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Virial Theory For Self Action Effects Of Laser Beams In Preformed Plasma Channels

Naveen Gupta[1], Sandeep Kumar, Rajendersingh

Department of Physics, Lovely Professional University, Phagwara
naveens222@rediffmail.com

Abstract: **Self action effects of intense laser beams interacting nonlinearly with preformed parabolic plasma channels have been investigated theoretically. Particularly the phenomena of self focusing, self channeling and self phase modulation of the laser beam arising as a consequence of the optical nonlinearity of the plasma channel have been investigated. Due to nonuniform irradiance over the laser beam, the ponderomotive force becomes active that alters the spatial density profile of the channel. As the index of refraction of the plasma depends on electron concentration, there is a resultant modification of the index of refraction of the channel in the presence of laser beam. This modification in the optical response of the plasma channel in turn alters the propagation dynamics of the beam envelope. Following Virial theory, a semi analytical solution of the wave equation for beam envelope has been obtained. Emphases are put on investigation of beam with and axial phase of the laser beam.**

Keywords: q-Gaussian, ponderomotive force, virial theory.

1.Introduction

The expansion of the transverse dimensions of a propagating beam is ubiquitous for all kind of waves including electro-magnetic waves and even matter waves. It would seem that this kind of spreading is inevitable and therefore irreducible, since it originates at a fundamental level from light's natural wave property of diffraction. However, in 1964 Chio et al[1,2] showed that in media whose index of refraction depends on the intensity of light, the spreading of an optical beam in principle can be obviated. Hence, the expansion of optical beam due to diffraction is neither inevitable nor irreducible. In nonlinear media, the optical properties like index of refraction, dielectric function, absorption coefficient etc get modified in presence of an intense optical beam. The resulting change in index of refraction resembles to that of a graded index fiber and thus the beam automatically gets accumulated towards its axis. This phenomena is known as self-focusing[3]. When self focusing gets exactly balanced by diffraction the laser beam then propagates in a stable self trapped mode.

Since its discovery by Askaryan[3], the phenomenon of self focusing is at the vanguard of experimental as well as theoretical investigations due to its relevance in number of applications. In the design of ultra intense laser systems such as those being used in laser driven nuclear fusion the phenomenon of self focusing plays an important role. By producing intra cavity losses it can limit the cavity intensity or can slow down the release of optical energy. It can also alter the transverse profile of the laser beam by producing wave front distortions.

It is well known fact that with respect to a reference on axis plane wave, a converging/diverging electromagnetic beam undergoes modulation of its axial phase[7]. This self-phase modulation of the axial phase of a laser beam is also known as Gouy phase shift, which is a matter of debate over past few years. Since its discovery[7], various theories

(ranging from classical[4] to quantum [6, 11] have been used to explain its origin. Classically self phase modulation of the laser beam arises due to the contribution of an additional phase per unit length in the neighborhood of beam focal spot arising from the second order derivative of field amplitude with respect to transverse coordinates. However, in quantum mechanical terms the longitudinal phase shift is considered as a purely geometrical effect, resulting as a consequence of modification of volume of space available for the propagation of the beam. The consequent change in the transverse momentum of the photon changes the longitudinal momentum as well that in turn modifies the longitudinal phase of the laser beam. Precise knowledge of the behavior of the overall phase of an optical beam is important to control and understand laser driven processes. In laser cavities it plays a significant role in determining the resonant frequencies of the transverse modes[26]. By increasing the frequency degeneracy of the resonant modes of the cavity, axial phase shift can influence the beam quality achieved in the laser resonator.
Plasmas can respond nonlinearly to the intense laser beams mainly through three mechanisms. These mechanisms are: (1). nonlinearity due to Ponderomotive force[16, 23] (2).nonlinear Ohmic heating[8, 18]. Relativistic mass nonlinearity[1,22]. First two mechanisms give nonlinear response of the plasma via modification of electron density due to ponderomotive force and Ohmic heating of plasma electrons, respectively. In relativistic mechanism modification in optical properties of plasma occurs without any modification in electron density. This nonlinearity immediately comes into picture when the power of incident laser is greater than the threshold for self-focusing and thus it does not show any transient behavior. In this mechanism optical properties of plasma become a function of intensity of the electromagnetic beam due to change in electron mass whenit starts oscillating at a velocity comparable to that of light.
Laser beams differing in intensity profile behave differently in nonlinear media. From literature review[2, 5, 9, 10, 12-17, 24] it has been seen that most of the earlier investigations on self action effects of laser beams have been carried out under the framework of paraxial theory by assuming that the irradiance over the beam wavefronts is ideal Gaussian. Paraxial theory over simplifies the analysis by Taylor expansion of the nonlinear index of refraction of the medium up to second power of the transverse coordinates. Thus, this approximation may be valid up to some extent for Gaussian beams where most of the intensity is concentrated in a narrow region around the axis. However, by experimentally analyzing the irradiance over the cross section of Vulcan laser, Patel et al[20] have reported that although the laser is operating in TEM00 mode, its irradiance is not exactly Gaussian. A significant amount of energy lies in regions away from the axis. By putting into the data reported by Patel et al, it was suggested by Nakatsutsumi et al[19] that the actual irradiance over the beam can be modeled by Tsalli's[25-27] q-Gaussian distribution. The paraxial theory approach is not appropriate for q-Gaussian laser beams, as they contain significant amount of energy in their off axial region. The aim of this paper is to give theoretical investigation on self action effects of q-Gaussian laser beams in preformed plasma channels with the help of Virial theory[12, 28-30] which is free from the limitations of paraxial theory.

2. Ponderomotive Nonlinearity of Plasma Channel

Consider the interaction of an intense laser beam with a preformed plasma channel. The axis of the plasma channel is along the z axis and the laser beam axis is coinciding with the channel axis. The electron density of the plasma channel varies parabolically with radial coordinate as[18]

$$n(r) = n_0 + \Delta n \frac{r^2}{r_{ch}^2} \tag{1}$$

where, n_0 is the axial electron density of the channel, Δn is the difference in the electron density at the edge of the channel and that at the axis and r_{ch} if the radius of the plasma channel. Due to the transverse intensity gradient over the laser beam, the plasma electrons experience a ponderomotive force

$$\boldsymbol{F}_p = \frac{e^2}{4m\omega_0^2}\nabla(AA^*)$$

Due to this ponderomotive force the plasma electrons migrate from high intensity region of the illuminated portion of the channel towards its edges. This migration of electrons thus, increases the depth of the channel. The modified electron density is given by

$$n_e = n(r)e^{-\beta AA^*} \tag{2}$$

where, $\beta = \frac{e^2}{(8\,K_0 T_0 m\,\omega_0^2)}$is the constant associated with the strength of ponderomotive force. Hence, the dielectric function

$$\epsilon = 1 - \frac{4\pi e^2}{m\omega_0^2} n_e$$

of the plasma channel in the presence of the laser beam gets modified as

$$\epsilon = 1 - \left(\frac{\omega_{p0}^2}{\omega_0^2} + \frac{\omega_{pch}^2}{\omega_0^2 r_{ch}^2} r^2\right) e^{-\beta AA^*} \tag{3}$$

where,

$$\omega_{p0}^2 = \frac{4\pi e^2}{m} n_0$$

And

$$\omega_{pch}^2 = \frac{4\pi e^2}{m} \Delta n$$

Splitting the dielectric function of the plasma channel into linear and nonlinear parts as

$$\epsilon = \epsilon_0 + \phi(AA^*)$$

we can write

$$\epsilon_0 = 1 - \frac{\omega_{p0}^2}{\omega_0^2} e^{-\beta AA^*|_{r=0}} \tag{4}$$

and

$$\phi(AA^*) = \frac{\omega_{p0}^2}{\omega_0^2} e^{-\beta AA^*|_{r=0}} - \left(\frac{\omega_{p0}^2}{\omega_0^2} + \frac{\omega_{pch}^2}{\omega_0^2 r_{ch}^2} r^2\right) e^{-\beta AA^*} \tag{5}$$

Eq.(5) gives the optical nonlinearity of plasma channel due to the Ponderomotive force. Thus, it is also known as Ponderomotive dielectric function of the plasma channel.

3. Evolution of Beam Width

The model equation governing the evolution of an optical beam through a nonlinear medium is

$$i\frac{\partial A}{\partial z} = \frac{1}{2k_0}\nabla_\perp^2 A_0 + \frac{k_0}{2\epsilon_0}\phi(AA^*)A \tag{6}$$

As the potential function ϕ is a function of filed amplitude A, eq.(6) is nonlinear and nature and thus, it does not follow superposition principle. Hence, conventional methods of solving partial differential equations are not applicable to eq.(6). Thus, in order to have physical insight into the propagation dynamics of the laser beam, semi analytical methods such as variational method, method of moments, source dependent expansion method etc are used for this equation.

In the present study we have used Virial theory to obtain an approximate solution of eq.(6). The basic idea of moment theory is the selection of a trial function containing the physical parameters of interest. This trial function characterizes the actual solution of the problem as close as possible. The Virial theory then recasts the original problem of solving a PDE into that of solving a set of ODEs governing the evolution of these parameters. In the present analysis we assume A(r; z) takes the form of the function given by

$$A(r,z) = \frac{E_{00}}{f}\left(1 + \frac{r^2}{q r_0^2 f^2}\right)^{-\frac{q}{2}} e^{i\theta} \tag{7}$$

where, the parameter f(z) is currently undetermined and upon multiplication with initial beam width r_0 it gives the waist size of the laser beam at particular location inside the plasma channel. Also, upon dividing by axial intensity, f(z) also gives the estimate of instantaneous axial intensity of the laser beam. Hence, the parameter f(z) can be referred to as dimensionless beam width parameter. The phenomenological parameter q is related to the deviation of irradiance over the beam cross section from ideal Gaussian profile. The function $\theta(z)$is known as longitudinal phase of the laser beam which is also known as Gouy phase. The Virial theory defines the effective beam width of an optical beam in root mean square (r.m.s) sense as

$$\langle r^2 \rangle = \frac{1}{I_0}\int A r^2 A^* d^2 r \tag{8}$$

where,

$$I_0 = \int A A^* d^2 r$$

and

$$d^2 r = r dr d\theta$$

Differentiating eq.(8) twice with respect to z and making use of eq.(6) it can be shown that the effective beam width of the beam envelope evolve according to the equation

$$\frac{d^2 f}{d\xi^2} = \frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}\frac{1}{f^3} - \left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right) J - \frac{1}{f}\left(\frac{df}{d\xi}\right)^2 \tag{9}$$

Where,

$$J = \frac{\beta E_{00}^2}{f^3}\left(\frac{\omega_{p0}^2 r^2}{c^2} I_1 + f^2 \frac{\omega_{pch}^2 r_0^2}{c^2}\frac{r_0^2}{r_{ch}^2} I_2\right) + \frac{\omega_{p0}^2 r_0^2}{c^2}\frac{r_0^2}{r_{ch}^2} f^2 I_3$$

$$I_1 = \int_0^\infty t\left(1+\frac{t}{q}\right)^{-2q-1} e^{-\frac{\beta E_{00}^2}{f^2}\left(1+\frac{t}{q}\right)^{-q}} dt$$

$$I_2 = \int_0^\infty t^2\left(1+\frac{t}{q}\right)^{-2q-1} e^{-\frac{\beta E_{00}^2}{f^2}\left(1+\frac{t}{q}\right)^{-q}} dt$$

$$I_3 = \int_0^\infty t\left(1+\frac{t}{q}\right)^{-q} e^{-\frac{\beta E_{00}^2}{f^2}\left(1+\frac{t}{q}\right)^{-q}} dt$$

$$t = \frac{r^2}{r_0^2 f^2}$$

$$\xi = \frac{z}{k_0 r_0^2}$$

Eq.(9) is similar to the equation of motion of a driven oscillator with unit mass. Hence, it follows that Virial theory has converted the problem of nonlinear wave propagation to a simple Newton's like mechanical problem. Although, eq.(9) also cannot be solved analytically, but its approximate solution can be easily obtained with the help of simple numerical techniques.

4. Propagation in Self-Trapped Mode

If while entering into the plasma channel, the laser beam possesses a plane wavefront i.e., if f = 1 and $\frac{df}{d\xi} = 0$ at $\xi = 0$, then the condition $\frac{d^2f}{d\xi^2} = 0$ will maintain will maintain their values throughout the journey of the beam through the channel. Such mode of propagation, when there is no change in the beam width of the laser beam, is called self trapped mode or spatial optical soliton. Hence, for$\frac{df}{d\xi} = \frac{d^2f}{d\xi^2} = 0$, eq.(9) gives the relationbetween dimensionless beam width $r_e = \frac{\omega_{p0}r_0}{c}$and the criotical beam intensity βE_{00}^2as

$$r_e^2 = \frac{1}{I_1'}\left\{\left[\frac{1}{\left(1+\frac{1}{q}\right)} - \frac{\omega_{p0}^2 r_0^2}{c^2}\frac{r_0^2}{r_{ch}^2}I_3'\right]\frac{1}{\beta E_{00}^2} - \frac{\omega_{pch}^2 r_0^2}{c^2}\frac{r_0^2}{r_{ch}^2}I_2'\right\} \quad (10)$$

$$I_1' = I_1|f = 1$$
$$I_2' = I_2|f = 1$$
$$I_3' = I_3|f = 1$$

The βE_{00}^2vs. re curve is known as critical curve that for a given values of q an Δn, divides the (βE_{00}^2, re) plane into three regions shown in figs.(7) and (8). Each region of(βE_{00}^2, re) plane characterizes a different regime of propagation of the laser beam.

The laser beam for which the point (βE_{00}^2, re) lies on the critical curve defined by eq.(10),will$\frac{d^2f}{d\xi^2}$ vanish at $\xi = 0$. This simply means that during the journey of the laser beamthrough the channel there will be no change in the curvature of the wavefront i.e., $\frac{df}{d\xi}$willremain constant and value of this constant will be equal to initial value, that we havetaken to be zero. Hence, $\frac{d^2f}{d\xi^2} = \frac{df}{d\xi} = 0$ at $\xi = 0$ indicates that $\frac{df}{d\xi} = 0$ for $\xi > 0$ also.Physically, this means that there will be no change in the beam width of the laser beam during its propagation. In this mode the laser beam is said to constitute a spatial soliton. Thus, the region of space lying on the critical curve corresponds to self channeling of the laser beam.

If the point (βE_{00}^2, re) lies above the critical curve, then the initial value of $\frac{d^2f}{d\xi^2}$ will bepositive and hence f will increase monotonically with distance. This mode of propagation is known as self broadening of the beam.

If the point (βE_{00}^2, re)lies below the critical curve then initial value of $\frac{d^2f}{d\xi^2}$ will benegative and thus f will decrease with distance. This mode is known as self focusing of the laser beam. Thus, the region below the critical curve corresponds to self focusing.

5. Evolution of Axial Phase

Using eq.(7) in (6) and equating imaginary parts we get

$$-2k_0\frac{d\theta_p}{dz}=\frac{r^2}{r_0^2f^4}\left(1+\frac{2}{q}\right)\left(1+\frac{r^2}{qr_0^2f^2}\right)^{-2}-\frac{2}{r_0^2f^2}\left(1+\frac{r^2}{qr_0^2f^2}\right)^{-1}+\frac{\omega_0^2}{c^2}\phi(AA^*) \quad (11)$$

Taking zeroth order spatial intensity moment of this equation, we get

$$\frac{d\theta_p}{dz}=\frac{1}{f^2}\left(1-\frac{1}{q}\right)-\frac{1}{2f^2}\frac{\left(1-\frac{1}{q}\right)\left(1-\frac{2}{q}\right)}{\left(1+\frac{1}{q}\right)}-\frac{1}{2}\left(1-\frac{1}{q}\right)I_4 \quad (12)$$

where,

$$I_4=\int_0^\infty\left(1+\frac{1}{q}\right)\left\{\frac{\omega_{p0}^2r_0^2}{c^2}e^{-\frac{\beta E_{00}^2}{f^2}}-\left[\frac{\omega_{p0}^2r_0^2}{c^2}+\frac{\omega_{pch}^2r_0^2}{c^2}\frac{r_0^2}{r_{ch}^2}f^2t\right]e^{-\frac{\beta E_{00}^2}{f^2}\left(1+\frac{t}{q}\right)^{-q}}\right\}dt$$

Eq.(12) is the governing equation for the evolution of axial phase of the laser beam along the length of the plasma channel.

6. Results and Discussion

In solving eqs.(9) and (12) it has been assumed that while entering into the channel the laser beam is collimated and is having plane wave front. Mathematically these conditions define the boundary conditions f=1, $\frac{df}{d\xi}$=$\theta = 0$ at ξ=0. In the present study eqs.(9) and (12) have beensolved for a typical set of parameters: $\omega_0 = 1.78\times10^{14} rad\ sec^{-1}$, $r_0 = 15\mu m$, $n_0 = (1.5\times10^{18}, 2\times10^{18}, 2.5\times10^{18})cm^{-3}$, $\Delta n = (0, 10, 15)cm^{-3}$, $E_{00} = (3\times10^9, 6\times10^9, 9\times10^9)Vm^{-1}$ and q=(3,4,∞).The corresponding evolutions of beam with and axial phase of the beam are shown in the formof graphs in figs1-12.

The graphs in _g.1 illustrate the effect of irradiance over the laser beam i.e., the effect ofq on evolution of its effective beam width through the channel. Breather like behavior i.e., harmonic variations in the beam width during the propagation through the plasma channel, can be clearly seen from the plots in fig.1. These harmonic variations of the beam width can be explained by examining the role and origin of various terms contained in eq.(9). The first term on the R.H.S, that varies as f^{-3}, the spatial dispersive term that models the spreading of the laser beam in transverse directions occurring as a consequence of diffraction divergence. The second term which is having complex dependence on f, arises due redistribution of carriers and is responsible for nonlinear optical response of the plasma to pump beam. As a consequence of this optical nonlinearity of the plasma the resulting nonlinear refraction of the laser beam tends to counter balance the effect of diffraction. The third term that depends on beam width as well as on rate of change of beam with accelerates or decelerates the focusing/defocusing of the laser beam depending on its sign. Thus, during its journey through the plasma channel, the restarts a competition between the two phenomena of diffraction and nonlinear refraction of the laser beam. The winning phenomenon ultimately decides the behavior of the laser beam i.e., whether the beam will converge or diverge. Thus, there exists a critical value of beam intensity (that can be obtained by equating right hand side of eq.(9) with zero) above which the beam will converge. In the present investigation the initial beam intensity has been taken greater than the critical intensity i.e., why the laser beam is converging initially. As the beam width decreases, its intensity increases. When the laser intensity becomes too high, the illuminated portion of the plasma gets almost completely evacuated from plasma electrons and thus the nonlinear refraction vanishes, leaving only the diffraction effects to dominate. Hence, after focusing to minimum, the beam width bounces back to its original value. As the beam width of the laser beam starts increasing, the competition between diffraction and nonlinear refraction starts again. Now, this competition lasts till maximum value of f is obtained. These processes go on repeating themselves and thus give breather like behavior to the beam envelope.

Reduction in focusing of the laser beams with their irradiance closer to Gaussian profile can also be seen from fig.1. The reason behind this effect is that for laser beams with higher value of q most of the intensity is concentrated in a narrow region around the axis of the beam. Hence, these beams get a get a very less contribution from the off axial part in order to produce nonlinearity in the medium. As the self focusing is a homeostasis of nonlinear refraction of the laser beam due to optical nonlinearity of the medium, increase in the value of q reduces the extent of self focusing.

The plots in fig.1 also indicate that instead of their reduced focusing, the laser beams with higher value of q possess faster focusing character. This is due to the faster focusing character of axial rays. Being away from the axis, off axial rays take more duration to get self focused. As there are more number of off axial rays in laser beams with lower q values, hence by increasing the value of deviation parameter q, the focusing of the laser beam becomes faster. This result of faster focusing of laser beams with lower q value is contrary to that reported by Sharma and Kourakis where it was shown that laser beams with lower value of q possess slower focusing. This difference between the two results is mainly due to the reason that in their analysis Sharma and Kourakis have expanded the dielectric function of the plasma only up to r^4. However, by analyzing the q-Gaussian distribution it can be seen that change in the value of q is having hardly any effect on the irradiance in the regions closer to the axis. The change in the value of q affects the irradiance only in the regions away from the beam axis that has been eliminated in the analysis of Sharma and Kourakis. However, in our study the dielectric function of the plasma has been considered as a whole. Thus, it can be concluded that by optimizing the value of q, one can control focusing as well as the location of the focal spot of a laser beam.

The graphical curves in fig.2 depict the variations of beam wavefront curvature. The periodic changes in the sign of wave front curvature indicates that during the journey of the laser beam through the plasma channel, its wave front periodically changes form plane wave front to convex and then again from convex to plane and then from plane to concave. All these processes keep on repeating in an oscillatory fashion.

The similarity of eq.(9) with mechanical problem of an oscillator allows us to define a 2-D space spanned by coordinates, where f plays the role of displacement and $\dot{f}$ plays that of velocity. Hence, the space $\left(f, \dot{f}\right)$ is known as phase space for the beam width. The phase space trajectories (fig.3) depict the nature of oscillations of the beam width of the laser beam. The spiral trajectories indicate the quasi periodicity of the oscillations i.e., the oscillations containing several frequencies. It can be seen that as the irradiance over the laser beam converges towards Gaussian, the number of frequencies in the oscillations of the beam width increases i.e., the phase space trajectories are becoming more spiral. This is due to the faster focusing character of the laser beams with higher q value.

The curves in fig.(4) are showing the effect of electron density on focusing of the laser beam. It can been seen that increase in plasma density enhances the focusing of the laser beam. This is due to the fact that increase in electron density makes the ponderomotive force stronger. As the focusing of the laser beam is occurring as a consequence of ponderomotive force acting on the electrons, increase in electron density enhances the self focusing of the laser beam.

The graphical curves in fig.(5) show the effect of channel depth on focusing of the laser beam. It can be seen that for a given set of laser and plasma parameters minimum focusing is occurring in the absence of radial inhomogeniety i.e., for $\Delta n = 0$ and with increasing channel depth focusinggets enhanced and occurs earlier. This is due to the reason that the channel depth enhancesthe radial inhomogeniety of the index of refraction. As parabolic plasma

channels are analogous to graded index fibers based on total internal reflection, increase in channel depth enhances the focusing of the beam.
The effect of initial intensity of the laser beam on its self focusing has been depicted in fig.6.It can be seen that laser beams with higher focusing possess higher focusing character. This is due to that fact that like the electron density, the intensity of the laser beam also makes the ponderomotive force stronger. Thus, initial laser intensity also enhances its focusing.
Figs. 7 and 8 portray the critical curves of the laser beam for different values of q and Δn, respectively. It can be seen that initially with slight increase in beam intensity the critical beam width decreases abruptly. This indicates that laser beams with higher intensity can be self trapped at smaller spot size. Further, it can be seen that if we keep on increasing the intensity the beam width becomes almost independent of the intensity. This is because at very high intensity, the portion of the illuminated region of the plasma channel is almost evacuated from the electron density. Hence, the laser beam propagates as if it is propagating through a region of vacuum which is being surrounded by a medium whose index of refraction is less than1. Upward shifting of the critical curves with increase in the deviation parameter q can also be seen from fig.7. This indicates that for a given radius the laser beams with higher q require higher intensity to get self trapped. This is because, laser beams with higher q get a very little off axial contribution towards nonlinear refraction.
The curves in fig.8 shows that with increase in the channel depth the critical curves shift downwards. This indicates that for given set of parameters laser beams can be easily self trapped in deeper plasma channels. This easiness of self trapping increases with increase in depth of the channel. This is because the radial inhomogeniety of the channel gets added to the nonlinear refraction and thus enhances its effect. Figs.(9)-(12) illustrate the evolution of axial phase of the laser beam with distance along the plasma channel. It can be seen that the axial phase of the laser beam decreases monotonically with distance, showing step like behavior. This is due to the periodical self focusing/defocusing of the laser beam. As the laser beam undergo self focusing, its intensity increases and hence, the laser phase fronts start experiencing larger refractive indices. This results in decreased phase velocity of the phase fronts that leads to decreased spacing between the phase fronts. This fact can be explained in another way. The axial phase shift of the laser beam occurs due to the transverse momentum gained by the photons due to reduction in the volume of space available for their propagation. As the transverse spatial confinement of the laser beam occurs due to self focusing, the photons gain additional transverse momentum (k_x; k_y) due to position momentum uncertainty $\Delta k_x \Delta x = constant$ and $\Delta k_y \Delta y = constant$. As the over all momentum ofthe laser beam is conserved, the increase in the transverse momentum reduces the longitudinalmomentum. Thus during the propagation of laser beam its longitudinal momentum reduces asthe beam keep on focusing. This results in monotonic decrease in its axial phase.
Step like behavior of the axial phase can also be seen from figs.(9)-(12). These steps occur at the periodical positions of the minima of the beam width i.e., at the positions of focal spots of the beam. This indicates that there is slowest decrement in axial phase near its focal point. This is quite contrary to the behavior of the phase in graded index fibers where the phase decreases slowest at the positions of minimum intensity i.e., at the positions of maximum beam width. This difference in the behavior of phase in graded index fibers and that in plasmas can be explained qualitatively by the fact that plasma as a nonlinear medium behave like a linear wave guide. In linear wave guides the growth rate of the axial phase is inversely proportional to the beam width.
It has been seen from fig.(9) that with the increase in the value of deviation parameter q there is a reduction in the rate of change of axial phase of the laser beam with distance. This is due to the one to one correspondence between the extent of focusing of the laser beam and the

rate of decrease of its axial phase. As with increasing q the focusing of the laser beam gets reduced and hence, the rate of change of axial phase also reduces with increase of deviation parameter q.
The curves in figs.(10)-(12) indicate that with increase in either of plasma density, channel depth or laser intensity, the rate of decrease of axial phase increases. This is due to enhancement of focusing of the laser beam with increase in these parameters.

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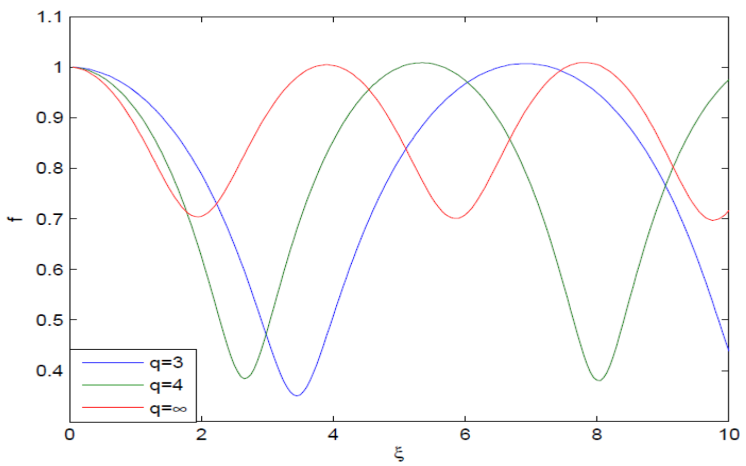

Figure 1. Effect of deviation parameter ‘q’ on self-focusing of laser beam.

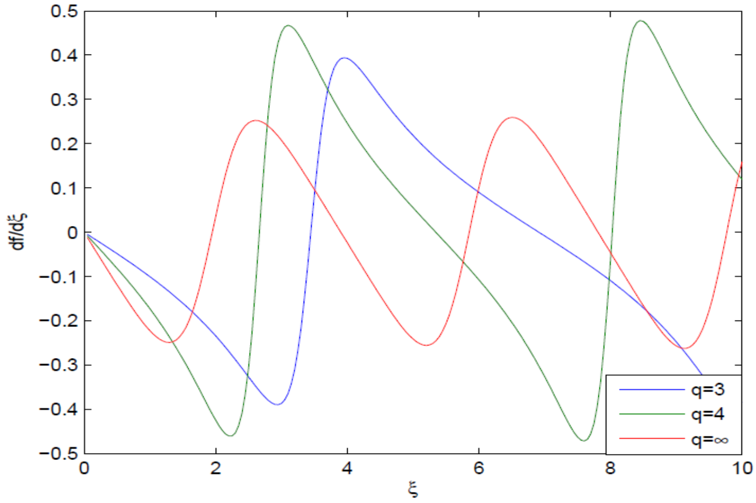

Figure 2. Effect of ‘q’ on laser beam wavefront curvature during its propagation.

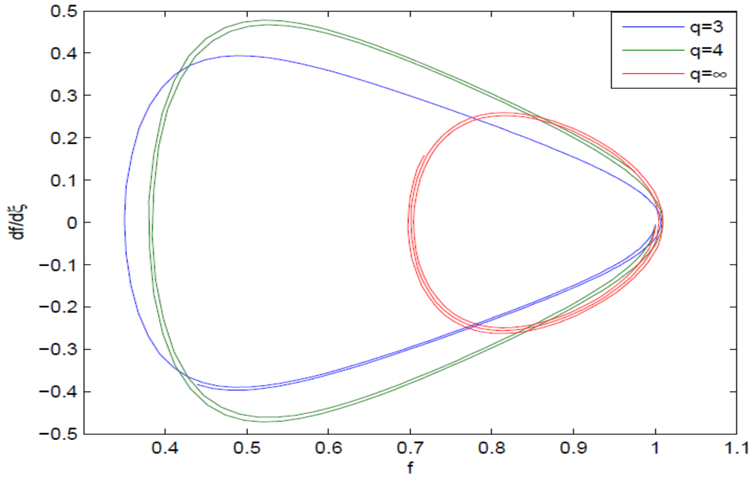

Figure 3. Phase space trajectories for beam width of laser beam.

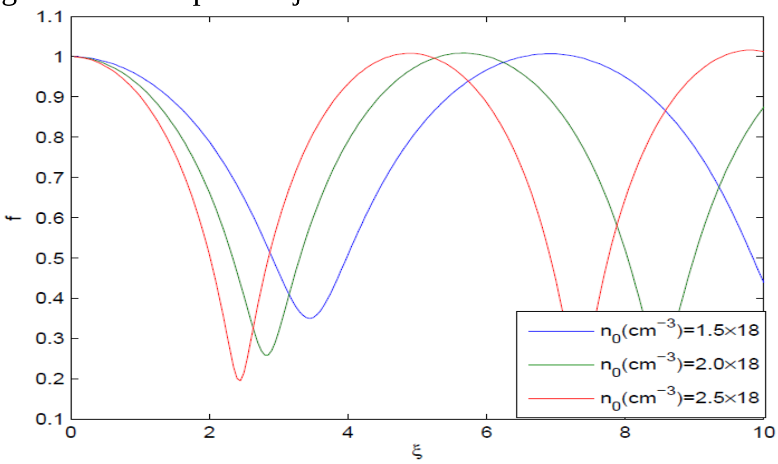

Figure 4.Effect of plasma density on self-focusing of laser beam.

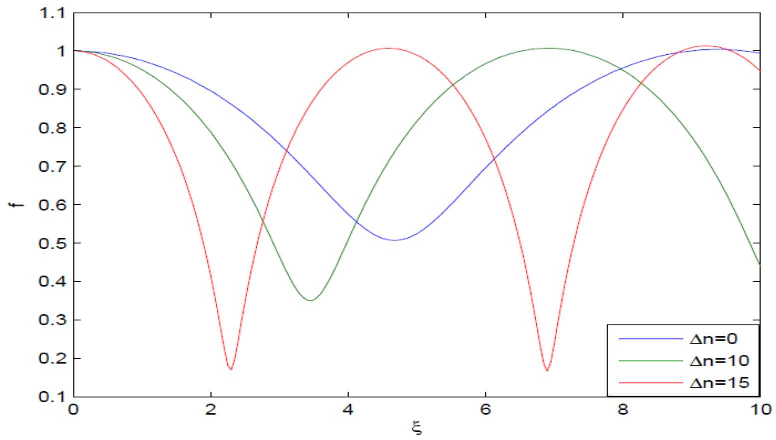

Figure 5. Effect of channel depth on self-focusing of laser beam.

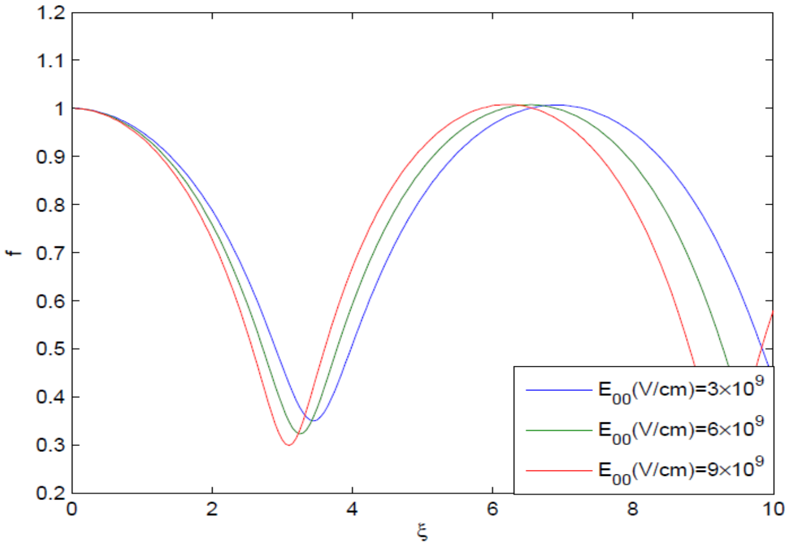

Figure 6. Effect of laser field amplitude on its self-focusing

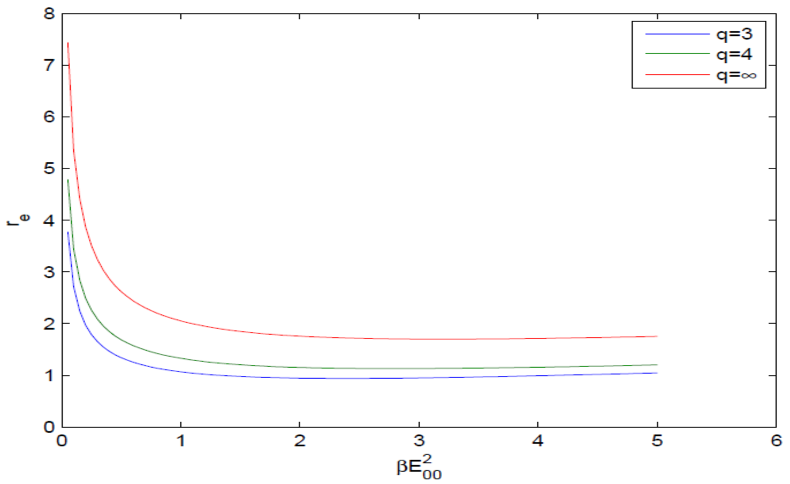

Figure 7. Effect of deviation parameter q on self trapping of the laser beam.

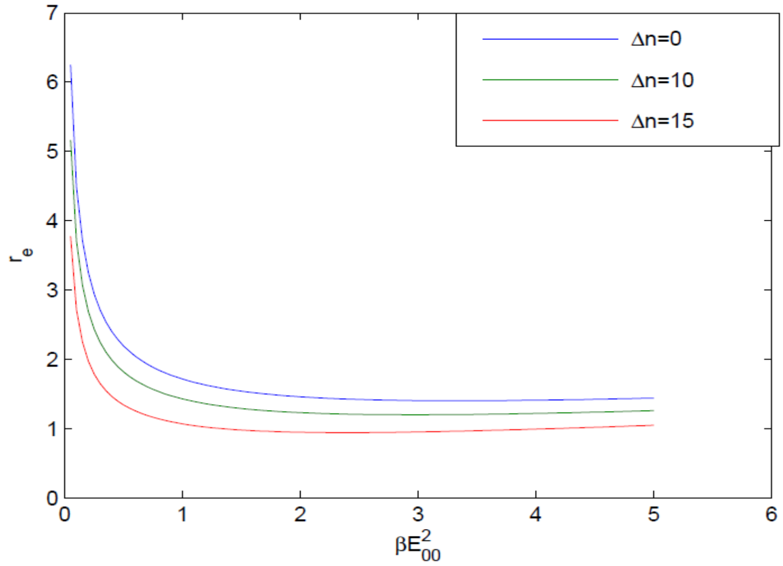

Figure 8. Effect of channel depth on self trapping of laser beam.

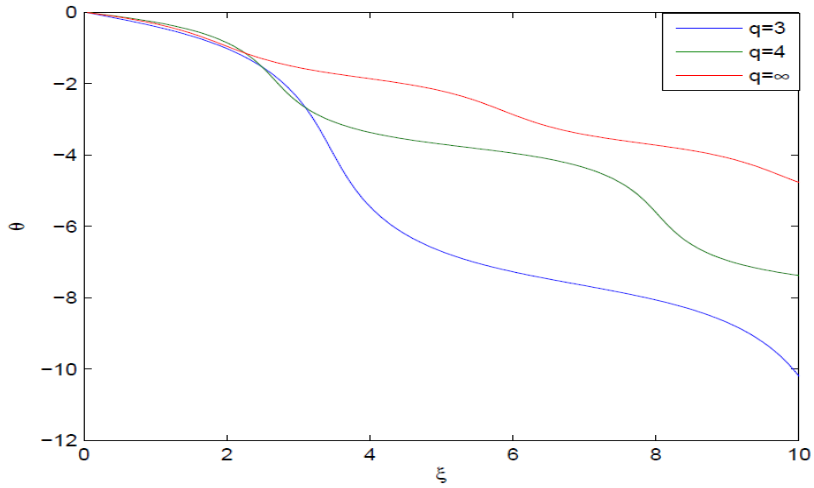

Figure 9. Effect of deviation parameter q on axial phase shift of laser beam.

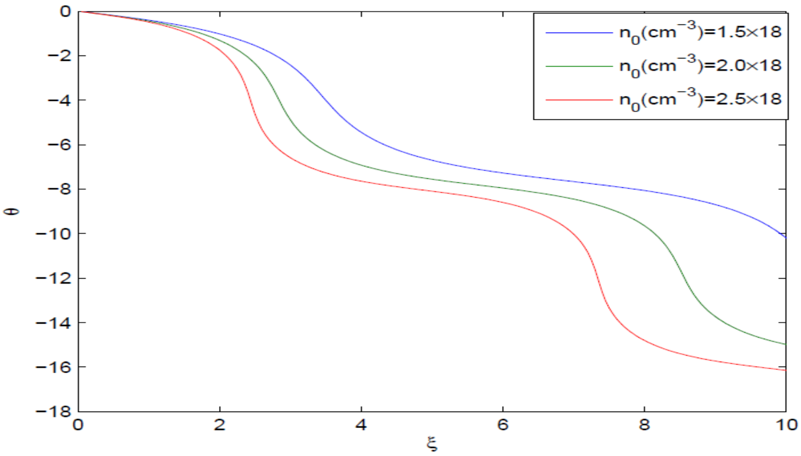

Figure 10. Effect of plasma density on axial phase shift of laser beam.

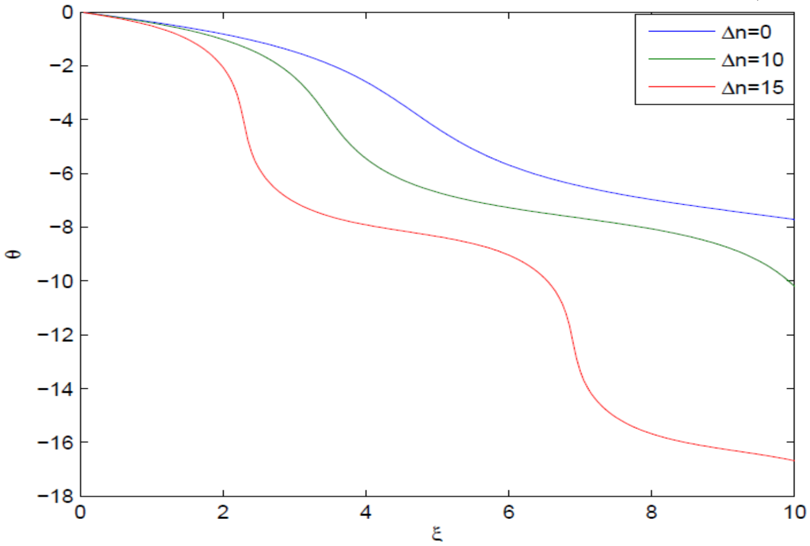

Figure 11. Effect of channel depth on axial phase shift of laser beam.

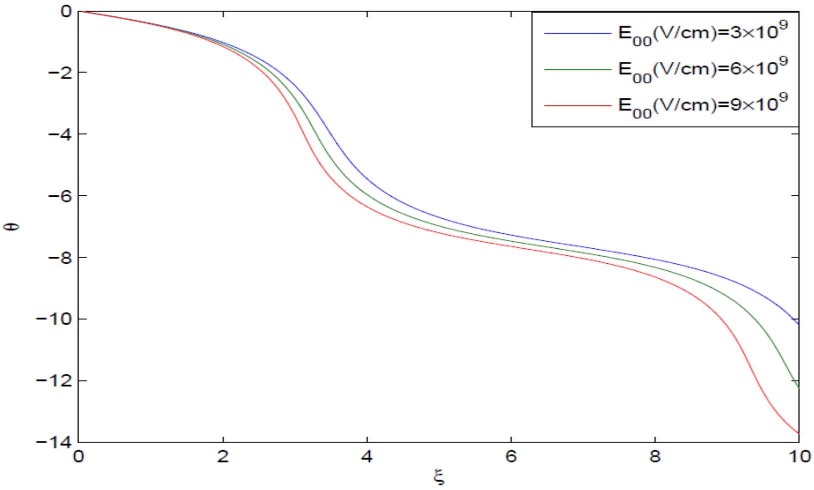

Figure 12. Effect of laser field amplitude on its axial phase shift.