M/G/1 QUEUEING MODEL WITH ARBITRARY SERVICE TIME BASED ON FUZZY RANDOMNESS

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ABSTRACT

Fuzzy waiting line models venture diversified footprints galvanizing on uncertain realm of the world. This study explains a fuzzy queuing model which signifiesthe arrival distribution$\lambda_0$asPoisson and the service time$\mu_0$is arbitrarily distributed, FCFS discipline. The synchronization of this paper is to extract the membership functions of the execution proportions like the average and variance of queue length for two fuzzy queues based on Randomness principle and the efficacy of different parameters of the system performance measuresis fathomed by numerical exploration.

Keywords: Fuzzy, Queuing system, arbitrary service time, Average queue length, Variance of queue length.

INTRODUCTION

Queuing theory has blossomed in many diverse areas since its initial conception.We use various platforms in our everyday life, for example at service-related activities such as customers waiting for service in hypermarkets, cars searching for parking or road crossing, flights awaiting at an international terminal and broken equipment's desperately looking for facilities to be repaired.Inwaiting line models,generally the inter-arrival and service times are invariably probabilistic, but more possibilisticin realistic scenarios.

Fuzzy waiting line structures are more legitimate and productive than deterministic modeling techniques. In the perspective of conventional evolutionary computation, all distributions will enforce the inter-arrival times and service times. In practicability, exploration is described based on the pattern of arrival and service which are defined by linguistic quantifiers such as rapid, tolerable, rather than by probabilities.

In 2015,Hemanta K. Baruah enunciated the variation in probability and possibility space formulated on Randomness-Fuzziness consistency principle.AparnaKushwaha, Anshula Pandey,Varun Kumar Kashyap (2018) researched on general queuing model with arbitrary service time distribution. The parametric idea is engraved to shape the membership functions for two specific counters FM / M/1 and M / FM/1 centered on the Randomness-Fuzziness consistency concept. The average and variance production measuresof the queue length are evaluated to identify the parameter values of the membership functions.

The enactment proportions of the expounded model are:

i) Average Queue Length: $L_p = \left(\frac{\left(\frac{\lambda_0}{\mu_0}\right)}{\left(1-\frac{\lambda_0}{\mu_0}\right)}\right)^2$

ii) Average Length of Non -Empty Queue: $E(K: K > 0) = \left(\frac{1}{1-\frac{\lambda_0}{\mu_0}}\right) = \frac{\mu_0}{\mu_0-\lambda_0}$

iii) Variance of Queue Length: $V(L) = \frac{\frac{\lambda_0}{\mu_0}}{\left(1-\frac{\lambda_0}{\mu_0}\right)^2}$

THE FM/M/1 QUEUE

Consider aqueuing system with single server, inwhich the arrivals and departures followpoisson process with fuzzy parameter λ_0 and the crisp parameter μ_0. FM/M/1 queue is same as the M/M/1 queue with their membership function as:

$$\varphi_f(\lambda_0,\mu_0)(\omega) = \sup \{\mu_0(\sigma) : \omega = f(\sigma,\mu_0)\}$$

The arrival rate is $\lambda_0 = \{(\sigma, \varphi_{\lambda_0}(\sigma)) : \sigma \in R^+ \}$

The membership function of the arrival rate is $\varphi_{\lambda_0}(\omega)$

Theα- level cut is $\lambda_0(\alpha) = \{\sigma \in R^+ : \varphi_{\lambda_0}(\sigma) \geq \alpha\}$, where $\lambda_0(\alpha)$ is a crisp set.

$$[\mathrm{Min}_{\sigma \in R^+}\{\sigma: \varphi_{\lambda_0}(\sigma) \geq \alpha\}, \mathrm{Max}_{\sigma \in R^+}\{\sigma: \varphi_{\lambda_0}(\sigma) \geq \alpha\}]$$

In this queue type the lower and upper bounds of intervals are denoted as,

$\chi' f(\alpha) = \min f(\sigma, \mu 0)$; subjected to: $\chi'_{\lambda_0}(\alpha) \leq \sigma \leq \chi''_{\lambda_0}(\alpha)$

$\chi''_f(\alpha) = \max f(\sigma, \mu_0)$;subjected to: $\chi'_{\lambda_0}(\alpha) \leq \sigma \leq \chi''_{\lambda_0}(\alpha)$

Then, $\chi'_f(\alpha)$ and $\chi''_f(\alpha)$ are invertible with respect to α.

According to the Randomness-Fuzziness consistency principle, reference of left function is $L(\omega)= \chi'_f(\alpha)^{-1}$ is a distribution function and the reference of right function is

$R(\omega)= \chi''_f(\alpha)^{-1}$ and this is a complementary distribution function.

The membership function of $\varphi_f(\lambda_0,\mu_0)$ is given by,

$$\varphi_f(\lambda_0,\mu_0)(\omega)=\begin{cases} L(\omega), \omega_1 \le \omega \le \omega_2 \\ R(\omega), \omega_2 \le \omega \le \omega_3 \\ 0, otherwise \end{cases}$$

such that, $L(\omega_1)= R(\omega_3)= 0$ and $L(\omega_2)= R(\omega_2)= 1$

The classical queuing theory is given by $W_0= \frac{(\lambda_0)^2}{(\mu_{0-\lambda_0})\mu_0}$ and $L_0= \frac{(\lambda_0)^2}{(\mu_{0-\lambda_0})\mu_0}$ where W_0 denote the arriving customers of expected waiting time in the queue and L_0 denote the length of the queue of expected system.The membership function of L_0 and W_0 are,

$$\phi_{L_0}(\omega) = sup_{\sigma,\mu_0,\in R^+,\frac{\sigma}{\mu_0}<1} \{\phi_{\lambda_0}(\sigma) : \omega= \frac{\sigma^2}{\mu_{0(\mu_0-\sigma)}}\}$$

$$\phi_{W_0}(\omega) = sup_{\sigma,\mu_0,\in R^+,\frac{\sigma}{\mu_0}<1} \{\phi_{\lambda_0}(\sigma) : \omega= \frac{\sigma^2}{\mu_{0(\mu_0-\sigma)}}\}$$

THE M/FM/1 QUEUE

Consider the queuing system of single server, in this queue the arrivals and departures follows the poisson process with fuzzy parameter μ_0 and the crisp parameter λ_0. M/FM/1 queue is same as the M/M/1 queue and their membership function is,

$$\varphi_f(\lambda_0,\mu_0)(\omega)= \sup \{\mu_0(\sigma) : \omega=f(\sigma,\mu_0)\}$$

The service rate μ_0 is $\mu_0= \{(\sigma, \phi_{\mu_0}(\sigma)):\sigma \in R^+\}$

The membership function of the service rate is $\phi_{\mu_0}(\omega)$

The α- level cut $\mu_0(\alpha)= \{\sigma \in R^+:\phi_{\mu_0}(\sigma) \ge\alpha\}$, where $\mu_0(\alpha)$ is a crisp set.

The α- cut is $[\min_{\sigma \in R^+}\{\sigma: \phi_{\mu_0}(\sigma)\ge\alpha\}, \max_{\sigma \in R^+}\{\sigma: \phi_{\mu_0}(\sigma)\ge\alpha\}]$

In this queue type the lower and upper bounds of intervals are

$\chi'_f(\alpha)= \min f(\lambda_0,\sigma)$ subject to: $\chi'_{\mu_0}(\alpha)\le \sigma \le \chi''_{\mu_0}(\alpha)$

$\chi''_f(\alpha)= \max f(\lambda_0,\sigma)$ subject to: $\chi'_{\mu_0}(\alpha)\le \sigma \le \chi''_{\mu_0}(\alpha)$

Then, $\chi'_f(\alpha)$ and $\chi''_f(\alpha)$ are invertible with respect to α.

According to the Randomness-Fuzziness consistency principle, reference of left function is $L(\omega)= \chi'_f(\alpha)^{-1}$ is a distribution function and the reference of right function is $R(\omega)= \chi''_f(\alpha)^{-1}$ and this is a complementary distribution function.

The membership function of $\varphi_f(\lambda_0,\mu_0)$ is given by,

$$\varphi_f(\lambda_0,\mu_0)(\omega)=\begin{cases} L(\omega), \omega_1 \leq \omega \leq \omega_2 \\ R(\omega), \omega_2 \leq \omega \leq \omega_3 \\ 0, otherwise \end{cases}$$

Such that, $L(\omega_1)= R(\omega_3) = 0$and $L(\omega_2)= R(\omega_2)= 1$

The classical queuing theory is $W_0= \frac{(\lambda_0)^2}{(\mu_{0-\lambda_0})\mu_0}$and $L_0= \frac{(\lambda_0)^2}{(\mu_{0-\lambda_0})\mu_0}$ where W_0 denote the arriving customers of expected waiting time in the queue and L_0 denote the length of the queue of expected system.

The membership function of L_0 and W_0 are,

$$\phi_{L_0}(\omega)=sup_{\sigma,\lambda_0,\in R^+,\frac{\lambda_0}{\sigma}<1} \{\phi_{\mu_0}(\sigma) : \omega= \frac{{\lambda_0}^2}{\sigma(\sigma-\lambda_0)}\}$$

$$\phi_{w_0}(\omega)=sup_{\sigma,\lambda_0,\in R^+,\frac{\lambda_0}{\sigma}<1} \{\phi_{\mu_0}(\sigma) : \omega= \frac{{\lambda_0}^2}{\sigma(\sigma-\lambda_0)}\}$$

NUMERICAL EXAMPLE

1. Consider the FM/M/1 queue with arrival rate of fuzzy number is given by λ_0= [7.5,8,8.5] and service rate with mean exponentially distributed as μ_0= 9. Then the confidence interval of α at the possibility level will be as $[\frac{(15+\alpha)}{2},\frac{(17-\alpha)}{2}]$.

To derive the membership function of average length for L_0 is,

$$\chi'_{L0}(\alpha)= \min \frac{(\lambda_0)^2}{(\mu_{0-\lambda_0})\mu_0} \quad \text{s.t,} \frac{(15+\alpha)}{2} \leq \lambda_0 \leq \frac{(17-\alpha)}{2} \qquad \ldots\ldots(1)$$

$$\chi''_{L0}(\alpha)=\max\frac{(\lambda_0)^2}{(\mu_{0-\lambda_0})\mu_0} \text{subject to:} \frac{(15+\alpha)}{2}\leq\lambda_0\leq\frac{(17-\alpha)}{2} \qquad \ldots\ldots(2)$$

Then λ_0 reaches its lower bound $\chi'_{L0}(\alpha)=\frac{(15+\alpha)^2}{18(3-\alpha)}$ attains its minimum. Consequently λ_0 reaches its upper bound as $\chi''_{L0}(\alpha)= \frac{(17-\alpha)^2}{18(1+\alpha)}$ attains its maximum.

The membership function of $\mu_{L0}(\omega)$ is,

$$\mu_{L_0}(\omega)=\begin{cases} -15-9\omega-162(\omega^2+4\omega)^{\frac{1}{2}}, \frac{225}{54}\le\omega\le\frac{256}{36} \\ 17+9\omega+18(9\omega^2+32\omega)^{\frac{1}{2}}, \frac{256}{36}\le\omega\le\frac{289}{18} \\ 0 \quad , \ otherwise \end{cases}$$

$$\mu_{w_0}(\omega)=\begin{cases} -15-9\omega-162(\omega^2+4\omega)^{\frac{1}{2}}, \frac{225}{54}\le\omega\le\frac{256}{36} \\ 17+9\omega+18(9\omega^2+32\omega)^{\frac{1}{2}}, \frac{256}{36}\le\omega\le\frac{289}{18} \\ 0 \quad , \ otherwise \end{cases}$$

For variance of queue length: The membership function of V (L) is,

$$\chi'_{V(L)}(\alpha)=\min \frac{\lambda_0\mu_0}{\mu_0{}^2-\lambda_0{}^2} \quad \text{s.t,} \frac{(15+\alpha)}{2}\le \lambda_0\le\frac{(17-\alpha)}{2} \qquad \text{……(1)}$$

$$\chi''_{V(L)}(\alpha)=\max \frac{\lambda_0\mu_0}{\mu_0{}^2-\lambda_0{}^2} \text{s.t,} \frac{(15+\alpha)}{2}\le \lambda_0\le\frac{(17-\alpha)}{2} \qquad \text{……(2)}$$

Then λ_0 reaches its lower bound $\chi'_{V(L)}(\alpha)=\frac{270+18\alpha}{99-30\alpha-\alpha^2}$ attains its minimum. Consequently λ_0 reaches its upper bound as $\chi''_{V(L)}(\alpha)=\frac{306-18\alpha}{35+34\alpha-\alpha^2}$ attains its maximum.

The membership function of $\mu_{L0}(\omega)$ is,

$$\mu_{V(L)}(\omega)=\begin{cases} \frac{-9-15\omega-3(14\omega^2+9)^{\frac{1}{2}}}{z}, \frac{270}{99}\le\omega\le\frac{288}{68} \\ \frac{9+17\omega+\frac{3}{\sqrt{2}}(4\omega^2-1)^{\frac{1}{2}}}{z}, \frac{288}{68}\le\omega\le\frac{306}{35} \\ 0 \quad , \quad otherwise \end{cases}$$

2. Consider the M/FM/1 queue with arrival rate of fuzzy number is given by λ_0= 5 and service rate with mean exponentially distributed as μ_0= [6.5, 7, 7.5]. Then the confidence interval of α at the possibility level will be as $[\frac{(13+\alpha)}{2},\frac{(15-\alpha)}{2}]$.To derive the membership function of average length for L_0 is,

$$\chi'_{L0}(\alpha)=\min \frac{(\lambda_0)^2}{(\mu_{0-\lambda_0})\mu_0} \text{s.t,} \frac{(13+\alpha)}{2}\le \mu_0\le\frac{(15-\alpha)}{2} \qquad \text{……(1)}$$

$$\chi''_{L0}(\alpha)=\max \frac{(\lambda_0)^2}{(\mu_{0-\lambda_0})\mu_0} \text{s.t,} \frac{(13+\alpha)}{2}\le \mu_0\le\frac{(15-\alpha)}{2} \qquad \text{……(2)}$$

Then μ_0 reaches its lower bound $\chi'_{L0}(\alpha)=\frac{100}{(13+\alpha)(3+\alpha)}$ attains its minimum. Consequently $\mu_0$reaches its upper bound as

$\chi''_{L0}(\alpha)=\frac{(17-\alpha)^2}{(15-\alpha)(5-\alpha)}$ attains its maximum.

The membership function of $\varphi_{L0}(\omega)$ is,

$$\mu_{L0}(\omega)=\begin{cases} -8\omega, -50(\omega^2+4\omega)^{\frac{1}{2}}, \frac{100}{39} \le \omega \le \frac{100}{56} \\ 10\omega + 50(\omega^2+4\omega)^{\frac{1}{2}},, \frac{100}{56} \le \omega \le \frac{100}{75} \\ 0, otherwise \end{cases}$$

$$\mu_{W0}(\omega)=\begin{cases} -8\omega, -50(\omega^2+4\omega)^{\frac{1}{2}}, \frac{100}{39} \le \omega \le \frac{100}{56} \\ 10\omega + 50(\omega^2+4\omega)^{\frac{1}{2}},, \frac{100}{56} \le \omega \le \frac{100}{75} \\ 0, otherwise \end{cases}$$

For variance of queue length,the membership function for V(L) is,

$$\chi'_{V(L)}(\alpha)= \min \frac{\lambda_0\mu_0}{{\mu_0}^2-{\lambda_0}^2} \quad \text{s.t, } \frac{(13+\alpha)}{2} \le \mu_0 \le \frac{(15-\alpha)}{2} \quad \text{......(1)}$$

$$\chi''_{V(L)}(\alpha)= \max \frac{\lambda_0\mu_0}{{\mu_0}^2-{\lambda_0}^2} \text{s.t, } \frac{(13+\alpha)}{2} \le \mu_0 \le \frac{(15-\alpha)}{2} \quad \text{......(2)}$$

Then μ_0 reaches its lower bound $\chi'_{L0}(\alpha)=\frac{130+10\alpha}{\alpha^2+26\alpha+69}$ attains its minimum. Consequently $\mu_0$reaches its upper bound as

$\chi''_{L0}(\alpha)= \frac{150-10\alpha}{\alpha^2-30\alpha+125}$ attains its maximum.The membership function of $\varphi_{L0}(\omega)$ is,

$$\mu_{V(L)}(\omega)=\begin{cases} \frac{5-13\omega-5(4\omega^2+1)^{\frac{1}{2}}}{z}, \frac{130}{69} \le \omega \le \frac{140}{96} \\ \frac{-5+15\omega+5(4\omega^2+1)^{\frac{1}{2}}}{z}, \frac{140}{96} \le \omega \le \frac{150}{125} \\ 0 \quad , \quad otherwise \end{cases}$$

CONCLUSION

The parametric programming method is applied to derive the membership functions of two simple fuzzy queues namely FM/M/1 and M/FM/1 and is based on the Randomness-Fuzziness consistency principle where the service time is arbitrary and the arrival is poisson. This paper analyzes the varied production measures to yield the maximum and minimum bound with numerical validity. It helps us to manipulate arbitrary service time based on randomness criterion. In addition, classical queues when extended to uncertainty throw realistic approach in fuzzy grounds.

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