the sign in the form (1) as we undertake only re currying dimensions.

RESULT AND DISCUSSION

Taking into account the symmetry of the flow, we look at the section $h \geq 0$ and considering that, we place frontier terms for horizontal and vertical velocity. The following approach will be reliable because of frontier terms in pipe center.

$$\begin{cases} y = h \ \ \partial a, \ \ \upsilon = 0, \ \ u = 0 \\ y = 0 \ \ \partial a, \ \ \frac{\partial u}{\partial y} = 0, \ \ \upsilon = 0 \end{cases} \tag{3}$$

By dropping the terms of small quantities, used in the equation, mentioned above, we will discover the following simple equation.

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \tag{4}$$

Here, the gradient of the pressure $\frac{\partial p}{\partial x}$ should consist of harmonic scales of time.

$$-\frac{\partial p}{\partial x} = A_0 + realAe^{i\omega t} = A_0 + A\cos\omega t \tag{5}$$

In that case, we try to find the solution to the equation like $u = u_0 + u_1 e^{i\omega t}$ and as a result the above mention equation will be this form.

$$\begin{cases} \frac{\partial^2 u_0}{\partial y^2} = \frac{1}{\mu} A_0 \\ \frac{\partial^2 u_1}{\partial y^2} - \frac{i\omega}{\nu} u_1 = \frac{1}{\mu} A \end{cases} \tag{6}$$

Here, the solution to the first equation will be related to Puazeyl sum and the second one, being in a trigonometric form, will be defined as follows:

$$\begin{cases} u_0 = \frac{A_0}{2\mu} h^2 (1 - \frac{y^2}{h^2}) \\ u_1 = \frac{Ah^2}{\mu} (\frac{1}{i\alpha_0^2} (1 - \frac{ch(\sqrt{i}\alpha_0 \frac{y}{h})}{ch(\sqrt{i}\alpha_0)})) \end{cases} \tag{7}$$

Using the first equality of the system (7), average velocity will be considered on the surface of the section.

$$Q = 2\int_0^h u_0 dy = 2\frac{1}{2\mu} A_0 \int_0^h h^2 (1 - \frac{y^2}{h^2}) dy = \frac{1}{\mu} A_0 h^2 (y - \frac{y^3}{3h^2})\Big|_0^h = \frac{2}{3\mu} A_0 h^3$$

(8)

$$U = \frac{Q}{2h} = \frac{\frac{2}{3\mu} A_0 h^3}{2h} = \frac{1}{3\mu} A_0 h^2 \tag{9}$$

And total saluting is mode by counting the quantities of u_0 and u_1

$$u = u_0 + u_1 e^{i\omega t} = \frac{A_0}{2\mu} h^2 \left[\left(\left(1 - \frac{y^2}{h^2}\right) + \frac{2A}{A_0} \left(\frac{1}{i\alpha_0^2} \left(1 - \frac{ch(\sqrt{i}\alpha_0 \frac{y}{h})}{ch(\sqrt{i}\alpha_0)}\right)\right) e^{i\omega t} \right) \right] \quad (10)$$

$$\frac{real(u)}{U} = \frac{3A}{A_0} real\left(\frac{1}{i\alpha_0^2}\left(1 - \frac{ch(\sqrt{i}\alpha_0 \frac{y}{h})}{ch(\sqrt{i}\alpha_0)}\right)\right) e^{i\omega t} \quad (11)$$

The final solution is made by dividing real part of the equality (11)

$$\frac{real(u)}{U} = \frac{3A}{A_0}\left(real\left(\frac{1}{i\alpha_0^2}\left(1 - \frac{ch(\sqrt{i}\alpha_0 \frac{y}{h})}{ch(\sqrt{i}\alpha_0)}\right)\right)(\cos\omega t + i\sin\omega t)\right) \quad (12)$$

$$\frac{real(u)}{U} = \frac{3A}{A_0}\left(\frac{M}{E}\sin\omega t + \frac{N}{E}\cos\omega t\right) \quad (13)$$

here coefficients such as $A, B, C, D, k_1, k_2, M, N, E$ are mode as the following.

$$\begin{cases} A = chk_1 \cos k_1 \\ B = shk_1 \sin k_1 \end{cases} \quad \begin{cases} C = chk_2 \cos k_2 \\ D = shk_2 \sin k_2 \end{cases} \quad \begin{cases} k_1 = \frac{\alpha_0}{\sqrt{2}} \frac{y}{h} \\ k_2 = \frac{\alpha_0}{\sqrt{2}} \end{cases} \quad (14)$$

$$\begin{cases} M = C(C - A) + D(D - B) \\ N = C(D - B) - D(C - A) \\ E = (C^2 + D^2)\alpha_0^2 \end{cases} \quad (15)$$

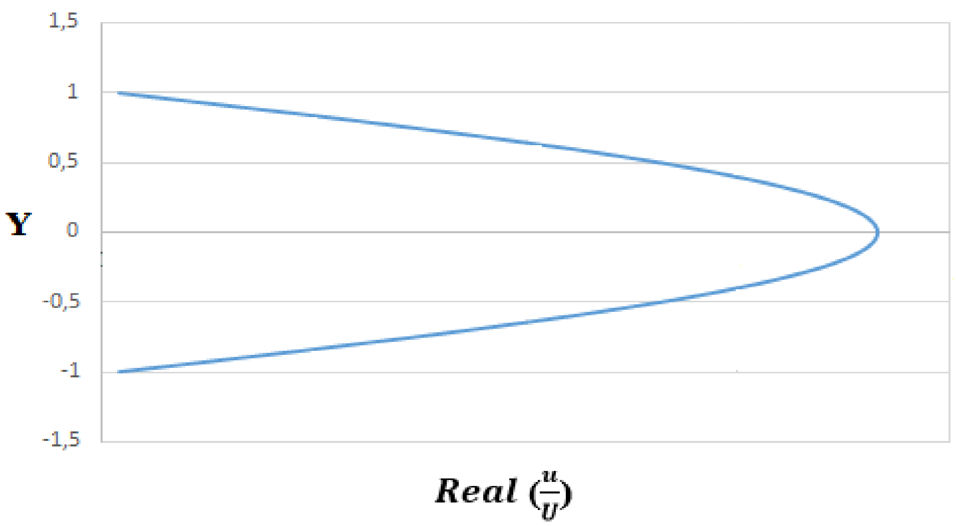

Figure 2. Distribution of velocity beyond the cross-section of the channel ($\alpha_0 = 1$)

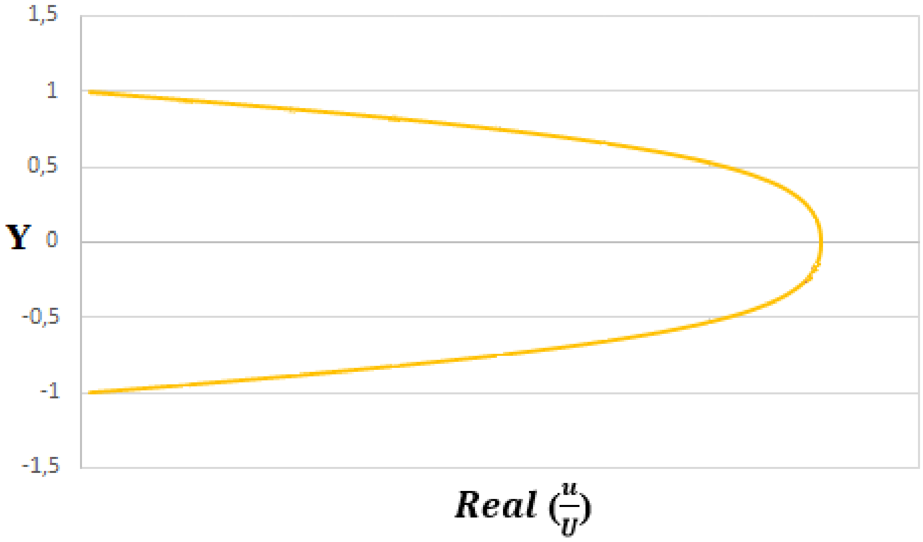

Figure 3. Distribution of velocity beyond the cross-section of the channel ($\alpha_0 = 4$)

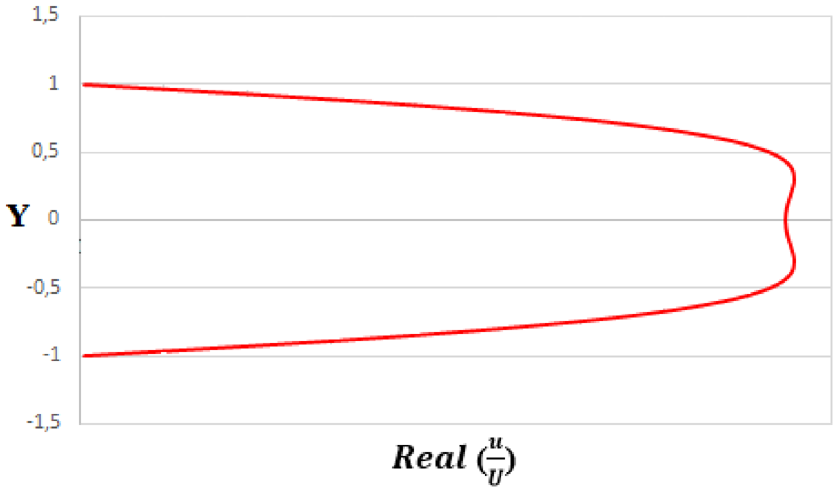

Figure 4. Distribution of velocity beyond the cross-section of the channel ($\alpha_0 = 5$)

Digital accounting has been carried out with the help of the determined formula and the law of distribution has been identified beyond the surface of the channel of the section of the velocity on the different quantity of time.

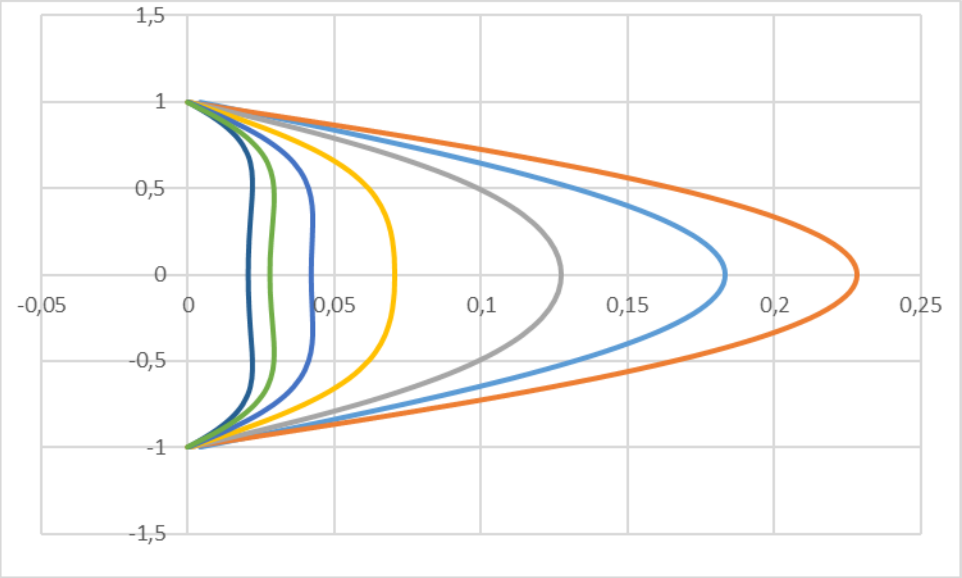

Figure 5. Distribution of velocity beyond the cross-section of the channel (all states)

CONCLUSION

Being obvious from the graph, when α_0=5, the distribution of the cross-section of velocity will be removed from parabolic distribution, and goes to the M-distribution. Large-scale fluid movement is in the $y_0 = h - \sigma$ section of the cross-section without the center of the channel. This y_0 will depend on the parameter of length and α_0 on the parameter of vibration. When α_0 <<1, will be $\sigma = h$, the motion of maximum velocity will be observed at the center $y_0 = 0$ and in cases α_0>1, will be $\sigma << 1$, high quantities of velocity are undergone around channel sides.

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