

# Mathematical Modeling of Ink Transition of Print Product

Ulbosin Eshbaeva<sup>1</sup>, Anvar Djalilov<sup>2</sup>, Fazliddin Turaev<sup>3</sup>, Safaeva Dilafuz<sup>4</sup>

<sup>1</sup>*Doctor of Technical Sciences, Professor, Department of Chemical Technology, Namangan Institute of Engineering and Technology, Namangan, Uzbekistan*

<sup>2</sup>*Senior teacher, Department of Technology of printing and packing industry, Tashkent Institute of Textile and Light Industry, Tashkent, Uzbekistan*

<sup>3</sup>*Assistant teacher, Department of Technology of printing and packing industry, Tashkent Institute of Textile and Light Industry, Tashkent, Uzbekistan*

<sup>4</sup>*Assistant teacher, Department of Technology of printing and packing industry, Tashkent Institute of Textile and Light Industry, Tashkent, Uzbekistan*

***Abstract:*** This article describes information about static simulation, a preliminary analysis of ink perception by offset printing method on papers, including synthetic polymers. The optimal choice is obtained based on the study of the model of ink perception during offset printing on printed materials, which allows predicting the ink perception of printed materials. The results of the study can facilitate orientation in determining the composition of experimental paper in its production.

***Keywords:*** ink film, print, ink transfer coefficient, print quality, regression equations.

## 1. INTRODUCTION

The printing process's main purpose is to transfer to the print in full the information contained in the original. In this case, the main difficulty lies in the fact that a large number of input parameters affect the result of the printing process. Correspondence of the accuracy of the copy to the original is characterized by indicators or quality criteria of the printed image. The quality of printed matter is largely determined by the properties of paper. The quality of printed matter is largely determined by the properties of paper. Since ink is applied to the paper, the quality of the prints substantially depends on the degree of correspondence between the ink and the paper on which it is applied. The properties of paper have a significant impact on the quality of printing processes, as well as on the operational characteristics of printing products [1-4].

The quality of the print, which ones proximity to the original in visual sensations directly depends on the most significant properties of the printed material, the quality of which is most often limited to measuring its parameters such as whiteness, smoothness, color brightness, opacity and transparency [5-7]. The printing properties of paper manifest themselves differently under different printing conditions (Fig. 1).

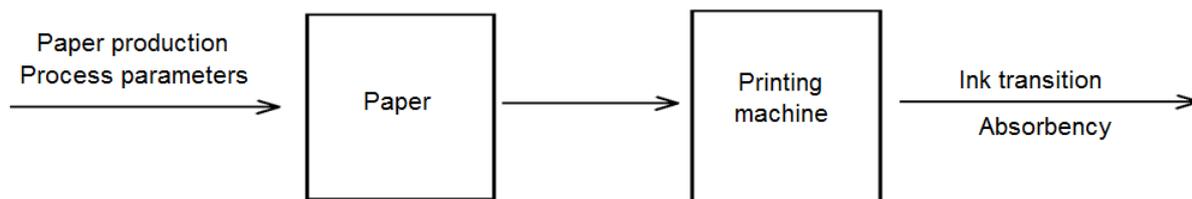


Fig. 1. Factors affecting print quality

The ability of paper to absorb ink under certain parameters of a particular printing process depends on the smoothness of the paper, the composition and structure. Consequently, under pressure ink penetrates into the pores and capillaries in the paper. The ink transfer (print) carries information about this distribution of ink on the surface of the paper, as well as about the whiteness and smoothness of the paper. Thus, the absorbency of paper is manifested as a result of the printing process [8-10].

The transfer coefficient of the ink on the printed material was determined by comparing the mass of the printed material before and after printing. The specifics of the interaction of the printing surface with the surface of the ink layer and the influence of technological factors on graphic image distortions when printing on paper also affects the gradation characteristics of the image [11-12].

In the formation of complex indicators of print quality subjective and objective methods are used in modern qualimetry of printing. The brightness of the print reflects the result of the paper absorbing ink, which manifests itself in the spatial filling of the surface and body of the paper with paint, determined by the composition of the effect of whiteness and smoothness of the paper [13-14].

## 2. RESEARCH METHODS

The aim of this work was to find patterns on the influence of paper production factors on print quality, to identify approaches to improving the printing properties of paper. It should be noted that the results for each specific paper, ink and type of printing process are specific and cannot be used directly for other combinations of paper, ink and printing. Therefore, it seems important to solve practical problems to improve the printing properties of papers.

We carried out a statistical analysis of calculating the distribution of the ink layer on the surface of the print, estimating the thickness of the ink layer on experimental papers (Table 1).

Table 1: Variants of the composition of two-layer castings

№ of options	Compositional structure		
	Toplayer 40-55 °shr		bottom layer 48-50 °shr
	CC *, %	SFW**, %	MC-2A, %
Sample №1	80	20	100
Sample №2	80	20	100
Sample №3	80	20	100
Sample №4	80	20	100

CC \* - cotton cellulose, SFW \*\* - synthetic fiber waste

To simplify the description of the process of transition of printing ink to the printed paper, we do not take into account the temperature of the printing ink. While maintaining the stability of the process parameters, we will consider only the following factors: the whiteness and smoothness of the paper. In order to establish the complex effect of various factors on the  $I_{trans}$  - ink transition of printed products, the method of mathematical design of the experiment was used [15-17]. The following parameters were taken as input parameters (factors):  $x_1$  - whiteness  $W$  (%) and  $x_2$  - smoothness  $G$  (s). Levels and intervals of variation of factors are presented in Table 2.

Table 2: Levels and intervals of variation of factors

№.	Factors	Code designation	Intervals of variation	Factor levels		
				top +1	primary 0	bottom -1
1	whiteness, %	$x_1$	10	60	50	40
2	Papers moothness, c	$x_2$	25	70	45	20

### 3. RESULTS AND DISCUSSION

The choice of the experimental area is based on the results of preliminary studies of the ink transfer and analysis of a priori information. In the experimental area, the main levels and ranges of variation of factors were established. The main (zero) levels of factors that are initial in the center of the plan are chosen so that their combination would correspond to the value of the optimization parameter ( $I_{trans}$ ), which is closest to the optimal one.

When constructing an experimental design, it is very important to choose variation intervals that cannot be chosen less than that with which the experimenter fixes the factor level and cannot be so large that the upper or lower levels go beyond the range of factors. An unreasonable increase in the intervals of variation can lead to the difficulty of a linear approximation of the response function [18-19].

The coded value of the quantitative factor  $x_i$  is determined by the formula

$$x_i = \frac{\hat{x}_i - \hat{x}_i^0}{\varepsilon_i},$$

where  $\hat{x}_i$  - is the natural value of the  $i$ -factor;  $\hat{x}_i^0$  - the natural value of the basic level of the  $i$ -factor;  $\varepsilon$  is the interval of variation of the  $i$ - factor.

In the present study, a complete factorial experiment was used in which all possible combinations of  $N$  levels of factors (the number of experiments) are determined by expression.

$$N = m^k, \tag{1}$$

where  $m$  - is the number of levels of each factor;  $k$  - is the number of factors.

For two factors, the full factorial experiment of type  $2^2$  is represented by the matrix shown in Table 3. The value of the response function ( $I_{trans}$  ink transfer) obtained during the experiments is denoted by  $y$ . At the first stage of mathematical planning, it is most expedient

to approximate the unknown response function  $y=f(x_1, x_2, \dots, x_k)$  by a polynomial of the first degree having the minimum number of coefficients  $b_i$  for a given number of factors  $b_i$  and, very importantly, contain the necessary information about the direction of the gradient - direction of the fastest optimization parameter optimization.

Table 3: Planning Matrix and Experiment Results

Experience Number	$x_0$	$x_1$	$x_2$	$x_1 \cdot x_2$	Optimization parameter, y
1	+	-	-	+	43
2	+	+	-	-	45
3	+	-	+		44
4	+	+	+	+	49

A polynomial of the first degree is expressed in general terms by the equation

$$y=b_0+b_1x_1+ b_2x_2+\dots + b_kx_k+ b_{12}x_1x_2 + b_{31}x_1x_3 + b_{12\dots k}x_1x_2\dots x_k,$$

for two factors, this equation has the form

$$y=b_0+b_1x_1+ b_2x_2+b_{12}x_1x_2, \quad (2)$$

where  $x_1, x_2$  - is the interaction effect characterizing the combined influence of two factors on the optimization parameter  $y$ .

The values of the coefficients of equation (2) are found from the following relationships:

1) free member  $b_0$

$$b_0 = \frac{1}{N} \sum_{j=1}^N y_j = \frac{1}{N} \sum_{j=1}^N x_{0j}y_j = \frac{182}{4} = 45,5$$

2) regression coefficients characterizing linear effects

$$b_i = \frac{1}{N} \sum_{j=1}^N x_{ij}y_j;$$

$$b_1=1,75; b_2=1,25,$$

3) regression coefficients characterizing the effects of interaction

$$b_{ij} = \frac{1}{N} \sum_{j=1}^N x_{ij}x_{lj}y_j;$$

$$b_{12}=0,75.$$

where  $i, l$  – are numbers of factors;  $j$  is the line or experience number in the planning matrix;  $y_j$  is the value of the optimization parameter in the  $j$  - experiment;  $x_{ij}, x_{lj}$  - are the encoded values ( $\pm 1$ ) of factors  $i$ -and  $l$  in the  $j$  experiment.

As a result of processing the experimental data, a regression equation with coded variables is obtained

$$y=45,5+1,75x_1+1,25x_2+0,75 x_{12}$$

The coefficients' statistical significance of the regression equation (3) is checked by comparing the absolute value of the coefficient with a confidence interval. To determine the

confidence interval, we preliminarily calculate the variance of the regression coefficients by the formula:

$$S^2\{b_i\} = \frac{1}{N} S_y^2,$$

where  $S^2\{b_i\}$  – is the variance of the  $I$  regression coefficient;  $S_y^2$  – dispersion of reproducibility of the experiment.

The confidence interval of the coefficients corresponds to

$$\Delta b_i = \pm t S\{b_i\},$$

where  $t$  - is the tabular value of the *Student* criterion, equal to 3.18 at a 5% level of significance and the number of degrees of freedom  $f = n_0 - 1 = 4 - 1 = 3$ .

The dispersion  $S_y^2$  of the optimization parameter is calculated from the results of four parallel experiments ( $n_0 = 4$ ) in the center of the plan, i.e. with  $x_1 = x_2 = 0$ . The calculation of the variance  $S_y^2$  is given in Table 4.

The dispersion of the regression coefficients and the confidence interval calculated by formulas (4) and (5), respectively, were

$$S^2\{b_i\} = 0,8325; \Delta b_i = \pm 0,92$$

In the table 4  $n_0$  - the number of responses in the center of the plan;  $y_u$  – is the value of the optimization parameter in the  $u$  - experiment in the center of the plan.

Table 4: Auxiliary table for calculating the variance of reproducibility  $S_y^2$  experiments

Experience number in the center of the plan	$y_u$	$\bar{y}$	$y_u - \bar{y}$	$(y_u - \bar{y})^2$	$S_y^2$
1	46	$\frac{\sum_{u=1}^4 y_u}{4} = 45,5$	0,5	0,25	$\frac{\sum_{u=1}^{n_0} (y_u - \bar{y})^2}{n_0 - 1} = \frac{\sum_{u=1}^4 (y_u - \bar{y})^2}{4 - 1} = \frac{1}{3} = 0,333$
2	45		-0,5	0,25	
3	46		0,5	0,25	
4	45		-0,5	0,25	
	$\sum_{u=1}^4 = 182$			$\sum_{u=1}^{n_0} (y_u - \bar{y})^2 = 1$	

Comparing the absolute values of the  $b_i$  regression coefficients with the confidence interval  $\Delta b_i (|b_1| > |\Delta b_i|, |b_2| > |\Delta b_i|, |b_{12}| > |\Delta b_i|)$ , we obtain the regression equation with coded variables:  
 $y = 45,5 + 1,75x_1 + 1,25x_2$  (6)

For the testing the adequacy hypothesis of the model represented by equation (6), we find the adequacy variance.

$$S_{ad}^2 = \frac{\sum_{j=1}^N (y_j - \bar{y}_j)^2}{f},$$

where  $y_j$  – is the experimental value of the optimization parameter in the  $j$  - experiment, calculated according to equation (6);  $f$  - is the number of degrees of freedom equal to  $f = N - (k + 1) = 4 - (2 + 1) = 1$ .

To make the amount included in expression (7), we will make an auxiliary table (table. 5). When calculating the values of  $\hat{y}_i$  in the equation (7), it is necessary to substitute the coded values of the factors.

Table 5: Auxiliary table for calculating  $S_{ad}^2$

Experiment number	$y_i$	$\hat{y}_i$	$y_i - \hat{y}_i$	$(y_i - \hat{y}_i)^2$
1	43	42,5	0,5	0,25
2	45	46	-1,0	1
3	44	44,5	-0,5	0,25
4	49	48,5	0,5	0,25

$$\sum_{j=1}^4 (y_j - \bar{y}_j)^2 = 1,75,$$

$$S_{ad}^2 = \frac{\sum_{j=1}^N (y_j - \bar{y}_j)^2}{f} = \frac{\sum_{j=1}^4 (y_j - \bar{y}_j)^2}{N - (k + 1)} = \frac{1,75}{4 - (2 + 1)} = 1,75$$

The model adequacy hypothesis was tested using the Fisher F-test. To do this, we determine the calculated value of the criterion

$$F_p = \frac{S_{ad}^2}{S_y^2} = \frac{1,75}{0,333} = 5,26 \quad (8)$$

At a 5% significance level and the number of degrees of freedom for the numerator (8)  $f1 = 1$  and for the denominator  $f2 = 3$ , the tabular value of the criterion is  $FT = 10.1$  [2]. As  $F_p < FT$ , the mathematical model represented by equation (6) is adequate.

Moving from coded  $x_1, x_2$  values of factors to natural, we obtain the dependence of the ink transfer  $I_{trans}$  of printed products on the above factors. The coded values of the factors are associated with the following natural relationships:

$$x_1 = \frac{W - W_0}{\varepsilon_1} = \frac{W - 50}{10}; \quad x_2 = \frac{G - G_0}{\varepsilon_2} = \frac{G - 45}{25}, \quad (9)$$

where  $W_0, G_0$  – are the basic levels of factors in physical terms;  $\varepsilon_1, \varepsilon_2$  - intervals of variation of factors.

Substituting expressions (9) into equation (6), we obtain

$$I_{trans} = 45,5 + 1,75 \frac{W - 50}{10} + 1,25 \frac{G - 45}{25}$$

and after the transformations we will present in the final form

$$I_{trans} = 34,5 + 0,175W + 0,05G \quad (10)$$

Equation (10), like equation (6), adequately describes the paint transfer model. Therefore, it can be used as an interpolation formula for calculating the values of the  $I_{trans}$  paint transition.

Thus, in the study of the ink transfer of printed products, equation (10) should be used to establish rational values for the whiteness and smoothness of paper (Table 6).

Table 6: Change in ink transfer from smoothness and whiteness of paper

W	G	$I_{trans}$	W	G	$I_{trans}$
40	20	42,5	60	20	46
	45	43,25		45	47,25
	70	45		70	48,5

G	W	$I_{trans}$	G	W	$I_{trans}$
20	40	42,5	70	40	45
	50	44,25		50	46,75
	60	46		60	48,5

To obtain a mathematical model for evaluating the ink transition of the sample of the printed material, when the ink is distributed between the smooth surface of the sample  $I_{trans} = I_{trans}(max)$  (ink is distributed almost completely in the surface layers of the paper, filling the hollows of the surface micro relief), we compiled graphs of the dependence of the transition on smoothness and whiteness (Fig. 2 and 3).

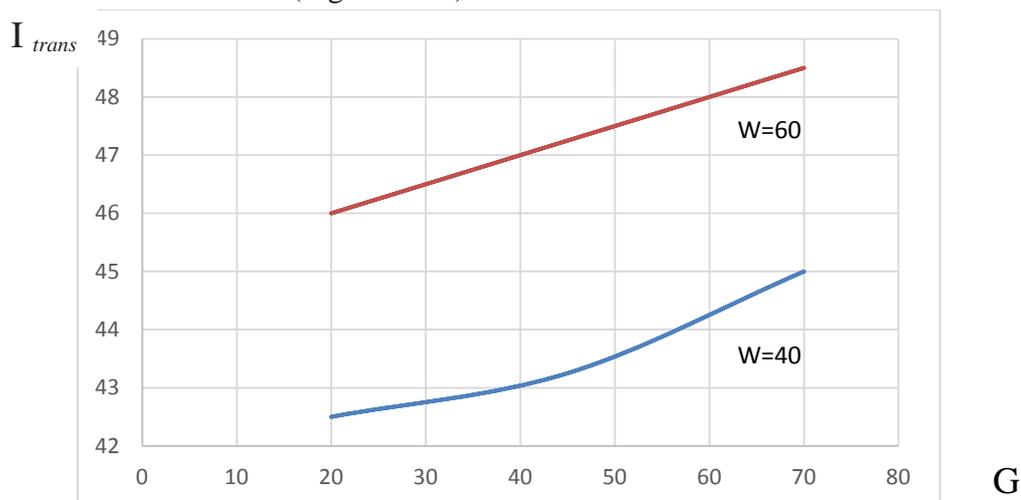


Fig. 2. Graphical modeling of the paint transition with varying smoothness

As can be seen from the table and figures, the paper arrangement according to the values of the ink transition varies depending on the smoothness of the paper. The smoothness of the paper also affects the fixing of ink on prints. Smoother papers give a lower depth of penetration of the ink, and therefore contribute to an increase in the optical density of the print. In this case, the paint will cover only the surface of the paper without falling into narrow recesses.

The relative amount of ink transferred to the paper ( $I_{trans}$ ), upon receipt of the print depends mainly on the size of the effective contact surface of the paper with the ink.

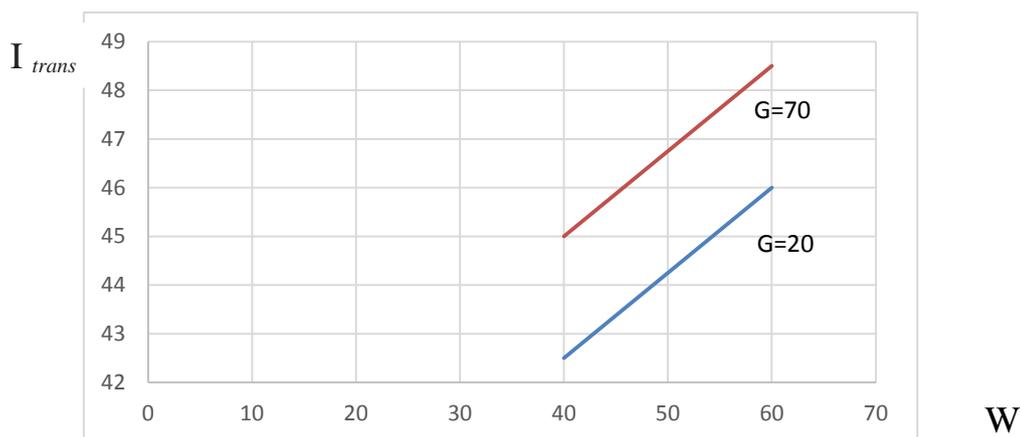


Fig. 3. Graphical modeling of the paint transition with varying whiteness

#### 4. CONCLUSIONS

Common to both curves is a sharp increase in  $I_{trans}$ ; it depends not only on the micro geometry of the paper surface, but also on the whiteness of the paper. An analysis of the results allows us to conclude that increasing the whiteness of paper increases the intensity and saturation of the chromatic color, which allows to obtain the desired optical effect with a smaller layer thickness. The optimal ink transition with the offset printing method  $I_{trans}= 48.5$  was achieved with a smoothness of 70 s, paper whiteness of 60%. Using the results of the study will help to obtain high-quality prints for given printing modes, i.e. at certain smoothness values of not less than 40 s, which can be achieved during paper casting by changing the composition or technological modes of the paper production process. The offered approach for assessing the characteristics of the properties of offset printing allows you to control the amount of transferred printing ink in the process of contact interaction with the ink-perceiving surface.

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