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Inverse Domination In Circular-Arc Graphs

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Abstract

The intersection graph of a set of arcs on the circle is called a circular-arc graph. Circular-arc has one vertex for each arc in the set and an edge between every pair of vertices corresponding to arcs that intersect. Let $C = \{c_1, c_2, ..., c_n\}$ ***be family of arcs on a circle. In this paper we are taking circular arcs such that if we remove*** c_1 ***then there will be a disconnection between left end side intersecting arc of*** c_1 ***and right end side intersecting arcs of*** c_1***.We are writing an algorithm to find an inverse of dominating set with respect to a minimum dominating set of a circular-arc family.***

Keywords— Inverse dominating set, Inverse domination number, Circulare-arc graph.

I. INTRODUCTION

The Concept of inverse domination was introduced by V.R. Kulli and C. Sigarkanti [1]. In [2] V Jude Anne Cynthia and A Kavitha [2] investigated the inverse domination in Circulant graph $G(n, \pm\{1,2\})$. Many other inverse domination parameters in domination theory were studied, for example[3,4,5,6] A dominating set D' is called an inverse dominating set with respect to a minimum dominating set D if D' is a minimum dominating set of $<V-D>$. The cardinality of a smallest inverse dominating set of G is called an inverse domination number $\gamma'(G)$ of G.

Let $C = \{c_1, c_2, ..., c_n\}$ or $C = \{1, 2, ..., n\}$ be family of arcs on a circle. Here we taking left end labelling.

Some Notations

Minimum dominating set = MDS_{ds},
Inverse dominating set = IDS_{ds} ,
$\max(A)$ = maximum number in set A,
$\min(A)$ = minimum number in set A,
$Nhod^-(j)$ = The set of all right intersecting arcs to arc j and less than j ,
$Nhod^+(j)$ = {The set of all left intersecting arcs to arc j} $\cup$ {Arcs which are contained in j},
$Nbd[i]$ = {The set of all intersecting arcs to arc i including i}
$NAJ(i)$ = First non intersecting arc to arc i left side.

A. An algorithm to find an inverse dominating set with respect to a minimum dominating set of a circular-arc graph

Input: Circular-arc family

Step 1: $MDS_{ds} = \{\}$
Step 2: $IDS_{ds} = \{\}$
Step 3: $i = 1$
Step 4: $Lsn1 = \{i\} \cup$ {The set of all circular-arcs $> i$ which are left side intersecting to arc $i\} \cup$
{arcs which are contained in i}
Step 5: $Lsn2 =$ {The set of all circular-arcs $\in Lsn1$ which are intersecting to all other arcs $\in Lsn1$ and
$\notin MDS_{ds}$ and $\notin IDS_{ds}$}
Step 6: If $|Lsn2| > 1$ then
Step 6.1: $a = \max(Lsn3)$
Step 6.2: $b = \min(Lsn3)$
Step 6.3: $MDS_{ds} = MDS_{ds} \cup \{a\}$
Step 6.4: $IDS_{ds} = IDS_{ds} \cup \{b\}$ go to step 11
Else
Step 7: If $|Lsn2| = 1$ and $Lsn2 \subseteq Nbd[k]$ for any $k \in MDS_{ds}$ then
Step 7.1: $a = p \in Lsn2$ go to step 9
Else
Step 8: If $|Lsn2| = 1$ then
Step 8.1: $a = p \in Lsn2$
Step 8.2: $MDS_{ds} = MDS_{ds} \cup \{a\}$
Step 9: Find $Nhod^{-}(a)$
Step 10: If $Nhod^{-}(a)$ is not null then
Step 10.1: $Isn1 = Nhod^{-}(a)$
Else
Step 10.2: $Isn1 = Nhod^{+}(a)$
Step 10.3: $Isn2 =$ {The set of all circular-arcs $\in Isn1$ which are intersecting
all other arcs $\in Isn1$ and $\notin MDS_{ds}$ and $\notin IDS_{ds}$}
Step 10.4: If $Isn2$ is null then
Step 10.4.1: $b = \min(Isn1)$
Else
Step 10.4.2: $b = \max(Isn3)$
Step 10.5: $IDS_{ds} = IDS_{ds} \cup \{b\}$
Step 11: Find $i = NAJ(a)$
Step 12: If i is greater than a
Step 12.1: go to step 4
Else
Step 12.2: Find $i = NAJ(b)$
Step 12.3: If i is greater than b
Step 12.3.1: a = i go to 10
Else
Step 12.3.2: go to step 14

$Step\ 13: \text{End}$

Output: IDS_{ds} is an inverse dominating set with respect to a minimum dominating set MDS_{ds}

Remark : In a circular-arc graph if we have one or two minimum dominating sets we may or may not get by using this algorithm.

B. Illustrations

Example 1

$Step\ 1: MDS_{ds} = \{\}$
$Step\ 2: IDS_{ds} = \{\}$
$Step\ 3: i = 1$
$Step\ 4: Lsn1 = \{1,2,3,4\}$
$Step\ 4: Lsn1 = \{1,2,3,4\}$
$Step\ 5: Lsn2 = \{1\}$
$Step\ 6: |Lsn2| = 2 > 1$
$Step\ 6.1: a = 3$
$Step\ 6.2: b = 1$
$Step\ 6.3: MDS_{ds} = \{3\}$
$Step\ 6.4: IDS_{ds} = \{1\}$
$Step\ 11: i = 5$
$Step\ 4: Lsn1 = \{5,6,7\}$
$Step\ 5: Lsn2 = \{5,6,7\}$
$Step\ 6: |Lsn2| = 2 > 1$
$Step\ 6.1: a = 7$
$Step\ 6.2: b = 5$
$Step\ 6.3: MDS_{ds} = \{3,7\}$
$Step\ 6.4: IDS_{ds} = \{1,5\}$
$Step\ 11: i = 10$
$Step\ 4: Lsn1 = \{10\}$
$Step\ 5: Lsn2 = \{10\}$
$Step\ 8: |Lsn2| = 1$
$Step\ 8.2: MDS_{ds} = \{3,7,10\}$
$Step\ 10.1: Isn1 = \{8,9\}$
$Step\ 10.3: Isn2 = \{8,9\}$
$Step\ 10.4.2: b = 9$
$Step\ 10.5: IDS_{ds} = \{1,5,9\}$
$Step\ 12: i = 1 < 10$
$Step\ 12.2: i = 1 < 9$
$Step\ 13: End$

Output: $IDS_{ds} = \{1,3,9\}$ is an inverse dominating set with respect to a minimum dominating set $MDS_{ds} = \{3,7,10\}$

Example 2

$Step\ 1: MDS_{ds} = \{\}$
$Step\ 2: IDS_{ds} = \{\}$

$Step\ 3: i = 1$

$Step\ 4: Lsn1 = \{1, 2, 3, 4\}$

$Step\ 5: Lsn2 = \{1\}$

$Step\ 8: \ |Lsn2| = 1$

$Step\ 8.2: MDS_{ds} = \{1\}$

$Step\ 10.2: Isn1 = \{2, 3, 4\}$

$Step\ 10.3: Isn2 = null$

$Step\ 10.4.1: b = 2$

$Step\ 10.5: IDS_{ds} = \{2\}$

$Step\ 11: i = 5$

$Step\ 4: Lsn1 = \{5, 6, 7\}$

$Step\ 5: Lsn2 = \{5, 6, 7\}$

$Step\ 6: |Lsn2| = 2 > 1$

$Step\ 6.1: a = 7$

$Step\ 6.2: b = 5$

$Step\ 6.3: MDS_{ds} = \{1, 7\}$

$Step\ 6.4: IDS_{ds} = \{2, 5\}$

$Step\ 11: i = 9$

$Step\ 4: Lsn1 = \{9\}$

$Step\ 5: Lsn2 = \{9\}$

$Step\ 6: |Lsn2| = 1$ and $9 \in Nbd[1]$

$Step\ 10.1: Isn1 = \{7, 8\}$

$Step\ 10.3: Isn2 = \{7, 8\}$

$Step\ 10.4.2: b = 8$

$Step\ 10.5: IDS_{ds} = \{2, 5, 8\}$

$Step\ 12: i = 1 < 9$

$Step\ 12.2: i = 1 < 8$

$Step\ 13: End$

Output: $IDS_{ds} = \{1, 7\}$ is an inverse dominating set with respect to a minimum dominating set $MDS_{ds} = \{2, 5, 8\}$

Special case

If there are three arcs j,k,l such that j is only one right side intersecting arc to k and l is only one left side intersecting arc to k and also there are another three arcs p, q,r such that p is only one right side intersecting arc to q and r is only one left side intersecting arc to q and if there is only one arc s which left side intersecting arc to l and right side intersecting arc to p then above algorithm directly we cannot find. But we have to use the following step. In the above algorithm in step 3 instead of i=1 we have to take i= j. Then we can use step 6.1 to step 6.4.

Example

From the above example

$Step\ 1: MDS_{ds} = \{\}$

$Step\ 2: IDS_{ds} = \{\}$

$Step\ 3: i = 2$

$Step\ 4: Lsn1 = \{2, 3, 4\}$

$Step\ 5: Lsn2 = \{2, 3, 4\}$

Step 6: If $|Lsn2| > 1$ then

$Step\ 6.1: a = 4$

$Step\ 6.2: b = 2$

$Step\ 6.3: MDS_{ds} = \{4\}$

$Step\ 6.4: IDS_{ds} = \{2\}$

$Step\ 11: i = 6$

$Step\ 4: Lsn1 = \{6,7,8\}$

$Step\ 5: Lsn2 = \{6,7,8\}$

Step 6: If $|Lsn2| > 1$ then

$Step\ 6.1: a = 8$

$Step\ 6.2: b = 6$

$Step\ 6.3: MDS_{ds} = \{4,8\}$

$Step\ 6.4: IDS_{ds} = \{2,6\}$

$Step\ 11: i = 10$ Here

$Step\ 4: Lsn1 = \{10,11,12\}$

$Step\ 5: Lsn2 = \{10,11,12\}$

Step 6: If $|Lsn2| > 1$ then

$Step\ 6.1: a = 12$

$Step\ 6.2: b = 10$

$Step\ 6.3: MDS_{ds} = \{4,8,12\}$

$Step\ 6.4: IDS_{ds} = \{2,6,10\}$

$Step\ 11: i = 1 < 12$

$Step\ 12.2: i = 1 < 10$

$Step\ 13: End$

$IDS_{ds} = \{2,6,8\}$ is an inverse dominating set with respect to a minimum dominating set $MDS_{ds} = \{4,8,12\}$

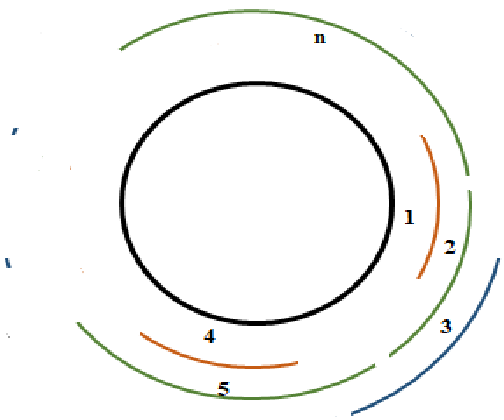


Figure 1: Circular-arc family

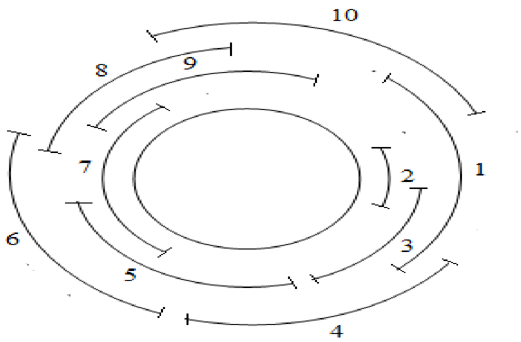
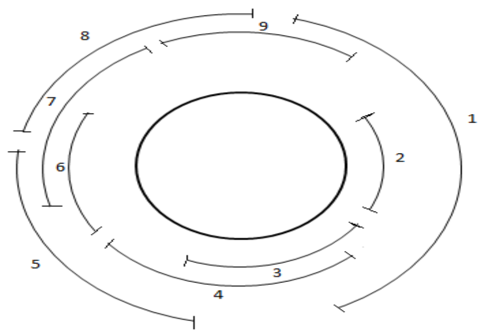


Figure 2 : Circular-arc

family 1



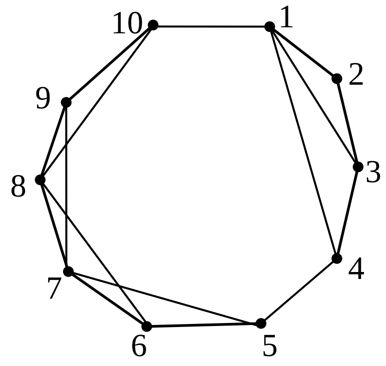
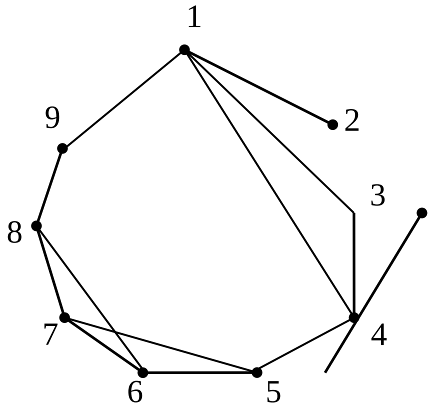


Figure 3: Circular- arc Graph 1 **Figure 4: Circular-arc family 2**



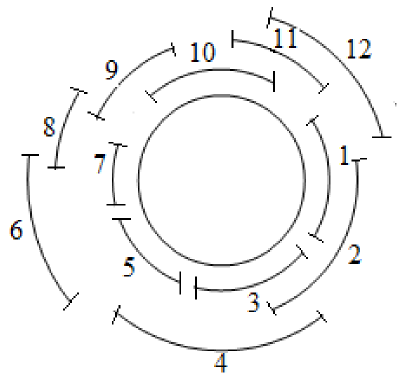


Figure 5: Circular-arc Graph 2 **Figure 6: Circular-arc family 3**

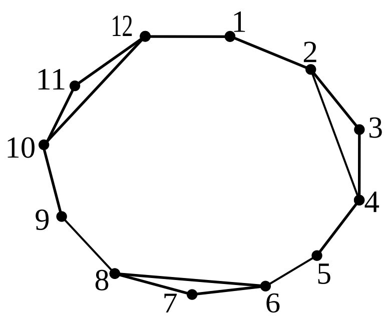


Figure 7: Circular-arc Graph 3

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