REGULAR DOMINATION IN INTUITIONISTIC FUZZY GRAPH

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Abstract: In this paper we define regular domination set and regular dominating number in Intuitionistic fuzzy graph and investigate some properties and bounds of regular domination number in various Intuitionistic fuzzy graphs.

Keywords: Intuitionistic fuzzy graph, regular dominating set and regular dominating number.

1. INTRODUCTION:

The first definition of fuzzy graphs was proposed by Kaufman, from the fuzzy relations introduced by Zadeh. Although Rosenfeld introduced another elaborated definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. The concept of domination in fuzzy graphs was investigated by A. Somasundaram, S. Somasundaram and A. Somasundaram present the concept of independent domination, total domination, connected domination of fuzzy graphs. C. Natarajanand S.K. Ayyaswamy introduce the strong (weak) domination in fuzzy graph. The first definition of intuitionistic fuzzy graphs was proposed by Atanassov. The concept of domination in Intuitionistic fuzzy graphs was investigated by R. Parvathi and G. Thamizhendhi.

In this paper we define regular domination set and regular dominating number in Intuitionistic fuzzy graph and investigate some properties and bounds of regular domination number in various Intuitionistic fuzzy graphs.

2. PRELIMINARIES

This section deals the some basic definitions of Intuitionistic fuzzy graphs. It is useful to construct the next section.

An Intuitionistic fuzzy graph (IFG) is of the form $G=(V,E)$, where
\[V = \{v_1, v_2, ..., v_n\}\] such that \(\mu_1: V \rightarrow [0,1], \gamma_1: V \rightarrow [0,1]\) denote the degree of membership and non-member ship of the element \(v_i \in V\) respectively and \(0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1\) for every \(v_i \in V\), \((i = 1, 2, ..., n)\). \(E \subseteq V \times V\) where \(\mu_2: V \times V \rightarrow [0,1]\) and \(\gamma_2: V \times V \rightarrow [0,1]\) are such that

\[
\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j),
\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \vee \gamma_1(v_j)
\]

and \(0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1\).

An arc \((v_i, v_j)\) of an IFG \(G\) is called an strong arc if

\[
\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j),
\gamma_2(v_i, v_j) = \gamma_1(v_i) \wedge \gamma_1(v_j).
\]

Let \(G = (V, E)\) be an IFG. The vertex cardinality of \(G\) is defined to be

\[
|v_i| = \left\lceil \frac{1 + \mu_1(v_i) - \gamma_1(v_i)}{2} \right\rceil
\]

for all \(v_i \in V\), \((i = 1, 2, ..., n)\).

Let \(G = (V, E)\) be an IFG. A set \(D \subseteq V\) is said to be a dominating set of \(G\) if every \(v \in V - D\) there exist \(u \in D\) such that \(u\) dominates \(v\).

An Intuitionistic fuzzy dominating set \(D\) of an IFG, \(G\) is called minimal dominating set of \(G\) if every node \(u \in D\), \(D - \{u\}\) is not a dominating set in \(G\). An Intuitionistic fuzzy domination number \(\gamma_f(G)\) of an IFG, \(G\) is the minimum vertex cardinality over all minimal dominating sets in \(G\).

3. **REGULAR DOMINATING SET**

In this section the idea of regular domination in Intuitionistic fuzzy graphs and also discusses some properties and bounds of a regular domination number in Intuitionistic fuzzy graphs.

**Definition 3.1** A set \(S \subseteq V\) is said to be a regular dominating set in Intuitionistic fuzzy graphs \(G(V, E)\) if

i) Every vertex \(u \in V - S\) is adjacent to some vertex in \(S\).

ii) Every vertex in \(S \subseteq V\) has the same degree.

Minimum cardinality among all the regular dominating sets is called the regular domination number \(\gamma_R(G)\) of \(G(V, E)\).

**Theorem 3.1:** In a regular Intuitionistic fuzzy graph \(G(V, E)\) then every dominating set is a regular dominating set of \(G(V, E)\).

**Proof:** Let \(G(V, E)\) be a regular Intuitionistic fuzzy graph. Therefore degree of every vertex in \(G(V, E)\) are unique. This implies every dominating set is a regular dominating set of \(G(V, E)\).
**Theorem 3.2:** Let $d_N(v) = \Delta_N(G)$ in an Intuitionisticfuzzy graph $G(V, E)$, then the degree of every vertex in $\gamma_R(G)$ set is equal to $\Delta_N(G)$.

**Proof:** Let $G(V, E)$ be an Intuitionisticfuzzy graph. Let $d_N(v) = \Delta_N(G)$ in $G(V, E)$. Therefore the vertex $v$ belongs to $\gamma_R(G)$ set. This implies the degree of every vertex in $\gamma_R(G)$ set is equal to $\Delta_N(G)$. Hence proved.

**Example 3.1**

In the figure 3.1, the degree of the vertices in $G(V, E)$ are $d(a) = 0.4, d(b) = 0.55 \quad d(c) = 0.3, d(d) = 0.55$, and $\Delta_N(G) = 0.55$. The regular dominating set of $G(V, E)$ is $D_1 = \{b, d\}$.

**Definition 3.2:** Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be intuitionistic fuzzy graphs on $V_1, V_2$ respectively with $V_1 \cap V_2 = \phi$. The union of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ denoted by $G_1 + G_2$ is the intuitionistic fuzzy graph $G(V, E)$ on $V_1 \cup V_2$ defined by $G = (G_1 \cup G_2) = ((\mu_1 \cup \mu_1), (\gamma_1 \cup \gamma_1), (\mu_2 \cup \mu_2), (\gamma_2 \cup \gamma_2))$ where

$$(\mu_1 \cup \mu_1) (u) = \begin{cases} 
\mu_1(u) & \text{if } u \in V_1 \\
\mu_1(u) & \text{if } u \in V_2 
\end{cases}$$

$$(\gamma_1 \cup \gamma_1) (u) = \begin{cases} 
\gamma_1(u) & \text{if } u \in V_1 \\
\gamma_1(u) & \text{if } u \in V_2 
\end{cases}$$

$$(\mu_2 \cup \mu_2) (uv) = \begin{cases} 
\mu_2(uv) & \text{if } uv \in E_1 \\
\mu_2(uv) & \text{if } uv \in E_2 \& (\gamma_2 \cup \gamma_2) (uv) = \\
0 & \text{otherwise} 
\end{cases}$$

$$(\gamma_2 \cup \gamma_2) (uv) = \begin{cases} 
\gamma_2(uv) & \text{if } uv \in E_1 \\
\gamma_2(uv) & \text{if } uv \in E_2 \\
0 & \text{otherwise} 
\end{cases}$$

**Theorem 3.2:** Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are two IFG. Let $D_1$ and $D_2$ be the minimal regular dominating sets of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. Then the regular dominating number of $G_1 \cup G_2$ is $\gamma_R(G_1 \cup G_2) = |D_1| + |D_2|$.
Proof: Let \( G_1(V_1, E_1) \) and \( G_2(V_2, E_2) \) are two IFG. Assume \( D_1 \) and \( D_2 \) be the minimal regular dominating sets of \( G_1(V_1, E_1) \) and \( G_2(V_2, E_2) \) respectively. If every vertex \( u \in G_1 \cup G_2 \) this implies \( u \in G_1 \) or \( u \in G_2 \) therefore there is a vertex \( v \in D_1 \) or \( v \in D_2 \) such that ‘v’ regularly dominates \( u \in G_1 \cup G_2 \). Since \( D_1 \) and \( D_2 \) be the regular dominating sets of \( G_1(V_1, E_1) \) and \( G_2(V_2, E_2) \) respectively. The regular dominating number of \( G_1 \cup G_2 \) is \( \gamma_R(G_1 \cup G_2) = |D_1| + |D_2| \). Hence proved.

Example 3.2

Figure 3.2

In the figure 3.2, the degree of the vertices in \( G_1(V_1, E_1) \) and \( G_2(V_2, E_2) \) are \( d(a) = 0.4, d(b) = 0.55 \) \( d(c) = 0.3, d(d) = 0.55 \), and \( d(e) = 0.55, d(f) = 0.4, d(h) = 0.5 \). The regular dominating set of \( G_1(V_1, E_1) \) and \( G_2(V_2, E_2) \) are \( D_1 = \{b, d\} \) and \( D_2 = \{g, h\} \). The regular dominating set of \( (G_1 \cup G_2) \) is \( D = \{b, d\} \) and the minimal dominating number of the graph \( (G_1 \cup G_2) \) is \( \gamma_{R_M}(G_1 \cup G_2) = 1.35 \).

Definition 3.3: Let \( G_1(V_1, E_1) \) and \( G_2(V_2, E_2) \) be intuitionistic fuzzy graphs on \( V_1, V_2 \) respectively with \( V_1 \cap V_2 = \emptyset \). The join of \( G_1(V_1, E_1) \) and \( G_2(V_2, E_2) \) is the intuitionistic fuzzy graph \( G \) on \( V_1 \cup V_2 \) defined by \( G = (G_1 + G_2) = ((\mu_1 + \mu_1), (\gamma_1 + \gamma_1), (\mu_2 + \mu_2), (\gamma_2 + \gamma_2)) \) where
Theorem 3.3: The sets $D_1, D_2$ be a regular dominating set of the Intuitionistic fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. Then $\gamma^*_k (G_1 + G_2) = \min \{ |D_1|, |D_2| \}$.

Proof: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be Intuitionistic fuzzy graphs and The sets $D_1, D_2$ be regular dominating sets of the fuzzy graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. In $G_1 + G_2$ every vertex in $G_1(V_1, E_1)$ is adjacent to every vertices in $G_2(V_2, E_2)$ and vise-versa. This implies the sets $D_1, D_2$ are dominating sets of $G_1 + G_2$ and degree of the vertices in $G_1 + G_2$ are

$$d(v) = \begin{cases} (d_{G_1}(v) + O(G_2)), & \text{if } v \in G_1 \\ (d_{G_2}(v) + O(G_1)), & \text{if } v \in G_2 \end{cases}$$

This implies $d(u) = d(v), \forall u, v \in D_1$ or $u, v \in D_2$ in $G_1 + G_2$. Hence $D_1$ or $D_2$ be a regular dominating sets of $G_1 + G_2$. Therefore the minimal dominating number of $G_1 + G_2$ is

$$\gamma^*_k (G_1 + G_2) = \min \{ |D_1|, |D_2| \}$$

Example 3.2:
In the figure 3.3, the degree of the vertices in $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ are $d(a)=0.4, d(b)=0.55, d(c)=0.3, d(d)=0.55,$ and $d(e)=0.55, d(f)=0.4, d(g)=0.5, d(h)=0.5$. The regular dominating set of $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ are $D_1=\{b,d\}$ and $D_2=\{g,h\}$. The degree of the vertices in $(G_1+G_2)$ are $d(a)=2.35, d(b)=2.5, d(c)=2.25, d(d)=2.5, d(e)=2.35, d(f)=2.2, d(g)=2.3, d(h)=2.3$. The regular dominating set of $(G_1+G_2)$ is $D=\{b,d\}$ and the minimal dominating number of the graph $(G_1+G_2)$ is $\gamma_{RF}(G_1+G_2)=0.7$.

Conclusion:

In this paper we define regular domination set and regular domination number in Intuitionistic fuzzy graph and investigate some properties and bounds of regular domination number in various Intuitionistic fuzzy graphs.

Reference: