

ENVIRONMENTALLY SUSTAINABLE INVENTORY MODEL WITH THE IMPACT OF TRADE CREDIT AND PARTIAL BACKORDERING

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ABSTRACT:

Sustainable management integration for reworking of faulty products provides long-lasting benefits. There are situations in global business when the products are purchased from a global supplier. There are chances that a fraction of defective goods can be in the lot earned. These defective goods are still important to save the world and can be fixed. Repairing faulty products in a nearby repair shop is affordable when contrasted with sending them back to the retailer. The cost of carbon emissions is also integrated into the effect on net income for the environment. The supplier, meanwhile, also offers the buyer a multi-credit-period. We built a sustainability model in this paper and are raising the effect on the climate. This paper seeks to optimize overall benefit by simultaneously considering multi-credit strategy, rework, scarcity, transportation costs, creating a synergic economic order quantity model. This model can help in making decisions to enhance sustainable inventory management efficiency by controlling the cycle time for a global supply chain. This also draws numerical comparison to provide organizational insights into actual procedures.

KEYWORDS: Inventory Model, Defective items, Shortages, Multi delay in payment.

I. INTRODUCTION:

Production managers incorporate and apply effective production planning under control structures in global business for a competitive environment to produce 100 per cent perfect products at an economized rate. The production system, however, still invents imperfect items. The imperfect items cut the buyer's income and also have a negative impact on the environment by focusing on the extra activity requiring the global supplier to exchange these imperfect items. This unauthorized supply will cause a loss of goodwill to the consumer. In a few days, in order to purchase recycled goods or renewable raw material at an economical cost in the global supply chain, the consumer figures out the way first, that often makes it difficult for a manufacturer to give the customer all the perfect things. Therefore, to ensure good quality and credibility for the company, it is

important for a customer to check a whole lot as soon as it reaches an inventory after the goods have been screened, some percentage of products may be classified as faulty goods with minor damages. Such defective goods are useful products. This is not acceptable to trade defective goods for an urgent shipment and it also has a detrimental effect on the environment because the supplier is located miles away from the customer. When the sale of faulty goods is to take place with the global manufacturer so the environmental impact will be increased, which in turn also raises the overall cost of the service. These minor damages can be repaired at a local repair shop to maintain sustainable product supply at an economic cost to reduce the impact on the environment, the sustainable approach is to repair these items at a local repair shop as compared to exchanging them with the supplier.

Throughout recent years, scholars and researchers have been drawing attention to the subjects of imperfect goods. It is worth noting that it is an unreasonable expectation that all items are up to the desired quality during fabrication or in the received lot. Biswajit Sarkar, Waqas Ahmed, Seok-Beom Choi and Muhammad Tayyab [1] studied the Partial Backordering and Multi-Trade-Credit – Time of Sustainable Resource Management for Environmental Impact. Compared to sending it back to the retailer, they suggested the sustainable inventory model to fix defective goods in a nearby repair store. The carbon emission costs are also included in the feature to reflect the environmental effect on the overall income. Marchi, B. [2] suggested a Green supply chain vendor-buyer model by considering both a decentralized and a centralized integrated approach. They recommend investing in technology development, and reducing the manufacturing process's carbon emissions. In an inadequate supply chain Kazemi [3] investigated the impact of carbon emissions. Under the sustainability viewpoint, they analyzed the inventory model with imperfect consistency. Kim and Sarkar [4] suggested a multi-stage cleaner method for manufacturing by removing all faulty products during production. W. Ritha, I. Antonitte Vinoline [5] were examining the supplier's optimum policy of acceptable delay in payment replenishment. Voros [6] suggested models of economic order and quantity of production without restriction to the fraction of imperfect products. Hsu [7] examined the EOQ model and indicated that the frequency of type – errors would occur due to inspection failure. Such poor consistency and processing errors often generate a shortage. Ahmed and Sarkar [8] describe the impact of carbon emissions in a sustainable management of the second generation biofuel supply chain. Eroglu and Ozdemir [9] further expanded the analysis and introduced the EOQ model in which they explored that each lot being inspected would contain faulty goods, and therefore, shortages and back-ordered goods. Sarkar [10] estimated the cumulative effects of variable pollution costs and multi – delay – in – compensation for a global supply chain sustainable.

We built a sustainability model in this paper and are raising the effect on the climate. This paper seeks to optimize overall benefit by simultaneously considering multi-credit

strategy, rework, scarcity, transportation costs, creating a synergic economic order quantity model. This model can help in making decisions to enhance sustainable inventory management efficiency by controlling the cycle time for a global supply chain. This also draws numerical comparison to provide organizational insights into actual procedures.

II. NOTATIONS AND ASSUMPTIONS

To develop the integrated model, the following notations and assumptions are defined throughout this paper.

- a** - Fixed cost of transportation
- Q_s** - Size of the order per cycle
- S_r** - Screening rate (units/ time unit)
- R_t** - Transportation, rework and return time for imperfect products
- T** - Cycle time (time unit)
- S_t** - Material screening time
- F_t** - Division of time that features a positive inventory level
- H_s** - Holding prices in the repair shop
- α** - Percentage for imperfect items
- D** - Demand rate per unit of time
- T_t** - Total transportation time for imperfect products
- R_r** - Rework rate (units/ time unit)
- O** - Buyer's Order Price
- R_s** - Setup cost for repair store
- H** - Holding cost for perfect items
- H'** - Cost of carbon emissions per item for holding perfect items
- H_s'** - Costs of carbon emissions per object in the repair shop
- H_r'** - Holding cost for rework products
- H_r'** - Cost of carbon emissions per item on the holding rework item
- S_c** - Screening cost per unit
- T_c** - Purchasing cost of one unit
- lm_c** - Labor and material cost required to repair a unit product
- L** - Cost incurred due to a loss of sales
- G** - Penalty cost incurred due to goodwill loss
- W** - Percentage of imperfect items passed on to customers
- β** - Backordered cost
- U** - Unit return cost for the imperfect products
- S** - Selling price of one unit
- γ** - Percentage of back ordered demand
- m_r** - Markup percentage by rework store
- M₁** - First permissible delay period for payment
- M₂** - Second permissible delay period for payment

- i_e - Interest earned (%)
 i_{e_1} - Interest charged by period M_1 %
 i_{e_2} - Interest charged by period M_2 %
 X' - Road construction cost
 f_c - Fuel cost
 d' - Distance travelled

Assumptions:

1. The inventory policy has a single type of product.
2. Emission of carbon from warehouse is due to the consumption of electricity per unit item. Consequently, carbon emission per unit item is considered for keeping perfect items according to carbon tax policy.
3. Shortages are permitted and these are partially backordered.
4. Inspection rates and demand are considered as known and constant.
5. The screening techniques and demand occur at the same time, but the screening price is higher than the price of production ($S_r > D$).
6. Defective products can be fixed in a controlled system and all defective goods are reprocessed.
7. The percentage of imperfect products are given and known
8. The relation between the purchasing price of consumer P_c and selling price of buyer S is $S \geq P_c$.
9. The holding price of reworked products exceeds the initial holding cost of perfect items ($H_r > H$).
10. Once the system inventory level is zero the reworked items are returned.
11. The dealer grants the buyer M_1 and M_2 multi-trade credit period. At some point in this time, the buyer sells the commodity and uses the profits to gain interest with i_e duration.
12. If the purchaser fails to pay the supplier during the first M credit period, then interest i_{c_1} will be charged, and later if the purchaser sells the product and uses its income to earn interest at i_{c_2} .
13. The amount of faulty products are sent to buyers, who in the next process are returned to the retailer. Of such returned goods, the consumer charges a cost per unit and a cost per unit as a refund expense incurred because of loss of goodwill.

III. MATHEMATICAL MODELLING:

This section describes and develops an integrated sustainable model of total profit inventory, with multiple delays in payments, partial back ordering, and repair of imperfect products and transport costs.

(i) Ordering cost = $\frac{O}{T}$

- (ii) Inspection cost = $S_c F_t D$
- (iii) Holding cost = $(H + H') \left[\frac{(1-\alpha)^2 F_t^2 TD}{2} + \frac{\alpha T (F_t D)^2}{S_r} \right] + (H_r + H'_r) \left[\frac{(\alpha F_t)^2 TD}{2} + \frac{\beta (1 - F_t)^2 \Gamma TD}{2} \right]$
- (iv) Rework cost = $\left[(1 + m_r) \frac{R_s + 2a}{\alpha F_t TD} + l m_c + 2T_c + (H_s + H'_s) R_t \right]$

The repaired products come back into the inventory when the initial inventory level becomes zero, therefore the level of inventory becomes $\alpha F_t TD$ Units. As the cycle ends, $(1-F_t)TD$ becomes the shortage of the system. The order quantity for a given cycle is considered to be

$$Q = FTD + \gamma(1 - F)TD$$

- (v) Shortage cost = $\gamma(1 - F_t)^2 TD + L(1 - \gamma)(1 - F_t)D$
- (vi) Goodwill penalty cost = $(U+G) W F_t D$
- (vii) Road construction cost = $\frac{X'}{T}$
- (viii) Fuel cost = $f_c \frac{d'}{T}$

INTEREST CHARGED AND INTEREST EARNED

If the allowable payment period is longer than the lead time, it will carry interest benefits to the purchaser according to the trade credit policy. When this permitted time is shorter than the lead time, otherwise the buyer can get more opportunity costs and less interest profits, whilst the supplier would pay fewer opportunity costs and the supplier would gain interest. Because of this the model of the supplier has the following two cases, based on this case allowable payment time and lead time length.

The cost differ between two cases:

Condition: 1

If the lead time T is less than or equal to the supplier's permissible payment M_1 duration, then only interest income is received as interest paid under such a condition is zero.

$$II = Si_e \left(DM_1 - \frac{TD}{2} \right)$$

Condition: 2

If the lead time T is greater than the supplier's first allowable payment time M_1 and less than or equal to the second allowable payment time M_2 provided to the buyer, then all interest costs will be paid and received.

$$II = Si_e \frac{(DM_1)^2}{2TD}$$

$$IC = P_{C_1} i_{c_1} \frac{(TD - DM_1)^2}{2TD}$$

Condition: 3

There is a special case where the buyer will be charged more interest if they fail to give the required payment in the first permitted time. In this case the lead time T is greater than the supplier's second permissible payment date M₂.

$$\Pi = Si_e \frac{(DM_1)^2}{2TD}$$

$$IC = P_C i_{c_2} \frac{(TD - DM_2)^2}{2TD} - P_C i_{c_1} \frac{D}{T} (M_2 T - M_2^2 - M_1 T + M_1 M_2) + P_C i_{c_1} \frac{D(M_2 - M_1)^2}{2T}$$

OPTIMUM SOLUTION

Total cost = Selling price-[Ordering cost+ product cost+ inspection cost+ holding cost+ rework cost+ shortage cost+ goodwill penalty cost+ interest earned-interest charged +fuel cost +road construction cost]

According to these different conditions of multi delay in payments, three cases are developed and the total profit function for all cases can be given as

CASE: 1

TOTAL PROFIT FUNCTION IF T ≤ M₁

$$TP(F_t, T) = (SD(F_t + \gamma(1 - F_t) - \left[\frac{O}{T} + P_c(F_t D + \gamma(1 - F_t)D + S_c F_t D + (H + H') \left(\frac{(1-\alpha)^2 F_t^2 TD}{2} + \frac{\alpha T(F_t D)^2}{S_r}\right) \left[\frac{(\alpha F_t)^2 TD}{2}\right] + \left[\frac{\beta(1-F_t)^2 TD}{2}\right] + (\alpha F_t D)(1 + m_r) \left[\frac{R_s + 2a}{\alpha F_t TD} + lm_c + 2T_c + (H_s + H'_s) \left(\frac{\alpha F_t TD}{R_r} + T_t\right) + L(1 - \gamma)(1 - F_t)D(U + G)W F_t D + Si_e \left(DM_1 - \frac{TD}{2}\right) + \frac{X'}{T} + f_c \frac{d'}{T}\right] \dots (1)$$

$$TP(F_t, T) = D(S - P_c) - Si_e DM_1 - Z_c D(1 - \gamma) - \left[\frac{1}{T} (O + (1 + m_r)(R_s + 2a)X' + f_c d' + F_t(S_c D + \alpha D(1 + m_r) + (lm_c + 2T_c + (H_s + H'_s)T_t) - Z_c D(1 - \gamma) + T \left(\frac{\beta \gamma}{2} + \frac{Si_e D}{2}\right) - F_t T(\gamma \beta D) + F_t^2 \frac{(H' + H')(1 - \alpha^2)D}{2} + \frac{(H_s + H'_s)(1 + m_r)\alpha^2 D^2}{R_r} + \frac{(H_s + H'_s)\alpha D^2}{S_r} + \frac{(H_r + H'_r)\alpha D^2}{S_r} + \frac{(H_r + H'_r)\alpha^2 D}{2} + \frac{\beta \gamma D}{2}\right] \dots (2)$$

$$Y'(F_t, T) = \frac{1}{T}(K_1) + T(K_2 - K_4 F_t + K_5 F_t^2) + K_3 F_t$$

$$= \frac{1}{T}(K_1) + T\Gamma(F_t) + \delta(F_t)$$

Where,

$$\Gamma(F_t) = (K_2 - K_4 F_t + K_5 F_t^2)$$

$$\delta(F_t) = K_3 F_t$$

The total cost equation reaches its least value with respect to T,

$$F_a^* = \frac{K_4 T - K_3}{2K_5 T}, T_a^* = \sqrt{\frac{K_1}{\Gamma(F_t)}}$$

The minimum value for the total cost by substituting T_a^* in the cost equation is

$$Y'(F_t, T) = \frac{1}{T}(K_1) + T\Gamma(F_t) + \delta(F_t)$$

$$= 2\sqrt{K_1}\Gamma(F_t) + \delta(F_t)$$

Place the values K_1, K_2, K_3, K_4, K_5 into equation

$$Y'(F_t, T) = \frac{(\gamma\beta D)T - (S_c D + \alpha D(1 + m_r)(lm_c + 2T_c + (H_s + H'_s)T_t) - Z_c(1 - \gamma))}{(H + H')(1 - \alpha^2)D + \frac{(H_s + H'_s)(1 + m_r)\alpha^2 D^2}{R_r} + \frac{(H_s + H'_s)\alpha D^2}{S_r} + (H_r + H'_r) + \alpha^2 D}$$

...(3)

$$T_a^* = \sqrt{\frac{K_1}{K_2 - K_4 F_t + K_5 F_t^2}}$$

$$T_a^* = \sqrt{\frac{K_1}{K_2 - K_4 \left(\frac{K_4 T - K_3}{2K_5 T}\right) + K_5 \left(\frac{K_4 T - K_3}{2K_5 T}\right)^2}}$$

$$T_a^* = \sqrt{\frac{K_1(2K_5 T)^2}{K_2(2K_5 T)^2 - K_4(K_4 T - K_3)(2K_5 T) + K_5(K_4 T - K_3)^2}}$$

$$K_1 = (O + (1 + m_r)(R_s + 2a) + X' + f_c d')$$

$$K_2 = \left(\frac{\beta\gamma D}{2} + \frac{S_i e D}{2}\right)$$

$$K_3 = (S_c D + \alpha D(1 + m_r)(lm_c + 2T_c + (H_s + H'_s)T_t) - Z_c D(1 - \gamma))$$

$$K_4 = \gamma\beta D$$

$$K_5 = \frac{(H' + H')(1 - \alpha^2)D}{2} + \frac{(H_s + H'_s)(1 + m_r)\alpha^2 D^2}{R_r} + \frac{(H_s + H'_s)\alpha D^2}{S_r} + \frac{(H_r + H'_r)\alpha^2 D}{2} + \frac{\beta\gamma D}{2}$$

CASE: 2**TOTAL PROFIT FUNCTION IF $M_1 < T \leq M_2$**

$$\begin{aligned} TP(F_t, T) = & SD[F_t + \gamma(1 - F_t) - \left(\frac{O}{T} + P_c(F_t D + \gamma(1 - F_t)D + S_c F_t D + (H + H')\left[\frac{(1-\alpha)^2 F_t^2 TD}{2} + \frac{\alpha T(F_t D)^2}{S_r}\right] + (H_r + H'_r)\left[\frac{(\alpha F_t)^2 TD}{2} + \frac{\beta(1-F_t)^2 \gamma TD}{2} + (\alpha F_t D(1 + m_r))\frac{R_s + 2a}{\alpha F_t TD} + \text{Im}_c + 2T_c + (H_s + H'_s) + \frac{\alpha F_t TD}{R_r} + T_t + L(1 - \gamma)(1 - F_t)D + (U + G)WF_t D + \text{Si}_e \frac{(DM_1)^2}{2TD} - P_c i_{c_1} \frac{(TD - DM_1)^2}{2TD} + \frac{X'}{T} + \frac{d'}{T}\right) \dots (4) \end{aligned}$$

$$Y_t(F_t, T) = \frac{1}{T}(K_1) + T(K_2 - K_4 F_t + K_5 F_t^2) + K_3 F_t$$

$$Y_t(F_t, T) = \frac{1}{T}(K_1) + T\Gamma(F_t) + \delta(F_t)$$

$$\Gamma(F_t) = K_2 - K_4 F_t + K_5 F_t^2$$

$$\delta(F_t) = K_3 F_t$$

$$T_a^* = \sqrt{\frac{K_1}{\Gamma(F_t)}}, \quad F_a^* = \frac{K_4 T - K_3}{2K_5 T}$$

$$\begin{aligned} Y'(F_t, T) &= \frac{1}{T}(K_1) + T\Gamma(F_t) + \delta(F_t) \\ &= 2\sqrt{K_1}\Gamma(F_t) + \delta(F_t) \end{aligned}$$

$$T_a^* = \sqrt{\frac{K_1}{K_2 - K_4 F_t + K_5 F_t^2}}$$

$$T_a^* = \sqrt{\frac{K_1(2K_5 T)^2}{K_2(2K_5 T)^2 - K_4(K_4 T - K_3)(2K_5 T) + K_5(K_4 T - K_3)^2}}$$

Finally, putting the values of K_1, K_2, K_3, K_4, K_5 in equation gives

$$K_1 = [(O + (1 + m_r))](R_s + 2a) + fd' + X' - \text{Si}_e \frac{DM_1^2}{2} + P_c i_{c_1} \frac{(DM_1)^2}{2}$$

$$K_2 = \frac{\beta\gamma D}{2} + \frac{P_c i_{c_1} D}{2}$$

$$K_3 = S_c D + \alpha D(1 + m_r) + (\text{Im}_c + 2T_c + (H_s + H'_s)T_t) - Z_c D(1 - \gamma)$$

$$K_4 = \gamma\beta D$$

$$\begin{aligned} K_5 = & \frac{(1 + m_r)(H_s + H'_s)\alpha^2 \beta^2}{R_r} + \frac{(1 - \alpha^2)(H + H')D}{2} + \frac{\alpha(H + H')D^2}{S_r} + \frac{H_r + H'_r}{2}\alpha^2 D \\ & + \frac{\beta\gamma D}{2} + (U + G)WF_t D \end{aligned}$$

CASE: 3

TOTAL OPTIMUM SOLUTION IF T>M₂

$$\begin{aligned}
 TP(F_t, T) = & SD(F_t + \gamma(1 - F_t) - \left[\frac{O}{T} + P_c(F_t D + \gamma(1 - F_t) D) + \right. \\
 & S_c F_t D + (H + H') \left[\frac{(1 - \alpha)^2 F_t^2 TD}{2} \right] + \frac{\alpha T(F_t D)^2}{S_r} + (H_r + H'_r) \\
 & \left. \left(\frac{(\alpha F_t)^2}{2} \right) + \frac{\beta(1 - F_t)^2 \gamma D}{2} \right. \\
 & + \alpha F_t D(1 + m_r) \left(\frac{R_s + 2a}{\alpha F_t TD} + lm_c + 2T_c \right) (H_s + H'_s) \left(\frac{\alpha F_t TD}{R_r} + T_t \right) \\
 & + L(1 - \gamma)(1 - F_t) D + (U + G) \\
 & W F_t D + Si_e \frac{(DM_1)^2}{2TD} - P_c i_{c_2} \left(\frac{(TD - DM_2)^2}{2TD} \right) - P_c i_{c_1} \frac{D}{T} (M_2 T - \\
 & M_2^2 - M_1 T + M_1 N) - P_u i_{c_1} \frac{D(M_2 - M_1)^2}{2TD} (M_2 T - M^2 - M^2 T + \\
 & M_1 N) - P_c i_{c_1} \frac{D(M_2 - M_1)^2}{2T} \left. \right] + f \frac{d'}{T} + \frac{x'}{T} \dots (5)
 \end{aligned}$$

$$\begin{aligned}
 Y_t(F_t, T) &= \frac{1}{T} (K_1) + T(K_2 - K_4 F_t + K_5 F_t^2) + (K_3 F_t)^2 + (K_3 F_t) \\
 Y_t(F_t, T) &= \frac{1}{T} (K_1) + T(K_2 - K_4 F_t + K_5 F_t^2) + (K_3 F_t)^2 + (K_3 F_t)
 \end{aligned}$$

$$\begin{aligned}
 Y_t(F_t, T) &= \frac{1}{T} (K_1) + T\Gamma(F_t) + \delta(F_t) \\
 \Gamma(F_t) &= K_2 - K_4 F_t + K_5 F_t^2 \\
 \delta(F_t) &= K_3 F_t
 \end{aligned}$$

$$T_a^* = \sqrt{\frac{K_1}{\Gamma(F_t)}}, \quad F_a^* = \frac{K_4 T - K_3}{2K_5 T}$$

$$\begin{aligned}
 Y'(F_t, T) &= \frac{1}{T} (K_1) + T\Gamma'(F_t) + \delta'(F_t) \\
 &= 2\sqrt{K_1} \Gamma(F_t) + \delta(F_t)
 \end{aligned}$$

$$\begin{aligned}
 Y'(F_t, T) &= \frac{\beta \gamma T - (S_c + \alpha(1 + m_r))(lm_c + 2T_c + (H_s + H'_s)T_t - Z_c(1 - \gamma))}{2 \left[\frac{(1 + m_r)(H_s + H'_s)\alpha^2 D}{R_r} + (1 - \alpha)^2 (H + H') + \frac{2\alpha(H + H')}{S_r} + (H_r + H'_r)\alpha^2 + \beta \gamma \right] T}
 \end{aligned}$$

$$T_a^* = \sqrt{\frac{K_1}{K_2 - K_4 F_t + K_5 F_t^2}}$$

$$T_a^* = \sqrt{\frac{K_1}{K_2 - K_4 \left(\frac{K_4 T - K_3}{2K_5 T} \right) + K_5 \left(\frac{K_4 T - K_3}{2K_5 T} \right)^2}}$$

$$T_a^* = \sqrt{\frac{K_1 (2K_5 T)^2}{K_2 (2K_5 T)^2 - K_4 (K_4 T - K_3) (2K_5 T) + K_5 (K_4 T - K_3)^2}}$$

$$\begin{aligned}
K_1 &= (O + (1 + m_r)(R_s + 2a) + X' + fd' + P_{c_1}i_{c_2} \frac{(DM_1^2)}{2} - P_{i_e} \frac{DM_1^2}{2} - P_{c_1}i_{c_1} DM_2^2 \\
&\quad + P_{c_1}i_{c_1} DM_1 M_2 + P_{c_1}i_{c_1} \frac{D(M_2 - M_1)^2}{2} \\
K_2 &= \left(\frac{\beta\gamma D}{2} + \frac{P_{c_1}i_{c_2}}{2} \right) \\
K_3 &= (S_c D + (1 + m_r)[lm_c + 2T_c + (H_s + H'_s)T_t] - Z_c D(1 - \gamma)) \\
K_4 &= (\gamma\beta D) \\
K_5 &= \frac{(1 + m_r)(H_s + H'_s)\alpha^2 D^2}{R_r} + \frac{(1 - \alpha)^2 (H + H') D}{2} \frac{\alpha (H + H') D^2}{S_r} + \frac{(H_r + H'_r)\alpha \beta \gamma D}{2} \\
&\quad + (U + G)WF_t D)
\end{aligned}$$

IV. NUMERICAL EXAMPLE:

1. Ordering cost = \$100/ order, Markup percentage = 2%, Setup cost of repair store=\$10/setup, Fixed cost(transportation) =\$20/trip, Road construction cost = Rs.750/trip, Fuel cost = Rs.50/liter, distance travelled = 1000Km, Back ordered cost = \$2/unit/year, percentage of back ordered demand = 15%, unit return cost for the imperfect products = \$2/ unit, Demand rate = 10,000 units/year, Selling price of one unit = \$5/unit, interest earned = 3%, inspection cost = \$ 0.5/ unit, percentage of imperfect items = 0.02%, labor and material cost = \$2/unit, transportation cost of the imperfect item on rework item =\$2/unit, Holding cost of rework products= \$2/unit/year, Carbon emission cost for holding perfect products = \$1/unit/year, Holding cost at repair store=\$1/unit/year, carbon emission cost at repair store = \$1/unit/year, Holding cost of perfect items = \$4/unit/year, carbon emission cost on holding perfect items=\$1/unit/year, Total transportation time of imperfect products=2/220 year, Lost sales of cost = 0.5\$/unit/year, Purchasing cost of one unit= \$20/unit, Repaired rate=10,000 units, Screening rate= 55,000/units/year, Return cost= \$2/unit, Penalty cost from goodwill loss = \$1/unit, Percentage of imperfect items returned=0.01%, Fraction of time= 0.22%.

Optimum values for different cases are given below:

Scenario	K ₁	K ₂	K ₃	K ₄	K ₅	T _a *
Case 1	50,910	9000	769.2	3000	26490.73	\$265
Case 2	38,301	51500	769.2	3000	26490.73	\$272
Case 3	18,223	1501.5	769.2	3000	26490.73	\$295

2. Ordering cost = \$100/order, Markup percentage = 2%, Setup cost of repair store=\$5/setup, Fixed cost(transportation) =\$10/trip, Road construction cost = Rs.750/trip, Fuel cost = Rs.50/liter, distance travelled = 1000Km, Back ordered cost = \$1/unit/year, percentage of back ordered demand = 10%, unit return cost for the imperfect products = \$2/ unit, Demand rate = 10,000 units/year, Selling price of one unit = \$3/unit, interest earned = 3%, inspection cost = \$ 0.1/ unit, percentage of imperfect

items = 0.01%, labor and material cost = \$1/unit, transportation cost of the imperfect item on rework item = \$0.1/unit, Holding cost of rework products = \$2/unit/year, Carbon emission cost for holding perfect products = \$1/unit/year, Holding cost at repair store = \$0.1/unit/year, carbon emission cost at repair store = \$0.5/unit/year, Holding cost of perfect items = \$1/unit/year, carbon emission cost on holding perfect items = \$1/unit/year, Total transportation time of imperfect products = 2/220 year, Lost sales of cost = 0.5\$/unit/year, Purchasing cost of one unit = \$20/unit, Repaired rate = 10,000 units, Screening rate = 55,000/units/year, Return cost = \$2/unit, Penalty cost from goodwill loss = \$1/unit, Percentage of imperfect items returned = 0.01%, Fraction of time = 0.22%.

Optimum values for different cases are given below:

Scenario	K_1	K_2	K_3	K_4	K_5	T_a^*
Case 1	50,876	1250	102.24	1000	20996.64	\$229
Case 2	50,881	2000	102.24	1000	20996.64	\$238
Case 3	50,950	4250	102.24	1000	20996.64	\$246

V. CONCLUSION

In this paper, with the integration of backordering, transportation, and multi-delay-in-payment, the sustainable inventory model was studied with synergistic effects of reworking for imperfect products for environmental impact. In addition, the carbon pollution costs are also included in the model to expand on the environmental effects in the benefit feature. In terms of multi-delay-in-payment the inventory method was also coordinated with a trade credit strategy. This multi-payment delay acts as a source of interim financial investment and can be used to boost sales. The optimal solution according to different scenarios of the cycle time with permissible delay-in-payment is derived. Finally, we illustrate a numerical example for expressing the inventory model that has been developed.

VI. REFERENCES

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