

Analysis Of Various Exponential Curve Fitting: Linearization Of Non-Linear Laws

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ABSTRACT: To express the given data approximately, there are several possible equations of different types, which one can attain. The best fitted curve may be of different degree polynomial or any exponential curve. The curve fitting plays a vital role in the study of theoretical and practical aspects of daily life problems. In experimental work, we repeatedly met the problem of fitting a curve to data which are subjected to errors. In present paper applications has been explored with help of MATLAB for linearization of non-linear laws and linear, quadratic and cubic fit with residuals are represented graphically.

1. INTRODUCTION

In fact, we are facing challenge to get the equation of the curve of “best fit” the process adopting is known as Curve fitting. The method of least squares is probably the best to fit a unique curve to a given data. The selection of the curve is matter of experience and practical considerations. Often engineers, scientists, organizers and sociologists have to take some decisions concerning the phenomenon of which they know only the behaviour from experimental measurements. In certain cases, for example in physics, the fundamental knowledge of phenomenon in questions allows us in proposing a precise, deterministic mathematical model which we call the model of knowledge. Hence the concept of curve fitting serves our purpose. In applied mathematics, for considered variables which are connected by some predefined laws, it is required to express the given data to attain best fit. Many authors [S. S. Sastry[1], Erwin Kreyszig [2], J.N. Sharma [3] have outlined the least square method with theoretical aspect . In the present paper, some examples have been explored with linearization of non-linear laws and linear, quadratic and cubic fit with residuals are represented graphically with help of MATLAB [4-6].

2. APPLICATIONS: LINEARIZATION OF NON-LINEAR LAWS

For $y = ax + \frac{b}{x}$ we can substitute $Y = xy; X = x^2$

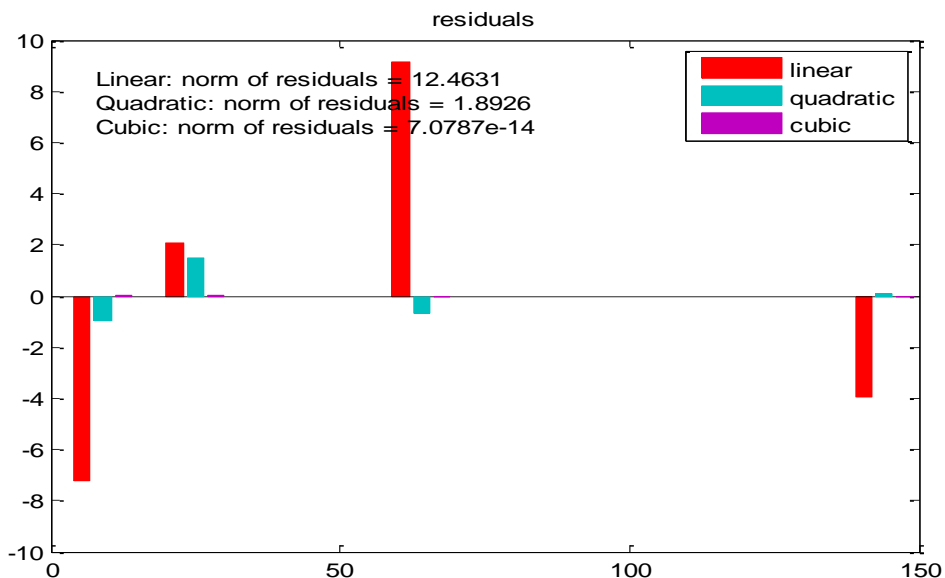
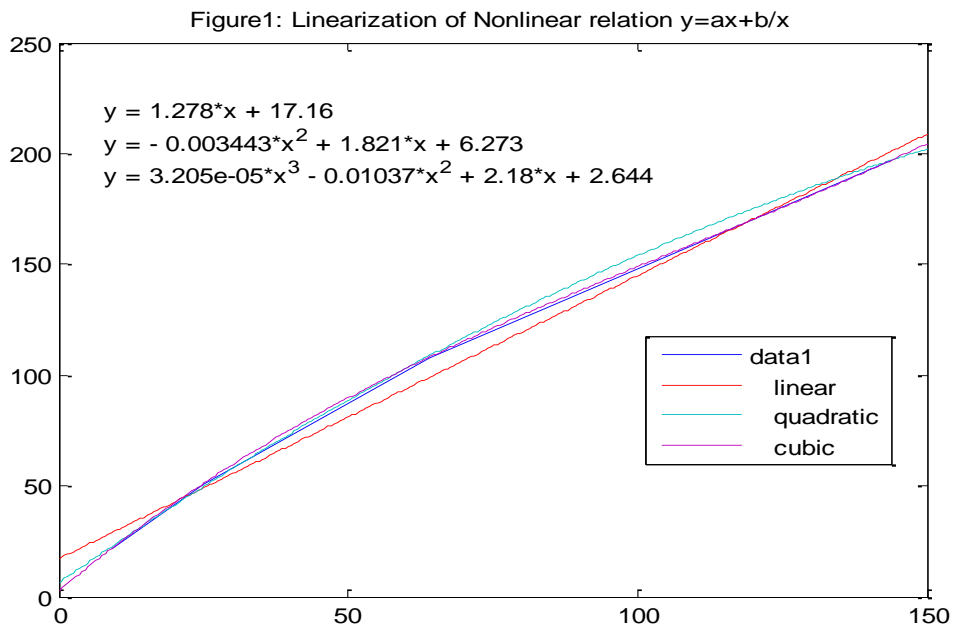
Example 1: Find best possible fit $y = ax + \frac{b}{x}$, for the following data given below:

$x:$	3	5	8	12
$y:$	7.148	10.231	13.509	16.434

```
>> x=[3 5 8 12];  
>> y=[7.148 10.231 13.509 16.434];  
>> X=x.^2  
X =
```

```
9 25 64 144
>> Y=x.*y
Y =
 21.4440  51.1550 108.0720 197.2080
>> polyfit(X,Y,1)
ans =
 1.2779 17.1597
>> plot(X,Y)
>> title('Figure1: Linearization of Nonlinear relation y=ax+b/x')
```

Hence the required curve is of the form $y = 1.2779x + \frac{17.1597}{x}$ and its linear fitting curve is $Y = 17.1597 + 1.2779X$.



For $y = ax^b$ we can substitute $Y = \log y$; $X = \log x$ and $\log(a) = A_0$; $A_1 = b$

Example 2: Find best possible fit $y = ax^b$, for data given below:

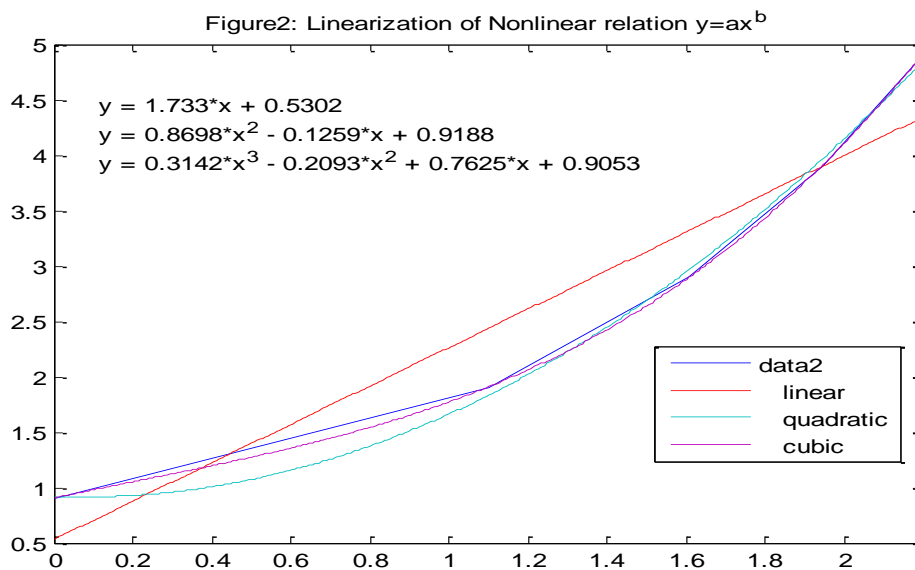
x :	1	3	5	7	9
y :	2.473	6.722	18.274	49.673	135.026

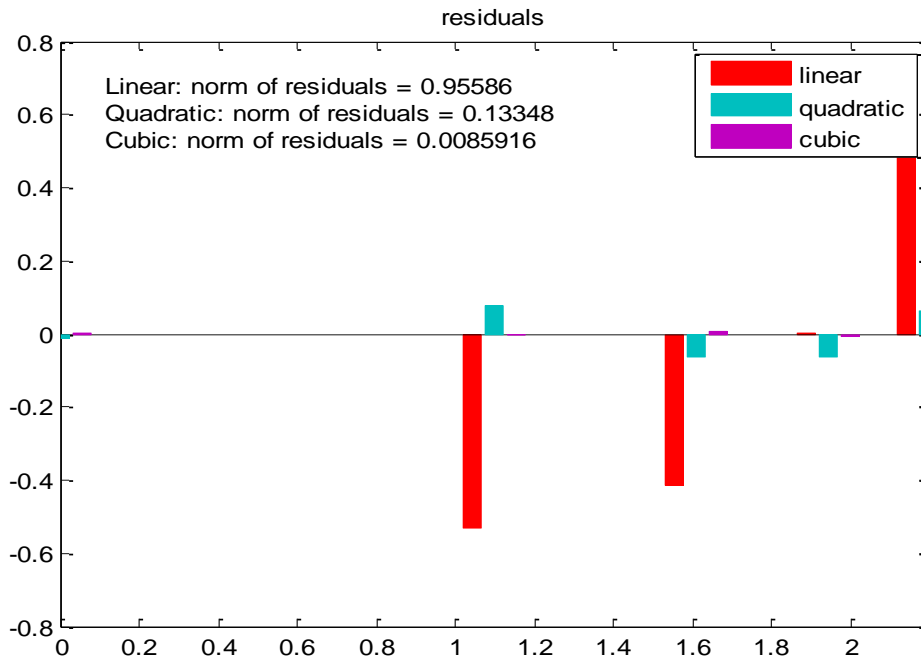
```
>> x=[1 3 5 7 9];
>> y=[2.473 6.722 18.274 49.673 135.026];
>> Y=log(y)
Y =
    0.9054    1.9054    2.9055    3.9055    4.9055

>> X=log(x)
X =
    0    1.0986    1.6094    1.9459    2.1972

>> polyfit(X,Y,1)
ans =
    1.7335    0.5302
>> plot(X,Y)
>> title('Figure2: Linearization of Nonlinear relation y=ax^b')
>> a=exp(0.5302)
a =
    1.6993
>> b=1.7335
b =
    1.7335
```

Hence the required curve is of the form $y = 1.6993x^{1.7335}$ and its linear fitting curve is $Y = 0.5302 + 1.7335X$.





Example3: Following are the data obtained in an experiment:

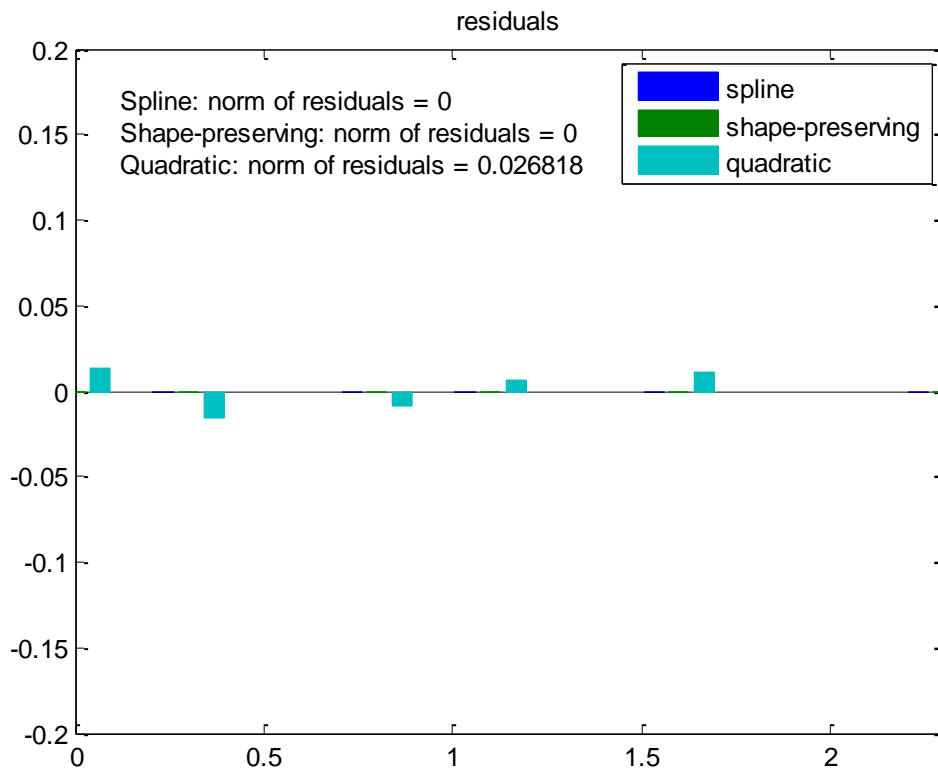
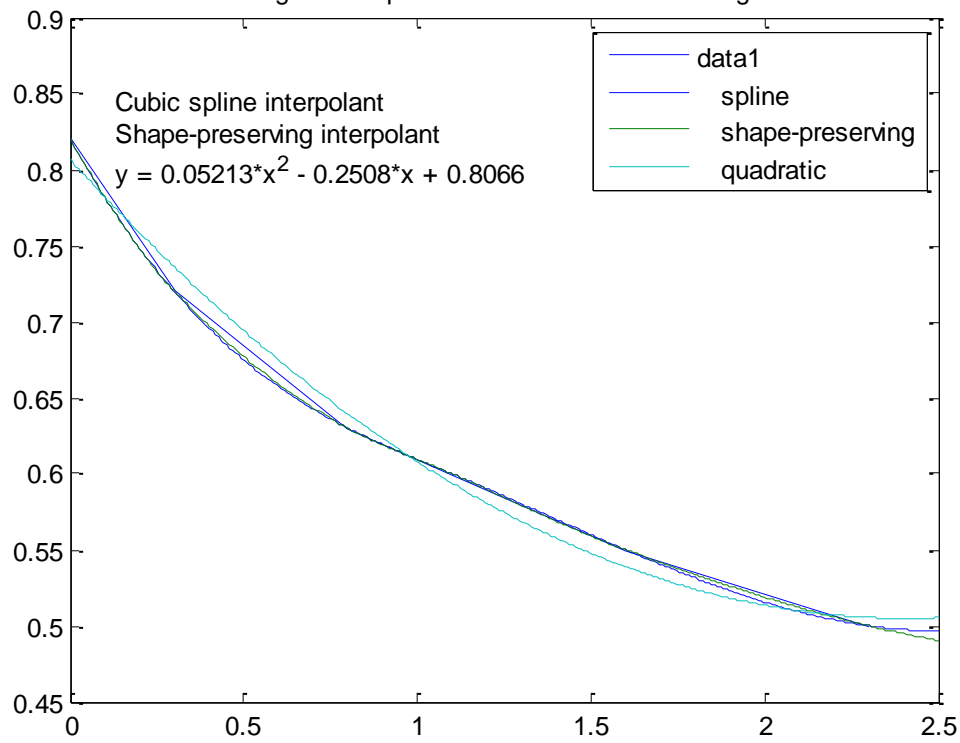
t:	0	0.3	0.8	1.1	1.6	2.3
y:	0.82	0.72	0.63	0.60	0.55	0.50

(i) Fit this data into an Exponent equation given by $y = a_0 + a_1e^{-t} + a_2te^{-t}$

```
>> t=[0 .3 .8 1.1 1.6 2.3]';
>> y=[.82 .72 .63 .60 .55 .50]';
>> x=[ones(size(t)) exp(-t) t.*exp(-t)]
x =
    1.0000    1.0000         0
    1.0000    0.7408    0.2222
    1.0000    0.4493    0.3595
    1.0000    0.3329    0.3662
    1.0000    0.2019    0.3230
    1.0000    0.1003    0.2306
>> a=x\y
a =
    0.4635
    0.3507
    0.0320
>> plot(t,y)
>> title('Exponential Function Curve Fitting')
>> title('Figure3: Exponential Function Curve Fitting')
```

Hence the polynomial is given by $y = 0.4635 + 0.3507e^{-t} + 0.0320te^{-t}$

Figure3: Exponential Function Curve Fitting



Example4: Fit the function $z = a_0 + a_1x + a_2y$ for the data given below:
 x: 0 1 2 4 6

y:	0	1	3	2	8
z	2	4	3	16	8

```
>> x=[0 1 2 4 6]';  
>> y=[0 1 3 2 8]';  
>> z=[2 4 3 16 8]';  
>> t=[ones(size(x)) x y]
```

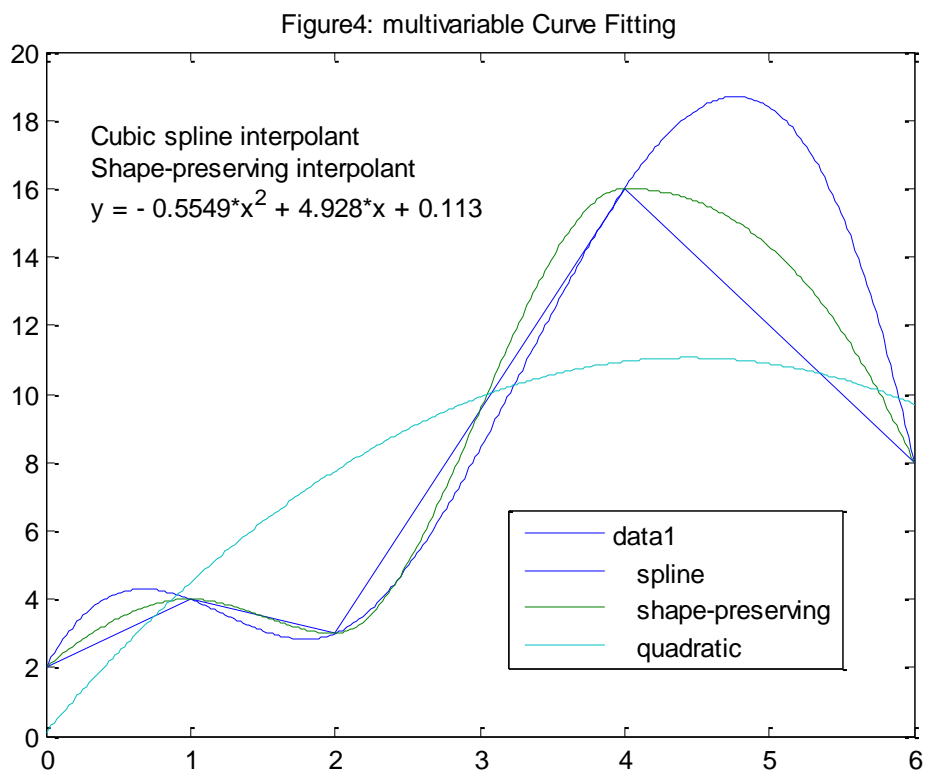
```
t =  
 1  0  0  
 1  1  1  
 1  2  3  
 1  4  2  
 1  6  8
```

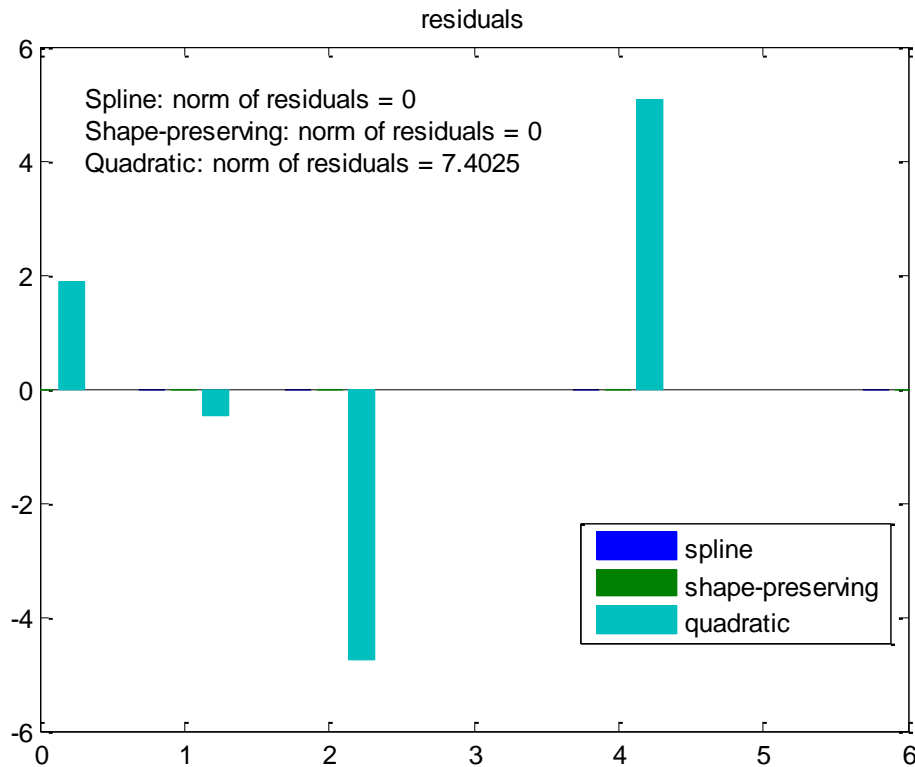
```
>> a=t\z  
a =
```

```
 2.0000  
 5.0000  
-3.0000
```

```
>> plot(x,z)  
>> title('Figure4: multivariable Curve Fitting')
```

Hence the polynomial is given by $z = 2 + 5x - 3y$





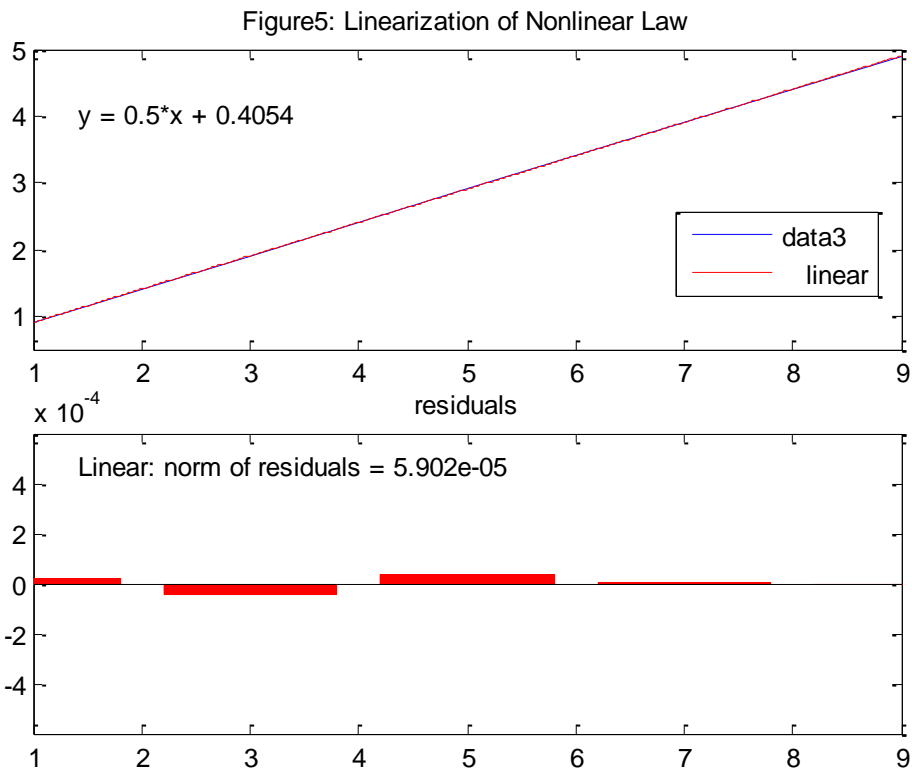
LINEARIZATION OF NON-LINEAR LAWS: For $y = ae^{bx}$ we can substitute $Y = \log y; X = x$ and $\log(a) = A_0; A_1 = b$

Example 5: Find best possible fit $y = ae^{bx}$, for data given below:

x:	1	3	5	7	9
y:	2.473	6.722	18.274	49.673	135.026

```
>> x=[1 3 5 7 9];
>> y=[2.473 6.722 18.274 49.673 135.026];
>> Y=log(y)
Y =
    0.9054    1.9054    2.9055    3.9055    4.9055
>> X=x
X =
     1     3     5     7     9
>> polyfit(X,Y,1)
ans =
    0.5000    0.4054
>> plot(X,Y)
>> title('Figure5: Linearization of Nonlinear Law')
>> a=exp(0.4054)
a =
    1.4999
>> b=0.5000
b =
    0.5000
```

Hence the required curve is of the form $y = 1.4999e^{0.5x}$ and its linear fitting curve is $Y = 0.4054 + 0.5X$.



LINEARIZATION OF NON-LINEAR LAWS: For $y = \frac{x}{a+bx}$ we can substitute $Y = \frac{1}{y}$; $X = \frac{1}{x}$ and $b = A_0$; $a = A_1$

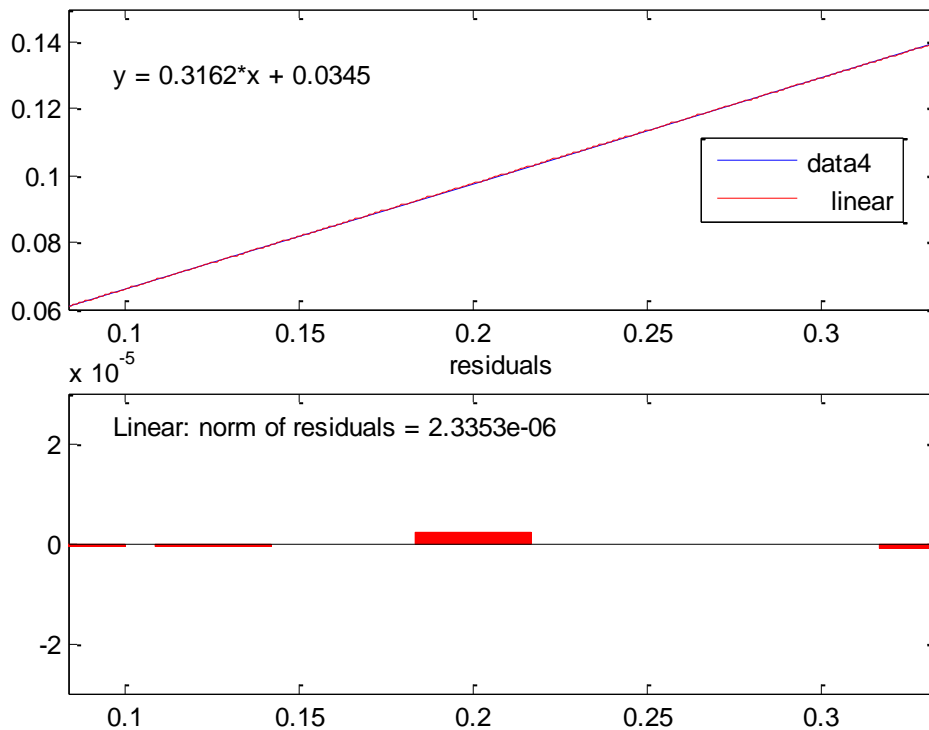
Example 6: Find best possible fit $y = \frac{x}{a+bx}$, for the following data given below:

x :	3	5	8	12
y :	7.148	10.231	13.509	16.434

```
>> x=[3 5 8 12];
>> y=[7.148 10.231 13.509 16.434];
>> X=1./x
X =
    0.3333    0.2000    0.1250    0.0833
>> Y=1./y
Y =
    0.1399    0.0977    0.0740    0.0608
>> polyfit(X,Y,1)
ans =
    0.3162    0.0345
>> plot(X,Y)
>> title('Figure6: Linearization of Nonlinear relation')
```

Hence the required curve is of the form $y = \frac{x}{0.3162+0.0345x}$
and its linear fitting curve is $Y = 0.3162X + 0.0345$.

Figure6: Linearization of Nonlinear relation



CONCLUSIONS

It has been observed that MATLAB is very useful tool for curve fitting. With help of this we can get the different degree equations for the curve fitting in terms of best fit. Hence we may compute slopes and residuals. The computed equations for linearization of non-linear laws and linear, quadratic and cubic fit with residuals same as analytical results available in literature. By using MATLAB we can save lot of time in comparison of theoretical calculations and computed results may be expressed in terms of multiple curve fits like linear, cubic and spline fit simultaneously.

REFERENCES

- [1] S. S. Sastry, Introductory Methods of Numerical Analysis. PHI Learning Private Limited Delhi, 5th Edition 2015.
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- [6] Mukherjee, R., Huang, Z. F., & Nadgorny, B. (2014). Multiple percolation tunneling staircase in metal-semiconductor nanoparticle composites. *Applied Physics Letters*, 105(17), 173104.