Simulation Modelling Process In The Analysis Of (M/M/2) Queueing Modelling

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Abstract:
This study helps to identify the effect on the waiting line and delay the time of passengers when the service rate increases or decreases. In this paper, we present an introduction of simulation and it's processed with the help of flowcharts of arrival and departure event for understanding and analysing various system performance measures of multiple server queueing modelling (M/M/s) which give service to passengers. Also, through the statistical approach, the significance of data (of particular bus route), which was collected from the AICTSL office, Indore (M.P.), India, is verified by the t-test. In this paper, we also determine and compare the system performance measure of the queueing model (M/M/2) and also calculate the delay time of the system and Erlang value. Simulation calculation of various performance measures is performed by the WinQSB software for (M/M/2) queueing model when the service rate is varying. Using MATLAB software, various system performance measures of the M/M/2 queueing model are graphically represented for optimizing the servers.

Keywords: simulation, queueing theory, statistical approach, arrival and departure event, system performance, winQSB& MATLAB.
Mathematics Subject Classification 2010: 60K25, 68M20, 34C60 34K60

INTRODUCTION:

From Statistics, Probability, Number Theory, and Computer Science by using these ideas and techniques simulation are called an interdisciplinary subject. Simulation is used in many sectors like biological system, routing algorithms, assembly line, weather forecasting, the atom bomb, atomic reactor, missile launching, the importance of variables, Verify analytic solutions (theories), and scale of impact on overall system behaviour defined by Averill, c. sharma, sushil [2][4][7]. Simulation programming tasks is represented with the aid of using Figure 1. In Figure 2, 3 step by step describe the simulation algorithm and the manner of simulation. This paper handling the passenger's records so the passenger's arrival and departure event are described with insideFigure 4, 5 the usage of the flow charts. M/M/S performance measures, transition density functions, difference equation of the gadget are defined with inside the performance measures section. Then the t-test takes a look at is applied with inside the statistical process and with the assist of the WinQSB software program calculates all of the performance measures.That'sproven with insideFigure 6thereconsequencesprovenwith the aid of using the pie charts with the aid of using Figure 7. All the performancemeasures whilstprovider charge increases or decreases theirconsequenceswrittenwithinside Table 1without problemsconcludingthe consequences. These values are represented with the aid of using the MATLAB software program inside Figure 8. Calling populations are often the nature of arrivals, service mechanism (Single queue single server, Single queue multiple servers, Multiple queue single server, multiple queues multiple servers, Service facility in series), system capacity(finite, infinite), queue

discipline (First come first serve, Last in first out, Service in random order, Priority service), customer behaviour (Balking, Reneging, Jockeying) define by Jerry, Aerill, Arizono [1][2][8]. However,the most real-world system is too complex to allow realistic models to be evaluated analytically(mathematical methods possible to use for finding the exact information such as algebra, calculus, or probability theory, this methodology is called an analytic solution) and these models must be studied throughsimulation define by C. Sharma, Arizono[4][8].
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Queueingtheoryrefersto the mathematical models are using to simulate these models define by Jerry [1]. In a simulation, we use a computer to evaluate a model numerically. Some imitation or representation of reality such as movies and animations are not simulations because they don't have an internal model. Similarly, in the simulation, most of the games can't be included because they use the internal model only for motivating devices define by Jerry, Geoffrey [1][3]. Simulation is a procedure in which random numbers are generated according to probabilities assumed to be associated with a source of uncertainty define by C. Sharma [4]. Monte Carlo method is developed by physicists working on the Manhattan project to study neutron scattering in 1940. In 1950 First special-purpose simulation languages developed (e.g. IMSCRIPT by Harry Markowitz at RAND Institute), 1970 research initiated on mathematical foundations of simulation. In 1980 PC-based simulation software developed, graphical user interfaces, object-oriented programming. Web-based simulation, fancy animated graphics, simulation-based optimization, Markov-chain Monte Carlo methods Simulation have become ever more prominent as a method for studying complex systems in which uncertainty is present in 1990. In various surveys, Simulation is the most frequently used tool of Operation Research practitioners.

MATERIAL AND METHODS OF SIMULATION:

Programming is doing for a problem. Firstly we need a problem, find or identify a suitable approach with the help of these problems can be solved. Some problems are analytical approach is rejected so we use simulation. Simulation is a modelling process; simulate means the simulation methods and techniques are used on it. Figure 1 shows how the simulation task is performed. Simulation programming task has three basic things first of all we need to generate the model, simulate through the simulation algorithm, and generate the report. First of all, we need to generate the model for applying the simulation. A simulation is a model-based approach in which the methods or technique is used for the number of runs is applied. Then simulation is applied to it to generate the report. When we generate a model the system will be created a set of numbers, this represents the state of the systems. All the sets of numbers reflect the state of the system. This set of numbers will be called system images. Routine represents the activity of the system will create discrete events making the change to the system images. second of the simulation tasks is prepared an algorithm this holds the procedures that execute the cycle of actions involved in caring out the simulation. For preparing an algorithm the process is described in Figure 2. In which we see the process, however, we collect the data and what operation is performed on it before gathering the statistics process. After that valid model is prepared for simulation runs is shown in Figure 3. The last task of the simulation programming is to generate the report. In the report, the statistical gathering during the simulation will be organized. In figure 2 described as, for performing a task, firstly we need to find the problem and then find the simulation technique. After that checking the simulation approach is appropriate or not, if the simulation is applicable then find its benefits. If not then stop the process else we create a new model and gather the statistics then generate the report. Otherwise, repeat the process of finding the simulation. We have resulted that the simulation process can be tested by the number of runs. Sometimes it is useful to repeat runs so that model parts have different random number while the rest use the same random numbers on each run, after the number of runs we verify the result and documentation and implementation will be prepared and then stop the simulation process. Figure 3 describes, in the simulation process, first of all, we need to describe the problem, firstly check the problem is appropriate or not if yes then plan and schedule for the model else stop the problem. After planning and scheduling, we need to define a model and simulate the model. If the simulation is not done properly then use the analysis methods if it is supportable then stop otherwise redefine a model.

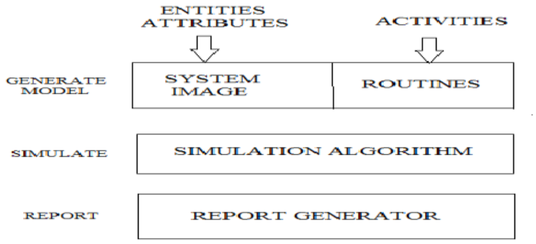

Figure 1: Simulation programming tasks

After that write a valid model, finally run the model if it is not supportable then again follow the process of redefining a model if yes then check the availability of units then more runs of a program if yes then repeat the process from run the model if not verify records, if it is not supportable then follow the process of redefining a program else document and implement it and stop the procedure.

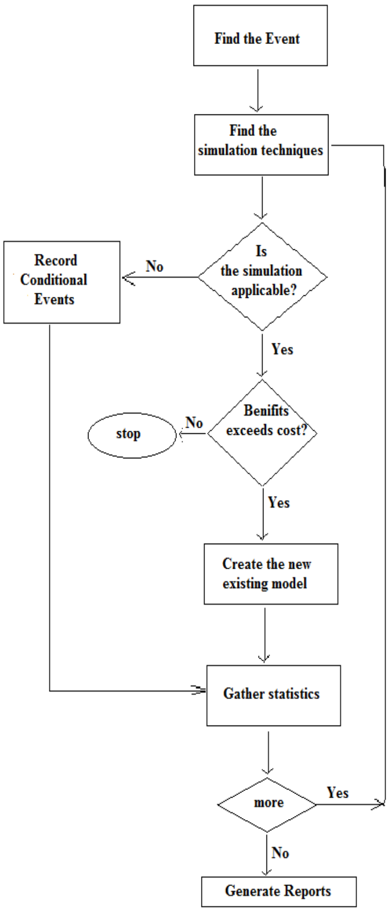

Figure 2: Execution of a simulation algorithm

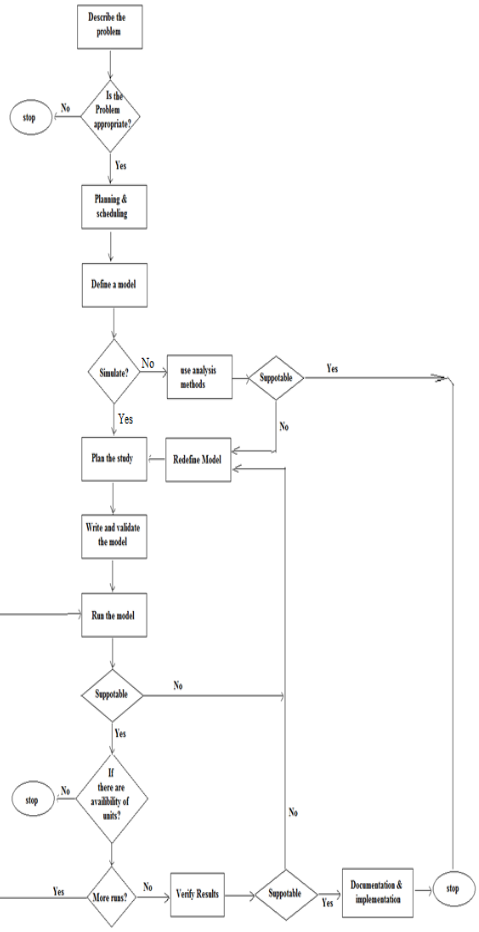

Figure 3: The process of simulating

In this paper, we are taking the real world particular bus route data for measuring waiting time, the queue length of the passengers for concluding how the most effective service can be provided to the passengers. Before analysing these points we need to understand how the passengers enter into the system and get the service and leave the system. For this, we are preparing the arrival and departure event to the passengers described in Figure 4, 5. After preparing a model, for implementing a model. Firstly we need to collect the data and analyse it. Next step is to identify the arrival event of the passengers whocome for getting the service then comes from a schedule for next passengers then passengers will check the server is busy or not, if yes then the passenger queue will be increased by the number one then check whether the queue is full or not if yes then display the error message then proceed for service, else the scheduled by the first passenger will be assigned by zero and start getting the service and reaming passengers who wait in the queue the value will be assigned by 1 when the server provides the server busy this time.

Now the departure event will be represented for the passengers who get the service. The final process for the passenger is the departure event firstly check the weather the queue is empty or not, if yes then the server will be idle for this time departure event is not exist else passenger get the service the queue length is not be reduced by the value one. Next passengers served by the server and reaming passengers will be assigned by the number 1 value. Then schedule for the reaming passengers. One by one every passenger in the queue inthe first place then finally the passengers go into the boarding event.

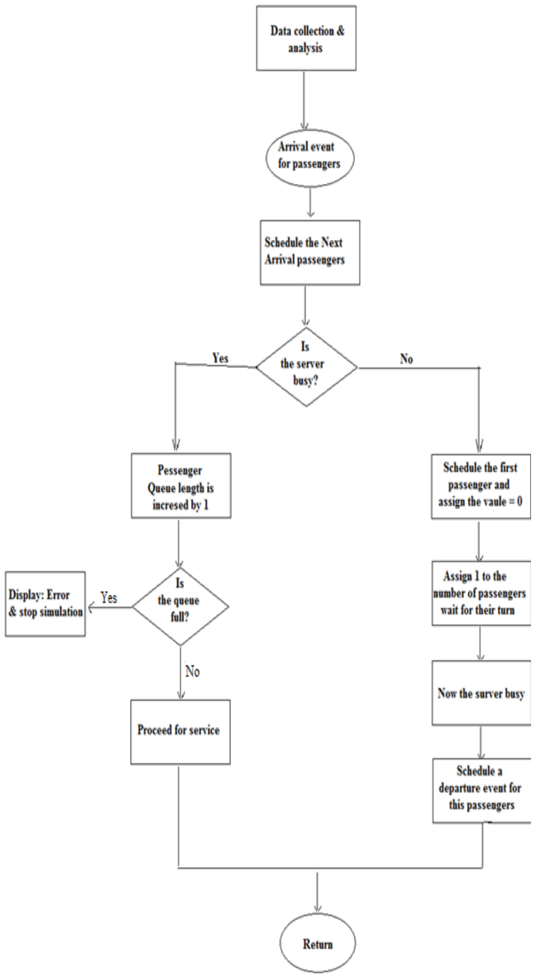

Figure 4: Flowchart for the arrival event

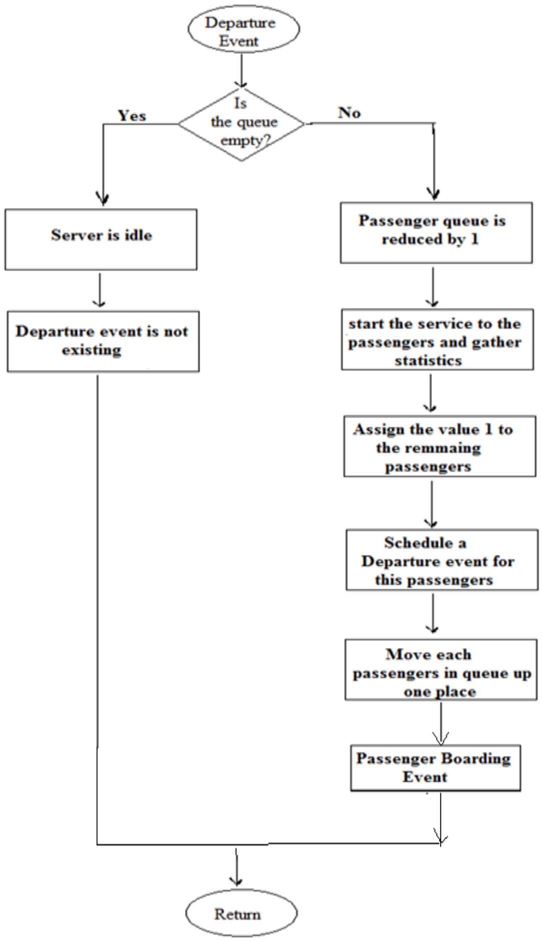

Figure 5: Flowchart for the departure even

Performance Measure:

Many authors are like [16], [17], [18], generalized the M/M/S model Poisson probability distribution, and come from an infinite population. The discipline of the queueing system is first come first serve in this paper. M/M/s queue, the buffer is limitless and the process of an arrival is Poisson with rate λ. The service time of each of the s ($1 \leq s \leq \infty$) is the server exponentially distributed with Parameter μ. The multi-server model is suitable for that problem in which storage or delay involves such as parking lord and warehouse.

Here the service rate of one server, we have the s number servers therefore $s\mu$ will be the service rate. If there are n passengers in the queueing system at any point in time, then the following two cases may arise:

(i) There will be no queue if the number of passengers in the server is less than the number of servers (n< S), however (s-n) numbers of passengers of the servers are not busy. The rate of combined service will be $\mu_n = n\mu$, n < S.

(ii) The case when all the servers will be busy it happened when the number of passengers in the system is more than or equal to the servers (n = s) then all servers will be the busy and the maximum number of passengers in the queue will be (n-s). The combined service rate will be $\mu_n = s\mu; n = s$. thus we have $\lambda_n = \lambda$ for all n = 0.

$$\mu_n = \begin{cases} n\mu, & n < s \\ \mu_n = s\mu, & n = s \end{cases}$$

Following notation

Q = In the system random variable reflects the total number of passengers (waiting and being served in the queue);

N_Q= The total number of passengers waiting in queue represented by a random variable (this does not include those passengers being served);

N_s= a random variable, reflecting the total number of passengers served;

D = a random variable reflecting the overall system delay(including the time a Passenger is waiting in the queue and in-service);

S = The service time from a representation of a random variable.

$\widehat{D}$= Reconsideration of a delayed passenger including the service time

$\widehat{W}_Q$= Wait in queue for delayed passengers except service time.

N(t) = with the instant t the number present in the system.

$$\rho = \frac{\lambda}{s\mu}; p_0 = \left[\sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{1}{s!}\frac{(s\rho)^s}{1-\rho}\right]^{-1}; p_n = \begin{cases} \frac{\rho^n}{n!} P_0; & 1 \le n < s \\ \frac{\rho^n}{s! s^{n-s}} P_0; & n \ge s;\ \rho = \lambda/\mu \end{cases}$$

The average number of passengers waits for their turn in the queue:$L_q = \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\lambda\mu}{(s\mu-\lambda)^2}\right] P_0$

The average number of passengers in the system:$L_s = \left[L_q + \frac{\lambda}{\mu}\right]$

Average waiting time of passengers in the queue:$W_q = \frac{L_q}{\lambda}$

Average waiting time that passengers spend in the system during service:$W_s = W_q + \frac{1}{\mu}$

The so-called Erlang C formulation is of particular interest. Its aim of C formula is to show all the busy servers in the time of proportion and is given by:

$\rho = \frac{\lambda}{\mu}$, $C_s(\rho) = \frac{\rho^2}{s!}\frac{s}{s-\rho}\pi_0$

Steady-State Equations: we get an equation for π_0, the solution of which is

$$\pi_0 = \left(\sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s!}\frac{s}{(s-\rho)}\right)^{-1}$$

Using the above notation, we have

Mean Delay and delayed passengers$E[\widehat{W}_Q] = \frac{1}{s\mu-\lambda}$, $E[\widehat{D}] = \frac{1}{s\mu-\lambda} + \frac{1}{\mu}$

We can verify this latter one using the Iterated Expectation rule as follows: $E[D] = \frac{C_s(\rho)}{\mu s-\lambda} + \frac{1}{\mu}$

Utilization: $\hat{u} = \frac{\rho}{s}$

FormulaErlang C:

The so-called Erlang C formulation is of particular interest. It reflects the amount of time that allthe servers are busy, and are given by:

$$C_s(\rho) = \frac{s\,\dfrac{\rho^2/_{s!}}{\sum_{n=0}^{s} \rho^n/_{n!}}}{s - \rho\left[1 - \rho^s/_{s!}\Big/ \sum_{n=0}^{s} \rho^n/_{n!}\right]}$$

$$E_s(\rho) = \frac{\rho^s/_{s!}}{\sum_{n=0}^{s} \rho^n/_{n!}}$$

Show that $C_s(\rho) \geq E_s(\rho)$.

Statistical Procedure:
In this section for identifying the significance of the data, the useful statistical approach is used to compare the set of real-world data. To full fill the purpose, the number of statistical test approaches is available like t-Test, Mann Whitney, Two-sample chi-square test, two-sample kolmogorov-Simirov, etc. In this paper full fill our need we select from statistical approach t-distribution.
In statistics t-test holds the number of applications, the following discussed below:
(i) For the single mean the significance by the t-test when the population variance is unknown.
(ii) The difference between the two sample means for the significance by the t-test when thepopulation variance being equal but unknown.
(iii) An observed sample correlation coefficient for the significance by the t-test.
If a given random sample x_1, x_2, x_3,……,x_n of size n has been drawn from the normal population with mean. Set the null hypothesis (H_0) and the alternative hypothesis (H_1). Under the hypothesis the test-statistic is
$\bar{x} = \frac{\sum x}{n}$, $s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$, t = $\frac{\bar{x}-1}{\sqrt{s^2/_n}}$.
Data:

Arrival	843	936	949	1098	1170	1216	1085	772	902	802	1063
Service	683	748	764	913	999	1044	950	661	812	684	945
Arrival	1193	1143	1092	868	922	1107	1027	984	1086	1122	1046
Serv	994	958	886	702	807	966	794	894	990	1013	932

i c e											

Set: H_0: $\mu = \mu_0$
H_1: $\mu > \mu_0$
μ = is the population means of the measured arrival, service, and queue of passengers. Where μ_0=1.
Tabular value: 2.0796
$\bar{X}$ =1019.3636, S^2 = 16331.9265, t = 37.3762
$\bar{Y}$ =869.9545, S^2 = 14921.47406, t = 33.3659
$\bar{d}$ =149.3636, S^2 = 1765.4805, t = 16.5618
Here we see that all the three calculated values of the t distribution are greater than the tabular value so the null hypothesis is rejected and the alternative hypothesis is accepted.

Table 1: System Performance

	$\lambda = 6, \mu = 5$	$\lambda = 6, \mu = 4$
P_0	0.270270	0.181818
L_s	1.201520	1.530303
L_q	0.001520	0.030303
W_s	0.400253	0.505051
W_q	0.200253	0.255051
$C_s(\rho)$	0.049958	0.204546
$E_s(\rho)$	0.035503	0.138462
$E(\widehat{W}_Q)$	0.166	0.5
$E(\widehat{D})$	0.366	0.666666

09-17-2018	Performance Measure	Result
1	System: M/M/2	From Sim
2	Customer arrival rate (lambda) per hour =	[illegible]
3	Service rate per server (mu) per hour =	[illegible]
4	Overall system effective arrival rate per hour =	[illegible]
5	Overall system effective service rate per hour =	[illegible]
6	Overall system utilization =	44.3
7	Average number of customers in the system (L) =	[illegible]
8	Average number of customers in the queue (Lq) =	
9	Average number of customers in the queue for a busy system (Lb) =	
10	Average time customer spends in the system (W) =	0.198
11	Average time customer spends in the queue (Wq) =	
12	Average time customer spends in the queue for a busy system (Wb) =	
13	The probability that all servers are idle (Po) =	34.
14	The probability an arriving customer waits (Pw) or system is busy (Pb) =	23.
15	Average number of customers being balked per hour =	[illegible]
16	Total cost of busy server per hour =	
17	Total cost of idle server per hour =	
18	Total cost of customer waiting per hour =	
19	Total cost of customer being served per hour =	
20	Total cost of customer being balked per hour =	
21	Total queue space cost per hour =	
22	Total system cost per hour =	
23	Simulation time in hour =	100
24	Starting data collection time in hour =	
25	Number of observations collected =	
26	Maximum number of customers in the queue =	
27	Total simulation CPU time in second =	[illegible]

09-17-2018	Performance Measure	Result
1	System: M/M/2	From Simulation
2	Customer arrival rate (lambda) per hour =	6.0000
3	Service rate per server (mu) per hour =	4.0000
4	Overall system effective arrival rate per hour =	4.0926
5	Overall system effective service rate per hour =	4.0916
6	Overall system utilization =	51.4753 %
7	Average number of customers in the system (L) =	1.0295
8	Average number of customers in the queue (Lq) =	0
9	Average number of customers in the queue for a busy system (Lb) =	0
10	Average time customer spends in the system (W) =	0.2516 hours
11	Average time customer spends in the queue (Wq) =	0 hour
12	Average time customer spends in the queue for a busy system (Wb) =	0 hour
13	The probability that all servers are idle (Po) =	27.9735 %
14	The probability an arriving customer waits (Pw) or system is busy (Pb) =	30.9241 %
15	Average number of customers being balked per hour =	1.8808
16	Total cost of busy server per hour =	$0
17	Total cost of idle server per hour =	$0
18	Total cost of customer waiting per hour =	$0
19	Total cost of customer being served per hour =	$0
20	Total cost of customer being balked per hour =	$0
21	Total queue space cost per hour =	$0
22	Total system cost per hour =	$0
23	Simulation time in hour =	1000.0000
24	Starting data collection time in hour =	0
25	Number of observations collected =	4092
26	Maximum number of customers in the queue =	0
27	Total simulation CPU time in second =	1.9380

Figure 6: Simulation Performance measures using WinQSB when service rate = 5& 4

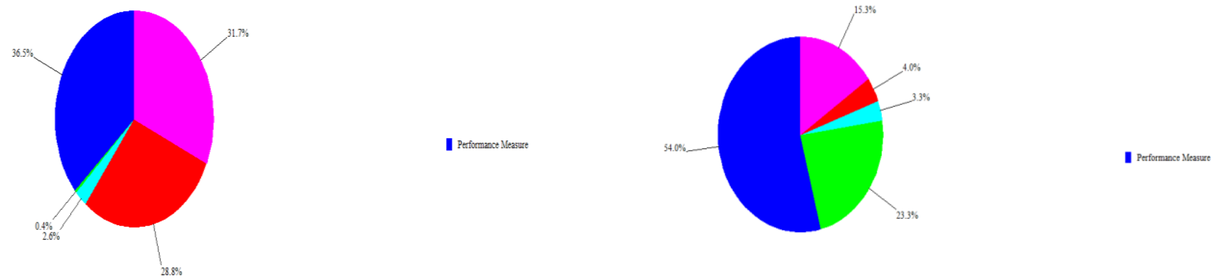

Figure 7: Pie chart using WinQSB when service rate = 5& 4

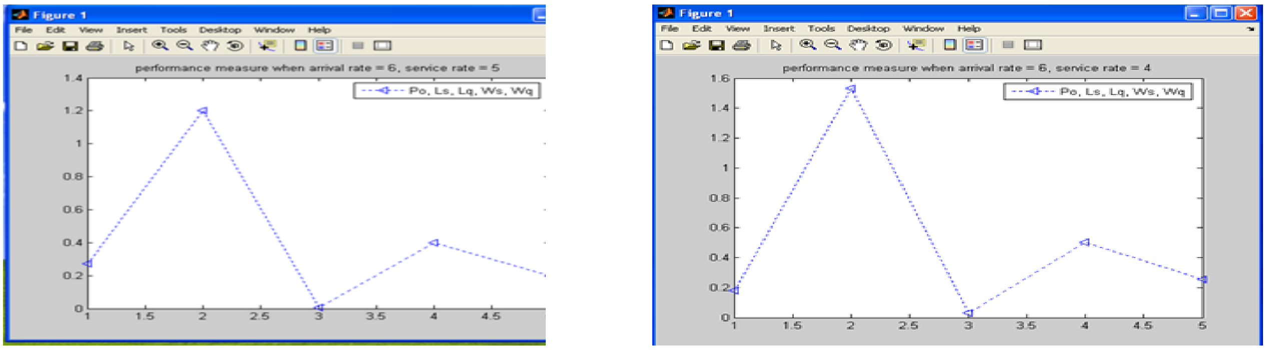

Figure 8: Simulation Performance measures using MATLAB when service rate = 5& 4

CONCLUSION:

This paper explains the basic structure of the queueing arrival event; the departure event and simulation process is modelled for efficient service to the passengers. For this purpose, several queue models are reviewed. These flow charts are important for all sectors. Through the t-test, data is verified that the collected data is significant or not. Here significant data we get, so we proceed to the next step for measuring the system performance. Here M/M/2 queueing model is used to measure the different system performances and compare the results of two different service rates (when the service rate is 5 and it reduced by the value one). Here we see that when the service rate is reduced then waiting time, service time of the system, and passengers who wait, delay, and delay time for the service is increased. For finding the more accuracy of the system we verify by the Erlang C formula, the result for both the situations for C formula is verified. In this paper we use WinQSB software for finding the system performance for both the service rate values; from these, we see the variation of both the service rate values shown the same results as the analytic method. The complete utilization of simulation for measuring the system performance by the pie chart and the analytical results are represented by graphically using MATLAB. Through all these work and results we concluded that, however, the service rate is increased system performance and the utilization of the system also increased and provide better to the passengers.

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