COMPARISON BETWEEN KUHN – TUCKER AND LAGRANGEAN METHODS ON FUZZY INVENTORY MODEL

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ABSTRACT

In this paper deals with an production inventory model without shortages in a fuzzy environment with fuzzy constraints for crisp production quantity or for fuzzy production quantity are rooted in fuzzy inventory management . We have applied graded mean method for defuzzifying the fuzzy general production inventory cost in trapezoidal fuzzy numbers. The aim of our work is to find optimal solution of these models by using Lagrangean method and Kuhn-tucker method finally a numerical example explained to show the uniqueness obtained in both crisp and fuzzy inventory model.

Keywords - Defuzzification,Extension principle,Trapezoidal Fuzzy Numbers.

1. INTRODUCTION:

Inventory management is to maintain enough inventories to meet the customers demand and is cost effective to yield profitability. EPQ model assume that all perfect quality items are produced. However, in a real-life production situation, due to unpredictable factors, generation of defective items is inevitable and the defective rate cannot be ignored in the production process.Uncertainties are dealt with as randomness in the traditional inventory model and are viewed using the theory of probability, assuming certain or unknown demand and supplyVarious uncertainties within inventory systems cannot be considered appropriately, fuzzy set concepts are used in modelling of inventory systems since 1980s.

Park (1987)[5] and Vujosevic et.al. (1996) developed the inventory models in fuzzy sense where ordering cost and holding cost are represented by fuzzy numbers. Chang(1999)[2]describethe production stock model wherein the product quantity is a fuzzy number.Also, based on the numerical example, he compared fuzzy and crisp approaches for solving this problem. Chih Hsun Hsieh (2002)[3] introduced two fuzzy production inventory models with fuzzy parameters for crisp production quantity, or for fuzzy production quantity. The authors found optimal solutions of these models by using Graded Mean Integration Representation method for defuzzifying fuzzy total production inventory cost and by using Extension of the Lagrangean method for solving inequality constraint problem. S.Sarkar, T.Chakrabarti(2013) an EPQ model of exponential deterioration with fuzzy demand and production with shortages.

2.1 THE FUNCTION PRINCIPLE:

Suppose $\widetilde{D}=\left(d_{1,}d_{2,}d_{3,}d_4\right)$ and $\tilde{E}=\left(e_{1,}e_{2,}e_{3,}e_4\right)$ are two trapezoidal fuzzy numbers then,

1. The addition of $\widetilde{D}$and $\tilde{E}$ is$\widetilde{D}\oplus\tilde{E}=\left(d_1+e_{1,}d_2+e_{2,}d_3+e_{3,}d_4+e_4\right)$.
2. The multiplication of $\widetilde{D}$and $\tilde{E}$ is $\widetilde{D}\otimes\tilde{E}$=$\left(d_1e_{1,}d_2e_{2,}d_3e_{3,}d_4e_4\right)$
3. The subtraction of $\widetilde{D}$and $\tilde{E}$ is $\widetilde{D}\ominus\tilde{E}$ =$\left(d_1-e_{4,}d_2-e_{3,}d_3-e_{2,}d_4-e_1\right)$

4. The division of $\widetilde{D}$and $\widetilde{E}$ is $\widetilde{D} \oslash \widetilde{E} = \left(\frac{d_1}{e_4}, \frac{d_2}{e_3}, \frac{d_3}{e_2}, \frac{d_4}{e_1}\right)$.

5. Let $\alpha \epsilon$R , Then

$\alpha \geq 0$, $\alpha \otimes \tilde{A} = (\alpha d_1, \alpha d_2, \alpha d_3, \alpha d_4)$

$\alpha < 0$, $\alpha \otimes \tilde{A} = (\alpha d_4, \alpha d_3, \alpha d_2, \alpha d_1)$

2.2 GRADED MEAN INTEGRATION REPRESENTATION METHOD:

Let $\tilde{\gamma}$ be a trapezoidal fuzzy numbers, and be denoted as$\tilde{\gamma} = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ .then we canget the graded mean integration representation of $\tilde{Y}$by formula

$$P(\tilde{\gamma}) = \int_0^1 \frac{h\left(\frac{\gamma_1+\gamma_4+(\gamma_2-\gamma_1-\gamma_4+\gamma_3)h}{2}\right)dh}{\int_0^{w_z} h dh} = \frac{\gamma_1+2\gamma_2+2\gamma_3+\gamma_4}{6}$$

2.3 KUHN TUCKER METHOD:

The Kuhn-Tucker Condition is,

(i)$\lambda \leq 0$

(ii)$\nabla fp(T_C) - \lambda \nabla g(Q) = 0$

(iii)$\lambda_i g_i(Q) = 0$, i = 1,2, … m

(iv) $g_i(Q) \geq 0, i = 1, 2, \dots, m$

2.4 LAGRANGIAN METHOD

By Minimizing, b = f (a)

Subject to $h_i(a) \geq 0$, i = 1,2,……,m.

The non-negativity constraints a ≥ 0, if any, are covered in the m constraints. Then, the procedure of Extension of the Lagrangean method involves the below steps.

Step 1: Solve the unconstrained problem,Minimize b = f (a).

If the resultant optimum satisfies all the constraints, end the step because all constraints are excessive. Otherwise, set k = 1 and move on to step 2.

Step 2: Activate any k constraints and optimize f(a) subject to the k active constraints by the Lagrangean method. If the resultant solution is feasible with respect to the enduring constraints, stop the process because it is a local optimum. Or else, activate another set of k - constraints and renew the step. If all remaining sets of active constraints taken k at a time are considered without confronting a feasible solution, move on to step 3.

Step 3: If k = m, stop; no feasible solution exists. Or else, set k = k+1 and go to step 2. By using the above Lagrangean method, we discuss the fuzzy inventory model by changing the crisp quantity into fuzzy quantity. As a result, we can get optimal solution.

3. NOTATIONS:

H_c – Holding cost, S_c – Setup cost, L_P – Length of the plan,D-Demand with time period, q*-Order quantity, T_c-Total cost, $\widetilde{H_C}$ − fuzzy Holding cost,$\widetilde{S_C}$ −fuzzy setup cost $\widetilde{L_P}$ − fuzzy length of the plan,$T_{\tilde{C}}$- Fuzzy Total cost, F(Q)- Defuzzified total cost Q*-optimal order quantity.

3.1 Crisp Sense:

First, we deal an inventory model without shortages in crisp sense, the economic size can be obtained by the following equations.

$T_c = \frac{H_C L_P Q}{2} + \frac{S_C D}{Q}$

Differentiate partially with respect to q and equal it to zero,

$\frac{\partial T_c}{\partial Q} = \frac{H_C L_P}{2} + \frac{S_C D}{Q^2} \Rightarrow$ Q = $\sqrt{\frac{2S_C D}{H_C L_P}}$

3.2 Fuzzy Sense:

$\widetilde{H_C} = (h_{c_1}, h_{c_2}, h_{c_3}, h_{c_4}\widetilde{S_C} = (s_{c_1}, s_{c_2}, s_{c_3}, s_{c_4})\widetilde{L_P} = (l_{p_1}, l_{p_2}, l_{p_3}, l_{p_4})\widetilde{D} = (d_1, d_2, d_3, d_4)$

$$T_{\tilde{C}} = \left[\frac{h_{c_1} l_{p_1} Q}{2} + \frac{s_{c_1} d_1}{Q}, \frac{h_{c_2} l_{p_2} Q}{2} + \frac{s_{c_2} d_2}{Q}, \frac{h_{c_3} l_{p_3} Q}{2} + \frac{s_{c_3} d_3}{Q}, \frac{h_{c_4} l_{p_4} Q}{2} + \frac{s_{c_4} d_4}{Q}\right]$$

By Graded mean integration representation method,

$$P(T_{\tilde{C}}) = 0 \Rightarrow \frac{1}{6}\left[\begin{array}{c}\left(\frac{h_{c_1} l_{p_1} Q}{2} - \frac{s_{c_1} d_1}{Q^2}\right) + 2\left(\frac{h_{c_2} l_{p_2} Q}{2} - \frac{s_{c_2} d_2}{Q^2}\right) \\ +2\left(\frac{h_{c_3} l_{p_3} Q}{2} - \frac{s_{c_3} d_3}{Q^2}\right) + \left(\frac{h_{c_4} l_{p_4} Q}{2} - \frac{s_{c_4} d_4}{Q^2}\right)\end{array}\right]$$

Differentiate partially with respect to Q and equate it to zero

$$\frac{\partial P(T_{\tilde{c}})}{\partial Q} = 0 \Rightarrow \frac{1}{6}\left[\begin{array}{c}\left(\frac{h_{c_1} l_{p_1}}{2} - \frac{s_{c_1} d_1}{Q^2}\right) + 2\left(\frac{h_{c_2} l_{p_2}}{2} - \frac{s_{c_2} d_2}{Q^2}\right) \\ +2\left(\frac{h_{c_3} l_{p_3}}{2} - \frac{s_{c_3} d_3}{Q^2}\right) + \left(\frac{h_{c_4} l_{p_4}}{2} - \frac{s_{c_4} d_4}{Q^2}\right)\end{array}\right]$$

$$Q^* = \sqrt{\frac{2(s_{c_1} d_1 + 2s_{c_2} d_2 + 2s_{c_3} d_3 + s_{c_4} d_4)}{(h_{c_1} l_{p_1} + 2h_{c_2} l_{p_2} + 2h_{c_3} l_{p_3} + h_{c_4} l_{p_4})}}$$

3.3 OPTIMAL SOLUTIONS OBTAINED BY KUHN TUCKER METHOD AND LAGRANGEAN METHOD

Suppose fuzzy order quantity $\tilde{Q} = (q_1, q_2, q_3, q_4)$

By function principle, the fuzzy total inventory cost is

$$T_{\tilde{C}} = \left[\frac{h_{c_1} l_{p_1} q_1}{2} + \frac{s_{c_1} d_1}{q_4}, \frac{h_{c_2} l_{p_2} q_2}{2} + \frac{s_{c_2} d_2}{q_3}, \frac{h_{c_3} l_{p_3} q_3}{2} + \frac{s_{c_3} d_3}{q_2}, \frac{h_{c_4} l_{p_4} q_4}{2} + \frac{s_{c_4} d_4}{q_1}\right]$$

We defuzzify the fuzzy total inventory cost by using graded mean integration representation formula is,

$$P(T_{\tilde{c}}) = \frac{1}{6}\left[\begin{array}{c}\left(\frac{h_{c_1} l_{p_1} q_1}{2} + \frac{s_{c_1} d_1}{q_4}\right) + 2\left(\frac{h_{c_2} l_{p_2} q_2}{2} + \frac{s_{c_2} d_2}{q_3}\right) \\ +2\left(\frac{h_{c_3} l_{p_3} q_3}{2} + \frac{s_{c_3} d_3}{q_2}\right) + \left(\frac{h_{c_4} l_{p_4} q_4}{2} + \frac{s_{c_4} d_4}{q_1}\right)\end{array}\right] ------(1)$$

With $0 \leq q_1 \leq q_2 \leq q_3 \leq q_4$

It can be written as $q_2 - q_1 \geq 0, q_3 - q_2 \geq 0, q_4 - q_3 \geq 0, q_1 \geq 0.$

KUHN TUCKER METHOD

Condition 1:

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \leq 0$$

Condition 2:

$$\frac{\partial}{\partial q_1}\left(P(T_{\tilde{c}})\right) - \lambda_1 \frac{\partial}{\partial q_1}\left(g_1(Q)\right) - \lambda_2 \frac{\partial}{\partial q_2}\left(g_2(Q)\right) - \lambda_3 \frac{\partial}{\partial q_3}\left(g_3(Q)\right) - \lambda_3 \frac{\partial}{\partial q_4}\left(g_4(Q)\right) = 0$$

Differentiate q_1, q_2, q_3, q_4 and equate them to zero

$$P(T_{\tilde{C}}) = \frac{1}{6}\left[\left[\frac{h_{c_1} l_{p_1}}{2} - \frac{s_{c_4} d_4}{q_1^2}\right] + 2\left[\frac{h_{c_2} l_{p_2}}{2} - \frac{s_{c_3} d_3}{q_2^2}\right] + 2\left[\frac{h_{c_3} l_{p_3}}{2} - \frac{s_{c_2} d_2}{q_3^2}\right] + \left[\frac{h_{c_4} l_{p_4}}{2} - \frac{s_{c_1} d_1}{q_4^2}\right]\right]$$

$$+\lambda_1 - \lambda_4 + \lambda_2 - \lambda_1 - \lambda_2 + \lambda_3 - \lambda_3 + \lambda_4 = 0$$

Condition 3:

The condition is, $\lambda_i g_i(Q) = 0$

$\lambda_1(q_2 - q_1) = 0$, $\lambda_2(q_3 - q_2) = 0$, $\lambda_3(q_4 - q_3) = 0$ and $\lambda_4 q_1 = 0$

Condition 4:

$$q_2 - q_1 \geq 0, q_3 - q_2 \geq o, q_4 - q_3 \geq 0, q_1 \geq 0.$$

Here we know that

$$q_1 \geq 0, \quad \lambda_4 q_1 = 0, \quad soweget\ \lambda_4 = 0$$

Then we replace $q_2\ by q_1, q_3\ by q_2\ and q_4\ by q_3$ (i.e) ($q_1 = q_2 = q_3 = q_4 = \tilde{Q}$)

By adding, we get,

$$\frac{1}{6}\left[\left(\frac{h_{c_1}l_{p_1}+2h_{c_2}l_{p_2}+2h_{c_3}l_{p_3}+h_{c_4}l_{p_4}}{2}\right) - \left(\frac{s_{c_1}d_1+2s_{c_2}d_2+2s_{c_3}d_3+s_{c_4}d_4}{q_1^2}\right)\right] = 0$$

$$Q^* = \sqrt{\frac{2(s_{c_1}d_1+2s_{c_2}d_2+2s_{c_3}d_3+s_{c_4}d_4)}{(h_{c_1}l_{p_1}+2h_{c_2}l_{p_2}+2h_{c_3}l_{p_3}+h_{c_4}l_{p_4})}} \quad .$$

LAGRANGEAN METHOD

Step 1:

Differentiate(1) Partially with respect to q_1, q_2, q_3, q_4 and equating them to zero.

$$\frac{\partial}{\partial q_1}(P(T_{\tilde{c}})) = 0 \Rightarrow \frac{1}{6}\left[\frac{h_{c_1}l_{p_1}}{2} - \frac{s_{c_4}d_4}{q_1^2}\right] = 0 \Rightarrow q_1 = \sqrt{\frac{2s_{c_4}d_4}{h_{c_1}l_{p_1}}} ------- (2)$$

$$\frac{\partial}{\partial q_2}(P(T_{\tilde{c}})) = 0 \Rightarrow \frac{2}{6}\left[\frac{h_{c_2}l_{p_2}}{2} - \frac{s_{c_3}d_3}{q_2^2}\right] = 0 \Rightarrow q_2 = \sqrt{\frac{2s_{c_3}d_3}{h_{c_2}l_{p_2}}} ------- (3)$$

$$\frac{\partial}{\partial q_3}(P(T_{\tilde{c}})) = 0 \Rightarrow \frac{2}{6}\left[\frac{h_{c_3}l_{p_3}}{2} - \frac{s_{c_2}d_2}{q_3^2}\right] = 0 \Rightarrow q_3 = \sqrt{\frac{2s_{c_2}d_2}{h_{c_3}l_{p_3}}} ------- (4)$$

$$\frac{\partial}{\partial q_4}(P(T_{\tilde{c}})) = 0 \Rightarrow \frac{1}{6}\left[\frac{h_{c_4}l_{p_4}}{2} - \frac{s_{c_1}d_1}{q_4^2}\right] = 0 \Rightarrow q_4 = \sqrt{\frac{2s_{c_1}d_1}{h_{c_4}l_{p_4}}} ------- (5)$$

It does not satisfy the conditions $0 < q_1 < q_2 < q_3 < q_4$.

Step 2: Set k = 1(fixed the constraints as 1)

Convert the inequality constraints $q_2 - q_1 \geq 0$ into an equality constraints $q_2 - q_1 = 0$ and Minimize P ($T_{\tilde{c}}$) with respect to $q_2 - q_1 = 0$ by the lagrangean method.

L ($q_1, q_2, q_3, q_4, \lambda$) = P ($T_{\tilde{c}}$) –λ ($q_2 - q_1$)

$$= \frac{1}{6}\left[\left(\left(\frac{h_{c_1}l_{p_1}q_1}{2} + \frac{s_{c_1}d_1}{q_4}\right) + 2\left(\frac{h_{c_2}l_{p_2}q_2}{2} + \frac{s_{c_2}d_2}{q_3}\right) + 2\left(\frac{h_{c_3}l_{p_3}q_3}{2} + \frac{s_{c_3}d_3}{q_2}\right) + \left(\frac{h_{c_4}l_{p_4}q_4}{2} + \frac{s_{c_4}d_4}{q_1}\right)\right)\right] -$$

$\lambda(□_2 - □_1) = 0$ ------(6)

Differentiate (6) partially with respect to $□_1, □_2, □_3, □_4$ and equating them to zero.

$$\frac{□□}{□□_1} = 0 \Rightarrow \frac{1}{6}\left[\frac{h_{□_1}□_{□_1}}{2} - \frac{s_{□_4}□_4}{□_1^2}\right] + \lambda = 0 \quad -------(7)$$

$$\frac{□□}{□□_2} = 0 \Rightarrow \frac{1}{6}\left[\frac{2h_{□_2}□_{□_2}}{2} - \frac{2□_{□_3}□_3}{□_2^2}\right] - □ = 0 \quad ------- (8)$$

$$\frac{□□}{□□_3} = 0 \Rightarrow \frac{1}{6}\left[\frac{2h_{□_3}□_{□_3}}{2} - \frac{2□_{□_2}□_2}{□_3^2}\right] = 0 \quad ------- (9)$$

$$\frac{□□}{□□_4} = 0 \Rightarrow \frac{1}{6}\left[\frac{h_{□_4}□_{□_4}}{2} - \frac{□_{□_1}□_1}{□_4^2}\right] = 0 \quad ------- (10)$$

$$\frac{□□}{□□} = 0 \Rightarrow -(□_2 - □_1) = 0 \quad -------(11)$$

$$(7) + (8) \Rightarrow \frac{1}{2}\left[h_{□_1}□_{□_1} + 2h_{□_2}□_{□_2}\right] = \frac{□_{□_4}□_4 + 2□_{□_3}□_3}{□_1^2}$$

From the equation (11) we obtain,

$\square_1 = \square_2 = \sqrt{\frac{2\square_{\square_4}\square_4+2\square_{\square_3}\square_3}{h_{\square_1}\square_{\square_1}+2h_{\square_2}\square_{\square_2}}}$

Using the equation (9) and (10) we obtain $\square_3$ and $\square_4$ respectively,

$\square_3 = \sqrt{\frac{2\square_{\square_2}\square_2}{h_{\square_3}\square_{\square_3}}}$, $\square_4 = \sqrt{\frac{2\square_{\square_1}\square_1}{h_{\square_4}\square_{\square_4}}}$.

It does not satisfy the local optimum $0 < \square_1 < \square_2 < \square_3 < \square_4$.

Step 3: Set k = 2(fixed the constraints as 2)

Convert the inequality constraints $\square_2 - \square_1 \geq 0$ and $\square_3 - \square_2 \geq 0$ into an equality constraints $\square_3 - \square_1 = 0 \square\square\square\square_3 - \square_2 = 0$ and Minimize P ($\square_{\tilde{\square}}$) with respect to $\square_2 - \square_1 = 0$ and by the $\square_3 - \square_2 = 0$ lagrangean method.

L ($\square_1, \square_2, \square_3, \square_4, \square_1, \square_2$) = P ($\square_{\tilde{\square}}$)–$\square_1$ ($\square_2 - \square_1$) $-$ $\square_2$ ($\square_3 - \square_2$)

$$= \frac{1}{6}\left[\left(\left(\frac{h_{\square_1}\square_{\square_1}\square_1}{2} + \frac{\square_{\square_1}\square_1}{\square_4}\right) + 2\left(\frac{h_{\square_2}\square_{\square_2}\square_2}{2} + \frac{\square_{\square_2}\square_2}{\square_3}\right) + 2\left(\frac{h_{\square_3}\square_{\square_3}\square_3}{2} + \frac{\square_{\square_3}\square_3}{\square_2}\right) + \left(\frac{h_{\square_4}\square_{\square_4}\square_4}{2} + \frac{\square_{\square_4}\square_4}{\square_1}\right)\right)\right] -$$

$\square_1 (\square_2 - \square_1) - \square_2 (\square_3 - \square_2) - - - - (12)$

Differentiate(12)partiallywithrespectto$\square_1, \square_2, \square_3, \square_4, \square_1, \square_2$and equating them to zero

$\frac{\square\square}{\square\square_1} = 0 \Rightarrow \frac{1}{6}\left[\frac{h_{\square_1}\square_{\square_1}}{2} - \frac{\square_{\square_4}\square_4}{\square_1^2}\right] + \lambda_1 = 0 \qquad ----(13)$

$\frac{\square\square}{\square\square_2} = 0 \Rightarrow \frac{1}{6}\left[\frac{2h_{\square_2}\square_{\square_2}}{2} - \frac{2\square_{\square_3}\square_3}{\square_2^2}\right] - \square_1 + \square_2 = 0 \qquad ---- (14)$

$\frac{\square\square}{\square\square_3} = 0 \Rightarrow \frac{1}{6}\left[\frac{2h_{\square_3}\square_{\square_3}}{2} - \frac{2\square_{\square_2}\square_2}{\square_3^2}\right] - \square_2 = 0 \qquad ----(15)$

$\frac{\square\square}{\square\square_4} = 0 \Rightarrow \frac{1}{6}\left[\frac{h_{\square_4}\square_{\square_4}}{2} - \frac{\square_{\square_1}\square_1}{\square_4^2}\right] = 0 \qquad ---- (16)$

$\frac{\square\square}{\square\square_1} = 0 \Rightarrow -(\square_2 - \square_1) = 0 \qquad ---- (17)$

$\frac{\square\square}{\square\square_2} = 0 \Rightarrow -(\square_3 - \square_2) = 0 \qquad ---- (18)$

(13) + (14) + (15)$\Rightarrow \frac{1}{2}\left[h_{\square_1}\square_{\square_1} + 2h_{\square_2}\square_{\square_2} + 2h_{\square_3}\square_{\square_3}\right] = \frac{\square_{\square_4}\square_4+2\square_{\square_3}\square_3+2\square_{\square_2}\square_2}{\square_1^2}$

From the equation (17) and (18) we obtain, $\square_1 = \square_2 = \square_3 = \sqrt{\frac{2\square_{\square_4}\square_4+2\square_{\square_3}\square_3++2\square_{\square_2}\square_2}{h_{\square_1}\square_{\square_1}+2h_{\square_2}\square_{\square_2}+2h_{\square_3}\square_{\square_3}}}$

Using the equation (16) and we obtain respectively,$\square_4 = \sqrt{\frac{2\square_{\square_1}\square_1}{h_{\square_4}\square_{\square_4}}}$.

From above equation $q_1 > \square_4$, which does not satisfy the constraints $0 < \square_1 < \square_2 < \square_3 < \square_4$.

Step 4:

Set k = 3(fixed the constraints as 3)

Convert the inequality constraints $\square_2 - \square_1 \geq 0$, $\square_3 - \square_2 \geq 0$ and $\square_4 - \square_3 \geq 0$ into equality constraints $\square_3 - \square_1 = 0$, $\square_3 - \square_2 = 0 and$ $\square_4 - \square_3 = 0$ and Minimize P($\square_{\tilde{\square}}$) with respect to $\square_2 - \square_1 = 0$, $\square_3 - \square_2 = 0 and$ $\square_4 - \square_3 = 0$ lagrangean method.

L ($\square_1, \square_2, \square_3, \square_4$, $\square_1$, $\square_2$, $\square_3$) = P($\square_{\tilde{\square}}$)–$\square_1$ ($\square_2 - \square_1$) $-$ $\square_2$ ($\square_3 - \square_2$) $-$ $\square_3$ ($\square_4 - \square_3$) = 0

$$= \frac{1}{6}\left[\left(\left(\frac{h_{\square_1}\square_{\square_1}\square_1}{2} + \frac{\square_{\square_1}\square_1}{\square_4}\right) + 2\left(\frac{h_{\square_2}\square_{\square_2}\square_2}{2} + \frac{\square_{\square_2}\square_2}{\square_3}\right) + 2\left(\frac{h_{\square_3}\square_{\square_3}\square_3}{2} + \frac{\square_{\square_3}\square_3}{\square_2}\right)\left(\frac{h_{\square_4}\square_{\square_4}\square_4}{2} + \frac{\square_{\square_4}\square_4}{\square_1}\right)\right)\right]$$

$-\square_1 (\square_2 - \square_1) - \square_2 (\square_3 - \square_2) - - - -(19)$

Differentiate (19) partially with respect to $\square_1, \square_2, \square_3, \square_4, \square_1, \square_2, \square_3$ and equating them to zero.

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$$\frac{\square\square}{\square\square_1} = 0 \Rightarrow \frac{1}{6}\left[\frac{h_{\square_1}\square_{\square_1}}{2} - \frac{\square_{\square_4}\square_4}{\square_1^2}\right] + \lambda_1 = 0 \qquad ----(20)$$

$$\frac{\square\square}{\square\square_2} = 0 \Rightarrow \frac{1}{6}\left[\frac{2h_{\square_2}\square_{\square_2}}{2} - \frac{2\square_{\square_3}\square_3}{\square_2^2}\right] - \square_1 + \square_2 = 0 \qquad ---- (21)$$

$$\frac{\square\square}{\square\square_3} = 0 \Rightarrow \frac{1}{6}\left[\frac{2h_{\square_3}\square_{\square_3}}{2} - \frac{2\square_{\square_2}\square_2}{\square_3^2}\right] - \square_2 + \square_3 = 0 \qquad ----(21)$$

$$\frac{\square\square}{\square\square_4} = 0 \Rightarrow \frac{1}{6}\left[\frac{h_{\square_4}\square_{\square_4}}{2} - \frac{\square_{\square_1}\square_1}{\square_4^2}\right] - \square_3 = 0 \qquad ----(23)$$

$$\frac{\square\square}{\square\square_1} = 0 \Rightarrow -(\square_2 - \square_1) = 0 \qquad ----(24)$$

$$\frac{\square\square}{\square\square_2} = 0 \Rightarrow -(\square_3 - \square_2) = 0 \qquad ----(25)$$

$$\frac{\square\square}{\square\square_3} = 0 \Rightarrow -(\square_4 - \square_3) = 0 \qquad ----(26)$$

(20) + (21) + (22) + (23)

$$\Rightarrow \frac{1}{2}\left[h_{\square_1}\square_{\square_1} + 2h_{\square_2}\square_{\square_2} + 2h_{\square_3}\square_{\square_3} + h_{\square_4}\square_{\square_4}\right] = \frac{\square_{\square_4}\square_4 + 2\square_{\square_3}\square_3 + 2\square_{\square_2}\square_2 + \square_{\square_1}\square_1}{\square_1^2}$$

From the equation (24), (25) and (26) we obtain,

$$\square_1 = \square_2 = \square_3 = \square_4 = \sqrt{\frac{2(\square_{\square_4}\square_4 + 2\square_{\square_3}\square_{3+} + 2\square_{\square_2}\square_2 + \square_{\square_1}\square_1)}{h_{\square_1}\square_{\square_1} + 2h_{\square_2}\square_{\square_2} + 2h_{\square_3}\square_{\square_3} + h_{\square_4}\square_{\square_4}}}$$

The above solution $\tilde{\square} = (\square_1, \square_2, \square_3, \square_4)$ satisfies all the inequalities constraints , this process terminates with $\tilde{\square}$ as a minimum solution to the problem.

Let $\square_1 = \square_2 = \square_3 = \square_4 = \tilde{\square}$

Then the optimal fuzzy Quantity, $Q^* = \sqrt{\frac{2(\square_{\square_4}\square_4 + 2\square_{\square_3}\square_{3+} + 2\square_{\square_2}\square_2 + \square_{\square_1}\square_1)}{h_{\square_1}\square_{\square_1} + 2h_{\square_2}\square_{\square_2} + 2h_{\square_3}\square_{\square_3} + h_{\square_4}\square_{\square_4}}}$

4 NUMERICAL EXAMPLES:

Murugan group of Organization producers food products in units. Retailer requires 20000 units of chocolates consistent with year. It's been expected that the price of puting an order is Rs. 500 and the cost of holding inventory is Rs. 50. Also the travelling charges is Rs. 10. Find the total inventory cost?

SOLUTION:

The Optimal solution is derived for both crisp and fuzzy numbers

Crisp sence:

$\square_{\square} = 500$, D = 20000 , $\square_{\square} = 50$, $\square_{\square} = 10$

$$Q = \sqrt{\frac{2\square_{\square}\square}{\square_{\square}\square_{\square}}} = 200$$

Total inventory cost (crisp):

$$T_c = \frac{H_C L_P Q}{2} + \frac{S_C D}{Q} = 10504.18$$

Fuzzy sense:

$\widetilde{\square_{\square}} = (350,400,600,650)$, $\tilde{\square} = (9500,10000,30000,30500)$, $\widetilde{\square_{\square}} = (35,40,60,65)$,S$\widetilde{\square_{\square}} = (5,8,12,15)$

$$\tilde{\square} = \sqrt{\frac{2\left(\square_{\square_1}\square_1 + 2\square_{\square_2}\square_2 + 2\square_{\square_3}\square_3 + \square_{\square_4}\square_4\right)}{\left(h_{\square_1}\square_{\square_1} + 2h_{\square_2}\square_{\square_2} + 2h_{\square_3}\square_{\square_3} + h_{\square_4}\square_{\square_4}\right)}} \Rightarrow 203.$$

Total inventory cost (fuzzy):

$$P(\widetilde{\square_{\square}}) = \frac{1}{6}\left[\left(\begin{array}{c}\left(\frac{h_{\square_1}\square_{\square_1}\square_1}{2} + \frac{\square_{\square_1}\square_1}{\square_4}\right) + 2\left(\frac{h_{\square_2}\square_{\square_2}\square_2}{2} + \frac{\square_{\square_2}\square_2}{\square_3}\right) \\ +2\left(\frac{h_{\square_3}\square_{\square_3}\square_3}{2} + \frac{\square_{\square_3}\square_3}{\square_2}\right) + \left(\frac{h_{\square_4}\square_{\square_4}\square_4}{2} + \frac{\square_{\square_4}\square_4}{\square_1}\right)\end{array}\right)\right] = 10498.78$$

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CONCLUSION:

Within the fuzzy environment to discuss the inventory version without scarcity is mentioned. In addition, the characteristic principle is the usage of the reduce operation with trapezoidal fuzzy quantity. We find that the most beneficial fuzzy order quantity $\widetilde{\square}^{*} = (\square^{*}, \square^{*}, \square^{*}, \square^{*})$ is the special kind of trapezoidal fuzzy variety. We obtained optimal solution by using Kuhn – Tucker method and Lagrangean Method.

References:

[1] Chan,Wang(1996), Backorder fuzzy inventory model under function principle, Information Science,95,1-2,71-79

[2] Chang S. C(1999), Fuzzy Production Inventory for Fuzzy Product Quantity with Triangular Fuzzy Number, Fuzzy Sets and Systems 107, 37-57.

[3] Hsieh, C.H.(2002), Optimization of Fuzzy Production Inventory Models, Information Sciences,146,1-4,29-40.

[4] Kalaiarasi .R , Sumathi.M and Sabina Begum.M,(2018) Optimization of fuzzy inventory model without shortage using Pentagonal fuzzy number, International Journal of Mechanical Engineering and Technology, vol 9, Issue 11.

[5] Park, K.S.(1987), Fuzzy Set Theoretical Interpretation of economic order quantity,IEEE Trans.Systems Man. Cybernet SMC, 17,1082-1084

[6] Yao,J.S.,Lee H.M. 1999, Fuzzy Inventory with or without backorder for fuzzy order quantity withtrapezoidal fuzzy number, Fuzzy sets and systems, 105,311-337