

A New Approach To Optimize The Membership Grade In Fuzzy Linear Programming Problem

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Abstract: *In this paper, an approach and the model for solving fuzzy linear programming (FLP) problems with some relevance's of the satisfaction of the fuzzy constraints are studied. The flexibility of the decision making (DM) for constructed model as a new approach is proposed. This encourages different weights to be assigned to limitations and to the goal purpose. A crisp question managed by a parameter has been resolved to find the required solution. The feasibility of the model suggested and its impact on the solution is addressed. The results show that the desired optimization can be obtained by admitting the decision making levels of preference for constraints. To illustrate the efficiency of the model for solving the FLP problem a numerical example is solved.*

Introduction

Operational analysis provides a broad variety of problem solving methods and strategies that enhance quantitative optimization decision-making and performance.. Linear programming problem is a method to attain best outcome through linear relationships in mathematical model. It provides better tools for solving practical problems in operation research by varying conditions. Optimization is methodology of making decisions that provides a scientific approach of adjusting a process so as to maximize desired factors and minimize the undesired factors under specified set of boundary.As optimization, appeared during 2nd world war when, transportation problems of resources was created methodically. The typical structure of the generalized linear problem can be translated as:

$$\text{Max or Min } Z = c_1x_1 + c_2x_2 + \dots + c_nx_n \text{ or } Z = \sum_{i=1}^n c_i x_i$$

$$\text{Subjected to } \sum_{i=1, j=1}^{m, n} a_{ij} x_j \leq \text{or } \geq b_i \text{ \& } x_i \text{'s } \geq 0 \quad (1)$$

Here x_i 's are decision variable and b_i 's represent the availability of m constraint.

In several realistic circumstances, the criteria or objective tasks in LPP can not be expected to be beneficial or precise.Many traditional optimization methods were producing good results to solve problems by providing hint. Such optimization problems analyze and reveal crisp-specific problems objective function and particular arrangement of constraints. Unfortunately, real world problems are not in the state of being determined. Traditional optimization deals with rigorous boundary of constraints. But due to existence of certain feasible changeability in industrial and economic surroundings it is hard to get required

degree of satisfactions from the crisp optimal problem. It is obvious or appropriate that many other LPP forms such as fuzzy-linear programming be used when coping with these conditions. The outcomes of this form of fuzzy Lpp are actual numbers that are a substitute for fuzzy figures. There are many effective method of solving an LPP. In this paper we are proposing triangular fuzzy Lpp and For example:

$$\text{Max } Z = \sum_{i=1}^n c_i x_i$$

Subjected to

$$\sum_{i=1, j=1}^{m, n} a_{ij} x_j \leq \text{or } \geq (b_i - p_i) \rightarrow b_i \rightarrow (b_i + p_i) \text{ And } x_i \geq 0. \quad (2)$$

If the availability, of constrains is fluctuating from basic (b_i) requirement to certain additive and minified availability, it can be done in a symmetric form or non-symmetric form then triangular fuzzy Lpp can estimate the required optimization under fuzzy conditions. The term fuzzy indicates "uncertain." Fuzziness happens where there is no definite border of a piece of knowledge. Uncertain mathematical knowledge was defined by the principle of the fuzzy set (Zadeh, 1965). Existing model decisions may also be implemented in different forms such as decision-making on entity, multi-level, multi-level and multi-criteria models. The classical linear programming problems are confined to optimize the objective under certain crisp restrictions. In this project we are using Fuzzy LPP to avoid the destruction in cost minimization under real time situations. The fuzzy Lpp to deal with probabilistic increment and decrement in the basic availability (b_i) of classical optimization and analyzing the result with targeted membership grade. Triangular fuzzy Lpp are used to interpret the feasible uncertainty and gives complete information in decision-making, risk rating, and expert systems. Both these Lpp are applied in many fields such as risk management, decision-making, and evaluation. Human decisions are generally affected with making a decision in existence of fuzziness, incomplete information. With the existence of the fuzzy linear programming many researchers introduced methods for the solution of this problem. J Reed and S. Leaven good (1998) cleared the concept of simplex method and how to solve linear programming maximization problem and to further use simplex method. This paper also clarifies the concept of objective function, decision variable and constraints set. Predrag Prodanovic (2001) proposed "fuzzy ranking techniques and numerous expert decisions modeling." This paper deals with the theory of fuzzy logic, represents imprecision by the fact that certain objects have poorly or ill-defined boundaries. Giorgio B. Dantzig (2002) addressed the origin of linear programming, and the historical importance and the course of its mathematical programming extensions.. He clears out the existence of linear programming and transportation problem. Yenilmez, K. & Gasimov, R. (2002) they concentrate on lpp with only fuzzy specified coefficients and in which both the right-hand side and the specified coefficients are fuzzy number. They contrast this method with well known "fuzzy decisive set technique." Rogers's et.all (2008) emphasized on linear fuzzy programming problems. This paper deals with the linear fuzzy programming problems which have fuzzy constraints with a varying objective function and varying constraints. Dr. Zaki. S Tewfik and Sabibha Fathil Jawed proposed (2010) a technique for optimizing and solving Fuzzy LPP. Dipti Dubey and Aparna Mehra (2011) presented "an approach to solve linear programming problem". This paper also clarifies the concept of ranking a fuzzy number and the concept of fuzzy triangular number. A research paper by Dipankar Chakra borty, Deepak Kumar Jana and Tappan Kumar Roy (2014) gave "A fresh beginning to fuzzy optimization

problem using unavoidably and integrity measures.” .Xinxiang Zhang Wiemin ma and Liping Chem. (2014) demonstrate and explained “the new resemblance of triangular fuzzy number and its application.” A new technique to compute triangular fuzzy number is presented, which takes the shape’s dissimilar area and midpoint of two triangular fuzzy numbers into consideration. UdaySharma[2015] clarified a modern approach for the resolution of the Fully Fuzzy Linear Programming Problem (FFLP) with three-angle Fuzzy Numbers and all drawbacks of Fuzzy Equality or Uniformité..Monalisha Pattnaik(2015) proposed Big-M Method in Fuzzy Based Linear Programming Problems for Post Optimal Analyses. A. Hosseinzadehet.all(2016) is working on a modern technique by utilizing the lexicography framework to overcome Totally Fuzzy Linear Programming. In this document they develop a new paradigm for FFLP resolution, by considering the (L-R) fuggish numbers and the system for lexicography along with crisp linear programming.

Methodology

Fuzzy Linear Programming

Classical LPPs are the minimum or maximum values under linear inequalities or linear function equations. The standard form of LPP is represented by

$$\text{Max /Min } Z = \sum_{j=1}^n c_j x_j$$

Subject to $\sum_{j=1}^n a_{ij} x_j \leq \text{or } \geq b_i$

Where, $x_j \geq 0, i, j \in \mathbb{N}$ (3)

The function to be Max Z or Min Z is called the an objective function. The c_j are called cost coefficients. The $A=[a_{ij}]$ matrix is called a restriction matrix and the $b= \langle b_1, b_2, \dots, b_m \rangle^T$ is called a vector on the right side. where $x= \langle x_1, x_2, \dots, x_n \rangle^T$ is the vector of variables.

The standard form fuzzy linear programming is represented by

$$\text{Max } Z = \sum_{j=1}^n c_j x_j$$

Subject to $\sum_{j=1}^n a_{ij} x_j \leq \tilde{b}_i$

Where, $x_j \geq 0, i, j \in \mathbb{N}$ (4)

Where, the \tilde{b}_i is the fuzzy number. With regard to the increase in the availability of restrictions, the fuzzy number can be presented in the above equation (2.5). The membership function would be described as follows.

$$\tilde{b}_i = \begin{cases} 1 & \text{when } x \leq b_i \\ \frac{b_i + p_i - x}{p_i} & \text{when } b_i \leq x \leq b_i + p_i \\ 0 & \text{when } x \geq b_i + p_i \end{cases} \quad (5)$$

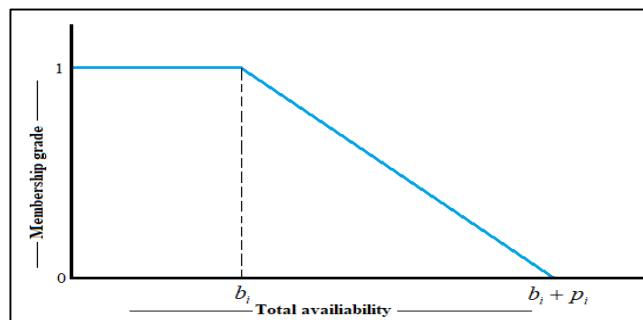


Figure 2.1: representation of membership function for \tilde{b}_i

The coefficient on the right is the membership function, i.e. the availability of restrictions. In order to optimize such a problem, we need to estimate the lower and upper boundaries of the optimum values. The lower bound (Z_l) value is

$$\begin{aligned} \text{Max } Z_l &= \sum_{j=1}^n c_j x_j \\ \text{Subject to } &\sum_{j=1}^n a_{ij} x_j \leq b_i \\ \text{Where, } &x_j \geq 0, i, j \in \mathbb{N}, x \in \mathbb{R}. \end{aligned} \quad (6)$$

The optimal values upper bound (Z_u) is as follows

$$\begin{aligned} \text{Max } Z_u &= \sum_{j=1}^n c_j x_j \\ \text{Subject to } &\sum_{j=1}^n a_{ij} x_j \leq b_i + p_i \\ \text{Where, } &x_j \geq 0, i, j \in \mathbb{N}, x \in \mathbb{R} \end{aligned} \quad (7)$$

Where, p_i is an increase in probabilistic availability of restrictions. In this case, the total probabilistic increase of access to restrictions is determined by the right coefficient.

The Simplex method can now be used to find a solution for both the lower and upper bounds of the LPPs. Using these lower and upper bounds, the optimized fuzzy LPP will be obtained as follows.

$$\begin{aligned} \text{Max } Z &= \lambda \\ \text{Subject to } &\lambda(Z_u - Z_l) - \sum_{j=1}^n c_j x_j \leq -Z_l \\ &\lambda(p_i) + \sum_{j=1}^n a_{ij} x_j \leq b_i + p_i \end{aligned} \quad (8)$$

Where, $x_j \geq 0, i, j \in \mathbb{N}$ and $\lambda \in (0,1)$ is membership grade

New approach

Many scholars have been investigating LP problems with fuzzy restrictions, and in particular many methods to solving question (1) have been suggested. Next, the clean linear programming problem for the two-phase solution,

$$\begin{aligned} \text{Max } v &= \sum_{j=1}^n v_j \\ \text{Subject to } &0 \leq v_j \leq \mu_j(x) \leq 1 \\ \text{Where, } &x_j \geq 0, i, j \in \mathbb{N}, x \in \mathbb{R} \end{aligned} \quad (9)$$

In the first step it is solved. If x^* is the appropriate answer to (9), the second stage solution is:

$$\begin{aligned} \text{Max } v &= \sum_{j=1}^n \alpha_j \\ \text{Subject to } &\mu_j(x') \leq \alpha_j \leq \mu_j(x) \leq 1 \\ \text{Where, } &x_j \geq 0, i, j \in \mathbb{N}, x \in \mathbb{R} \end{aligned} \quad (9)$$

We now define the ideal two-phase solution by (v^*, x^*) the optimum solution of (8) is (v^*, x^*) , the (α^*, x^*) is the appropriate solution of (9). The advantage of system (9) over the conventional max min operator in the two-phase method (8) is its capacity, where appropriate, to build on the solution with higher membership grade. We consider that we

should have $\alpha_i(x) = \alpha_i^*, i = 0, \dots, m$, the best solution for (9) Thus, problem(9) Could be described as equivalent:

$$\begin{aligned} \text{Max } &\sum_{j=1}^n \mu_j(x) \\ \text{Subject to } &\mu_j(x') \leq \mu_j(x) \leq 1 \\ \text{Where, } &x_j \geq 0, i, j \in \mathbb{N}, x \in \mathbb{R} \end{aligned} \quad (10)$$

DM's compliance with the goal function and restrictions are considered by the two stage approach[6] to problem(1) to be separate priorities with equivalent weights in the MOLP

model. Model (8) is solved for the optimum solution in Phase I using max-min theory. Phase II considers pattern (9) to boost the solution of (8). Here we propose a different strategy that seeks to both maximize the objective function and, where appropriate, to fulfill requirements at higher rates of achievement. We propose the following multi-target model: $Max(w_0\alpha_0, w_1\alpha_1 \dots, w_m\alpha_m)^T$

$$Subjected\ to\ A(x_i) \leq b_i \leq (1 - \alpha_i)p_i, i = 1 \dots, m \tag{11}$$

$$\alpha_0 = \frac{c^T x - z^0}{z^1 - z^0} \tag{12}$$

$$0 \leq \alpha_j \leq 1, x_i \geq 0, i, j \in \mathbb{N}, x \in \mathbb{R}$$

Where each $w_i \geq 0$ is a weight linked to α_i ; the following system taking a max-min solution to (11) is a given weight associated with

Max v

subjected to

$$w_0\alpha_0 \geq v$$

$$w_1\alpha_1 \geq v$$

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$$w_m\alpha_m \geq v$$

$$\alpha_0 = \frac{c^T x - z^0}{z^1 - z^0}$$

$$A(x_i) \leq b_i \leq (1 - \alpha_i)p_i, i = 1 \dots, m \tag{13}$$

Optimal goals of problem-related principles (6) and (7) are z^0 and z^1 and (v, α, x) is a prudent choice to (13)

Numerical example

Consider the following fuzzy linear programming problem.

$$Max\ z(x) \leq 4x_1 \leq 5x_2 \leq 9x_3 \leq 11x_4$$

$$s.t.\ g_1(x) \leq x_1 \leq x_2 \leq x_3 \leq x_4 \leq 15-20$$

$$g_2(x) \leq 7x_1 \leq 5x_2 \leq 3x_3 \leq 2x_4 \leq 80-120$$

$$g_3(x) \leq 3x_1 \leq 4.4x_2 \leq 10x_3 \leq 15x_4 \leq 100-130$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

(13)

We solve for lower bound and upper bound then we obtain the values from the following equations z^0 and z^1

Solution for upper bound z^0

Sol: - The standard form of this problem is as shown below. In this form, S_1, S_2 and S_3 are called as surplus variables which are introduced to balance the constraints

		C_j	4	5	9	11	0	0	0	
CB_i	Basic Variable	solution	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Minimum Ratio
0	S_1	15	1	1	1	1	1	0	0	15/1=15.

0	S_2	80	7	5	3	2	0	1	0	$80/2=40.$
0	S_3	100	3	4.4	10	15	0	0	1	$100/15=6.67$
		Z_j	0	0	0	0	0	0	0	
		$Z_j \square C_j$	-4	-5	-9	-11	0	0	0	

Table 1: table 1 shows the optimized value after the first iteration for lower bound.

CB_i		C_i	4	5	9	11	0	0	0	
	Basic V	Solution	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Minimum Ratio
0	S_1	8.34	0.8	0.71	0.34	0	1	0	-0.1	$8.34/0.8=10.43$
0	S_2	66.67	6.6	4.42	1.67	0	0	1	0.14	$66.67/6.6=10.01$
11	X_4	6.67	0.2	0.29	0.67	1	0	0	0.1	$6.67/0.2=33.35$
		$Z_j \square C_j$	-1.8	-1.77	-1.67	0	0	0	0.74	

Table 2: table 2 shows the optimized value after the second iteration for lower bound.

CB_i		C_j	4	5	9	11	0	0	0	
	Basic variable	solution	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Minimum Ratio
0	S_1	0.2525	0	0.17	0.131	0	1	-0.121	-0.050	$0.252/0.13=0.315$
4	X_1	10.101	1	0.668	0.252	0	0	0.151	-0.0202	$10.101/0.25=40.4$
11	X_4	4.6464	0	0.159	0.616	1	0	-0.030	0.070	$4.646/0.61=7.616$
		$Z_j \square C_j$	0	-0.569	-1.212	0	0	0.272	0.696	

Table 3: table 3 shows the optimized value after the third iteration for lower bound.

CB_i		C_j	4	5	9	11	0	0	0	
	Basic Variable	Solution	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Minimum ratio

9	X ₃	1.923	0	1.307	1	0	7.615	-0.923	-0.384	1.923/-0.92=2.0
4	X ₁	9.615	1	0.338	0	0	-1.923	0.384	0.076	9.615/0.38=25.3
11	X ₄	3.461	0	-0.646	1	-4.69	0.538	0.307	0	3.461/0.53=6.53
		Z _j □ C _j	0	1.015	0	0	9.230	-0.846	1	

Table 4: table 4 shows the optimized value after the fourth iteration for lower bound.

CB _i		C _j	4	5	9	11	0	0	0	
	Basic Variable	Solution	X ₁	X ₂	X ₃	X ₄	S ₁	S ₂	S ₃	Minimum ratio
9	X ₃	7.857	0	0.2	1	1.714	-0.428	0	0.142	
5	X ₁	7.142	1	0.8	0	-0.714	1.428	0	-0.142	
0	S ₂	6.428	0	-1.20		1.857	-8.714	1	0.571	
		Z _j □ C _j	0	0	0	1.571	1.857	0	0.714	

Table 5: table 5 shows the optimized value after the fifth iteration for lower bound.

The optimal solutions $z^0 = 4(7.142) + 5(0) + 9(7.85) + 11(0) = 99.218$.

Solution for upper bound z^1

CB _i		C _j	4	5	9	11	0	0	0	
	Basic Variable	Solution	X ₁	X ₂	X ₃	X ₄	S ₁	S ₂	S ₃	Minimum ratio
0	S ₁	20	1	1	1	1	1	0	0	20/1=20
0	S ₂	120	7	5	3	2	0	1	0	120/2=60
0	S ₃	130	3	4.4	10	15	0	0	1	130/15=8.667
		Z _j □ C _j	-4	-5	-9	-11	0	0	0	

Table 1: table 1 shows the optimized value after the first iteration for upper bound.

CB _i		C _j	4	5	9	11	0	0	0	
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CB_i	C_j	4	5	9	11	0	0	0		
Basic Variable	Solution	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Minimum ratio	
0	S_1	11.333	0.8	0.706	0.333	0	1	0	-0.066	11.333/0.8=14.16
0	S_2	102.66	6.6	4.413	1.666	0	0	1	-0.133	102.66/6.6=15.55
11	X_4	8.67	0.2	0.293	0.666	1	0	0	0.066	8.67/0.2=43.35
	$Z_j - C_j$	-1.8	-1.77	-1.67	0	0	0	0	0.734	

Table 2: table 2 shows the optimized value after the second iteration for upper bound.

CB_i	C_j	4	5	9	11	0	0	0		
Basic Variable	Solution	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Minimum ratio	
4	X_1	14.1667	1	0.883	0.417	0	1.25	0	-0.083	14.167/0.417=33.97
0	S_2	9.16667	0	-1.416	-1.08	0	-8.25	1	0.417	9.167/-1.08=-8.487
11	X_4	5.83333	0	0.117	0.584	1	-0.25	0	0.083	5.833/0.584=9.988
	$Z_j - C_j$	0	-0.184	-0.917	0	2.25	0	0	0.584	

Table 3: table 3 shows the optimized value after the third iteration for upper bound.

CB_i	C_j	4	5	9	11	0	0	0		
Basic Variable	Solution	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Minimum ratio	
4	X_1	10	1	0.8	0	-0.714	1.428	0	-0.142	
0	S_2	20	0	-1.20	1.857	-8.72	1	0.572		
9	X_3	10	0	0.2	1	1.714	-0.43	0	0.143	
	$Z_j - C_j$	0	0	0	1.5714	1.857	0	0.714		

Table 4: table 4 shows the optimized value after the fourth iteration for upper bound.

The optimal solutions for upper bound is

$$z(\text{optimal}) = 4x_1 + 5x_2 + 9x_3 + 11x_4 = 4(10) + 5(0) + 9(10) + 11(0) = 130.$$

Hence $z^0 = 99.218$ and $z^1 = 130$

Then the max-min operator is ready to solve.

max z

s.t. $4x_1 + 5x_2 + 9x_3 + 11x_4 + 30.71429z = 99.218$

$$4x_1 + 5x_2 + 9x_3 + 11x_4$$

$$z = 130$$

$$x_1 + x_2 + x_3 + x_4 + 5z = 20$$

$$x_1 + x_2 + x_3 + x_4$$

$$z = 15$$

$$7x_1 + 5x_2 + 3x_3 + 2x_4 + 40z = 120$$

$$7x_1 + 5x_2 + 3x_3 + 2x_4$$

$$z = 80$$

$$3x_1 + 4.4x_2 + 10x_3 + 15x_4 + 30z = 130$$

$$3x_1 + 4.4x_2 + 10x_3 + 15x_4$$

$$z = 100$$

$$x_1, x_2, x_3, x_4$$

$$z = 0,$$

$$z \in [0, 1]$$

As above we get the balanced constraints. Thus we have.

CB_i	C_j	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	
	B.V.Sol.	X_1	X_2	X_3	X_4	\square	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	Minimum Ratio.			
0	S_1	99.2	4	5	9	11	-30.7	-1	0	0	0	0	0	0	0	0	0	99.2/11=9.025974
0	S_2	130	5	5	9	11	0	0	1	0	0	0	0	0	0	0	0	130/11=11.81818
0	S_3	20	1	1	1	1	5	0	0	1	0	0	0	0	0	0	0	20/1=20
0	S_4	15	1	1	1	1	0	0	0	0	-1	0	0	0	0	0	0	15/1=15
0	S_5	120	7	5	3	2	4	0	0	0	0	1	0	0	0	0	0	120/4= 30
0	S_6	80	7	5	3	2	0	0	0	0	0	0	-1	0	0	0	0	80/0= \square
0	S_7	130	3	4.4	10	15	30	0	0	0	0	0	0	1	0	0	0	130/30=4.333333
0	S_8	100	3	4.4	10	15	0	0	0	0	0	0	0	0	0	-1	0	100/15=6.666667
	$Z_j - C_j$	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	

Table 1: table 1 shows the optimized value after the first iteration. After doing the same operation as above tables hence we get the optimal solution in the 4th iteration.

CB_i	C_j	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	
	B. Sol.	X_1	X_2	X_3	X_4	\square	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8				
0	S_4	2.5	0	0	0	-0.13	0	-0.08	0	0.8	1	0	0	0	0	0	0	-0.0580
0	S_2	15.35	0	0	0	-0.78	0	0.5	1	-0.9	0	0	0	0	0	0	0	-0.3570
0	X_2	5.65470	1	0	0	-2.1110	0	-0.3580	0	6.59	0	0	0.83	0	0	0	0	-0.7320
0	X_3	7.79760	0	1	0	2.081	0	0.036	0	-1.81	0	0	-0.166	0	0	0	0	0.264
0	S_5	38	0	0	0	-0.1020	0	-0.0650	0	-0.12	0	1	1	0	0	0	0	-0.0460
0	X_1	4.04761	0	0	0	0.90	0	0.24	0	-3.93	0	0	-0.66	0	0	0	0	0
0	S_8	15	0	0	0	-0.76	0	-0.48	0	-0.90	0	0	0	0	0	0	0	0.651
1	\square	0.5	0	0	0	0.025	1	0.016	0	0.030	0	0	0	0	0	0	0	0.011
			0	0	0	0.025	0	0.016	0	0.030	0	0	0	0	0	0	0	0.011

Table 2: table 2 shows the optimized value after the fourth iteration

Here, we get the values such that $X_1 = 4.0476$,

$$X_2 = 5.6547,$$

$$X_3 = 7.7976 \quad \text{and} \quad \alpha_0 = 0.5$$

Hence, the results obtained for this problem by two-phase method are

$X^* = (4.0476, 5.6547, 7.7976, 0)$. The optimal value $\alpha^* = 0.5$ and from (13), we get

$$Z = 4(4.0476) + 5(5.6547) + 9(7.7976) + 11(0) = \mathbf{114.642}$$

We note that, that the values of X_1, X_2 and X_3 in the membership grade of each constraints of (13) we achieve $\mu_0(x^*) = \mu_1(x^*) = \mu_3(x^*) = \alpha = 0.5$ and $\mu_2(x^*) = 0.8303$

The second phase is to solve the following problem.

$$\text{Max } \alpha = \alpha_0 + \alpha_1 + \alpha_2 + \alpha_3$$

$$\text{s.t.} \quad 0.5 \leq \alpha_i, i = 0, 1 \text{ and } 3$$

$$0.8303 \leq \alpha_2$$

$$4x_1 + 5x_2 + 9x_3 + 11x_4 + 30.71429\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 99.218$$

$$4x_1 + 5x_2 + 9x_3 + 11x_4$$

$$\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 130$$

$$x_1 + x_2 + x_3 + x_4 + 5\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 20$$

$$x_1 + x_2 + x_3 + x_4 = 15$$

$$7x_1 + 5x_2 + 3x_3 + 2x_4 + 40\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 120$$

$$7x_1 + 5x_2 + 3x_3 + 2x_4$$

$$\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 80$$

$$3x_1 + 4.4x_2 + 10x_3 + 15x_4 + 30\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 130$$

$$3x_1 + 4.4x_2 + 10x_3 + 15x_4$$

$$\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 100$$

$$x_1, x_2, x_3, x_4 \geq 0, 0.5 \leq \alpha_0 \leq 1, 0.5 \leq \alpha_1 \leq 1, 0.5 \leq \alpha_3 \leq 1 \text{ and } 0.8303 \leq \alpha_2 \leq 1$$

The standard form of linear of linear programming problem is shown below:

$$\text{Max } Z = 0x_1 + 0x_2 + 0x_3 + 0x_4 + 0\alpha_0 + \alpha_1 + 0\alpha_2 + 0\alpha_3$$

$$\alpha_i - s_i = 0, i = 0, 1 \text{ and } 3$$

$$\alpha_2 - s_4 = 0$$

$$4x_1 + 5x_2 + 9x_3 + 11x_4 + 30.71429\alpha_0 - s_5 = 99.218$$

$$4x_1 + 5x_2 + 9x_3 + 11x_4$$

$$+s_6 = 130$$

$$x_1 + x_2 + x_3 + x_4 + 5\alpha_0 + s_7 = 20$$

$$x_1 + x_2 + x_3 + x_4 + s_8 = 15$$

$$7x_1 + 5x_2 + 3x_3 + 2x_4 + 40\alpha_0 + s_9 = 120$$

$$7x_1 + 5x_2 + 3x_3 + 2x_4 + s_{10} = 80$$

$$3x_1 + 4.4x_2 + 10x_3 + 15x_4 + 30\alpha_0 + s_{11} = 130$$

$$3x_1 + 4.4x_2 + 10x_3 + 15x_4 + s_{12} = 100$$

$$x_1, x_2, x_3, x_4 \geq 0, 0.5 \leq \alpha_0 \leq 1, 0.5 \leq \alpha_1 \leq 1, 0.5 \leq \alpha_3 \leq 1 \text{ and } 0.8303 \leq \alpha_2 \leq 1$$

After solving above system we got The optimal solution is $X^{**} = (4.1, 5.58, 7.8, 0)$ and $Z = 114.5$

$$\text{and } \mu_0(x^{**}) = \mu_1(x^{**}) = \mu_3(x^{**}) = \alpha = 0.5 \text{ and } \mu_2(x^{**}) = 1$$

It is easy to see that not only achieves the optimal objective value but also attains higher grade of $\mu_2(x^{**})$. Thus, the satisfaction levels obtained by our method and the ones obtained by two phase method are

same. After tried other weights, we obtain following solutions

$$X^{**} = (0, 12.17, 6.39, 0) \text{ and } Z = 118.36$$

$$\text{and } \mu_0(x^{**}) = 0.63, \mu_1(x^{**}) = 0.29, \mu_3(x^{**}) = 0.43 \text{ and } \mu_2(x^{**}) = 1$$

Through having lower / higher degrees of choice for constraints, optimisation values may be obtained by the latter outcome. It should be done to the benefit of the DM effectively

Conclusion

In this paper, introduction, history, development and importance of operations research have been studied. Basic definitions of fuzzy sets and fuzzy linear programming problem have been reviewed. Membership functions of triangular fuzzy number have been given. The new fuzzy linear programming problem model is constructed that gives the Decision maker autonomy by enabling for the limitations and impartial roles of preferred different weight allocations. A method to solve fuzzy linear programming problem with grade of satisfaction in constraint has been discussed. Also the method has been illustrated with numerical example. The previous optimization found that optimal attributes can be obtained by having better / significantly larger choice ratios for the constraints. To the approval of the DM, such function may be easily utilized.

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