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FREQUENCY OF REPLACEMENT WITH MAXIMUM EFFECTS OF ELECTRICUTIONS AND CUMULATIVE DAMAGES IN RELIABILITY

Irudaya Monica Catherine J[1], Dhanalakshmi K[2], SelvaRani P[3]

[1]*Assistant Professor, Department of Mathematics,*
Holy Cross College (Autonomous), Tiruchirappalli-2, Tamilnadu, India.
[2]*Assistant Professor, Department of Mathematics,*
Holy Cross College (Autonomous), Tiruchirappalli-2, Tamilnadu, India.
[3]*Assistant Professor, Department of Mathematics,*
Holy Cross College (Autonomous), Tiruchirappalli-2, Tamilnadu, India.
[1]monicairudaya@gmail.com, [2]dhanamaths219@gmail.com, [3]selvabala08@gmail.com

ABSTRACT

Electricutions creating damages to the system is a common phenomenon in reliability theory, which creates damage to the system due to its magnitude will affect the functioning of a system. Adequate replacement of the system is not realistic since it involves cost. A stochastic model is constructed with three different cases of Electricutions and the time to replacement of a system is obtained, when the breakdown threshold for the cumulative damages and maximum magnitude of Electricutions crosses its threshold simultaneously. The numerical illustration is determined to the mean and variance of time to replacement and the conclusion is presented.

KEYWORDS:

Time to replacement, breakdown threshold, cumulative damages, magnitude of Electricutions , external and internal Electricutions , shock model approach.

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INTRODUCTION

A Electricutions creating damages to the system placed in the environment is a common phenomenal in reliability theory. The system receives Electricutions in many different categories and it is classified into two mutual exclusive Electricutions (i) Internal power supply or voltage problem. (ii) Electricutions due to circumstances. These shock will create damage to the system due to its magnitude.Focusing only on the damages alone is not reality. The impact of damages occurs by the magnitude of the Electricutions . So the concept of threshold is considered for the cumulative damages and the maximum magnitude of the Electricutions of the system. In this context author [2] has studied many reliability models with various assumptions on damages. Authors in [1], [3] and [4] have studied the concept of manpower planning and the shock model approach. Considering the shock model approach, the replacement of a system is carried out whenever the cumulative damages and the maximum magnitude of the Electricutions of the system crosses its threshold simultaneously.

In this paper, the mean and variance of time to replacement of a system is determined for the three different cases of Electricutions . In case-I: it is assumed that the damages to the system are created by the magnitude of the Electricutions produced internally or externally. There are some minute Electricutions that will never create damage to the system. In case-II: it is assumed that the system receives n+r Electricutions . Of these n Electricutions will create the damages with probability $0<p<1$ to the system. Some system receives more Electricutions internally than the Electricutions due to the external circumstances. In case-III: it is assumed that the Electricutions received by the system has been classified into two mutually exclusive types.

For these three cases, the mean and variance of time to replacement have been determined when the threshold of cumulative damages and the maximum magnitude of the Electricutions of the system crosses its threshold simultaneously. The results are numerically illustrated and the findings in the illustration coincide with the realistic observation.

MODEL DESCRIPTION:

Consider the system in which its functioning gets affected due to the magnitude of the Electricutions . LetA_i , (i=1,2,3,……) be a stochastic process that represents the magnitude of the i^{th} shock with exponential distribution function $G_i(.)$ of parameter $\alpha_1>0$. Let B_i, (i=1,2,3,……)be a stochastic process which represents the damage due to the i^{th}shock with exponential distribution function $H_i(.)$ of parameter $\alpha_2>0$.D_i be the stochastic process that represents the time between $i-1^{th}$ and i^{th}shock. Let T_k is a randomly indexed partial sum that represents the maximum of the magnitude of the first k Electricutions (i.e.) T_k= Max $\{A_1,A_2,\ldots A_k\}$. The randomly indexed partial sumS_ℓ represents the cumulative damages to the system by the first ℓ Electricutions . Let N(t) is the stochastic process that represents the number of Electricutions exerted to the system up to the time t.Let R is a random variable that represents time to replacement of the system, with distribution function L(.), density function $\ell(.)$ with Laplace transform $\bar{l}(.)$. Let c_1 represents the constant threshold for the maximum magnitude of the Electricutions and c_2 represents the constant threshold for cumulative damages due to the Electricutions . It assumed that magnitude of the Electricutions , damages created to the system, inter-shock times and the threshold are statically independent.

ANALYTICAL RESULTS:

The analytical results for the mean and variance of time to replacement has been derived for the three different cases of inter-shock times.

CASE - I

In this case, it is assumed that the damages to the system are created by the magnitude of the Electricutions produced internally or externally and the number of Electricutions up to the time t follows the Poisson process with rate $\lambda > 0$. According to the policy, the replacement occurs before the time t is equivalent to a maximum of the magnitude of shock and cumulative damages crosses its threshold simultaneously before the time t. Hence the distribution function of time to replacement is determined as

$$P(R<t) = P(T_{N(t)}>c_1 \cap S_{N(t)}>c_2) = P(T_{N(t)}>c_1).P(S_{N(t)}>c_2)$$

Using the law of total probability, the distribution function of time to replacement is given by

$$L(t)= (1-e^{-\lambda t(1-G(c_1))})\ (1-e^{-\lambda t(1-H(c_2))})$$

By differentiating with respect to t and taking Laplace to transform for the probability density function of time to replacement is

$$\bar{l}(s) = \frac{\lambda(1-H(c_2))}{S+\lambda(1-H(c_2))} + \frac{\lambda(1-G(c_1))}{S+\lambda(1-G(c_1))} - \frac{\lambda(2-G(c_1)-H(c_2))}{S+\lambda(2-G(c_1)-H(c_2))}$$

Now, differentiating the Laplace transform of time to replacement with respect to s, the mean time to replacement is determined at s=0.

$$E(R) = \frac{1}{\lambda e^{-\alpha_2 c_2}} + \frac{1}{\lambda e^{-\alpha_1 c_1}} - \frac{1}{\lambda(e^{-\alpha_1 c_1}+e^{-\alpha_2 c_2})}$$

The second moment of time to replacement is determined by differentiating twice the Laplace transform of time to replacement with respect to s and s=0. From these results, the variance of time to replacement is determined and it is given by

$$V(R) = \left[\frac{2}{(\lambda e^{-\alpha_2 c_2})^2} + \frac{2}{(\lambda e^{-\alpha_1 c_2})^2} - \frac{2}{\left(\lambda(e^{-\alpha_1 c_1}+e^{\alpha_2 c_2})\right)^2}\right] - \left[\frac{1}{\lambda e^{-\alpha_2 c_2}} + \frac{1}{\lambda e^{-\alpha_1 c_1}} - \frac{1}{\lambda(e^{-\alpha_1 c_1}+e^{-\alpha_2 c_2})}\right]^2$$

CASE - II

Now the analytical results for the mean and variance of time to replacement are determined by assuming that the system receives n+r Electricutions . Of these n Electricutions will create the damages with probability 0<p<1 to the system. By proceeding as in case – I, differentiating the Laplace transform of time to replacement with respect to s, the mean and variance of time to replacement are determined

$$E(R) = \frac{1}{\lambda p e^{-\alpha_2 c_2}} + \frac{1}{\lambda p e^{-\alpha_1 c_1}} - \frac{1}{\lambda p(e^{-\alpha_1 c_1}+e^{-\alpha_2 c_2})}$$

$$V(R) = \left[\frac{2}{(\lambda p e^{-\alpha_2 c_2})^2} + \frac{2}{(\lambda p e^{-\alpha_1 c_1})^2} + \frac{2}{(\lambda p(e^{-\alpha_1 c_1}+e^{-\alpha_2 c_2})^2}\right] - \left[\frac{1}{\lambda p e^{\alpha_2 c_2}} + \frac{1}{\lambda p e^{-\alpha_1 c_1}} + \frac{1}{\lambda p(e^{-\alpha_1 c_1}+e^{-\alpha_2 c_2})}\right]^2$$

CASE - III

In this case, the Poisson process N(t) that represents the number of Electricutions exerted to the system is considered as the sum of two [Internal and External] independent Poisson process with parameter $\lambda_1 > 0$ and $\lambda_2 > 0$

Now, the probability density function of time to replacement is derived by taking derivative for the distribution function with respect to t. Taking Laplace transform for the probability density function of time to replacement and differentiating the Laplace transform, the moments of time to replacement are determined.

$$E(R)= \frac{1}{(\lambda_1+\lambda_2)e^{-\alpha_2 c_2}} + \frac{1}{(\lambda_1+\lambda_2)e^{-\alpha_1 c_1}} - \frac{1}{(\lambda_1+\lambda_2)(e^{-\alpha_1 c_1}+e^{-\alpha_2 c_2})}$$

The variance of time to replacement is derived by using the first two moments of time to replacement. It is given by

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$$V(R)=\left[\frac{2}{((\lambda_1+\lambda_2)e^{-\alpha_2 c_2})^2}+\frac{2}{((\lambda_1+\lambda_2)e^{-\alpha_1 c_2})^2}-\frac{2}{\left((\lambda_1+\lambda_2)(e^{-\alpha_1 c_1}+e^{\alpha_2 c_2})\right)^2}\right]-\left[\frac{1}{(\lambda_1+\lambda_2)e^{-\alpha_2 c_2}}+\frac{1}{(\lambda_1+\lambda_2)e^{-\alpha_1 c_1}}-\frac{1}{(\lambda_1+\lambda_2)(e^{-\alpha_1 c_1}+e^{-\alpha_2 c_2})}\right]^2$$

NUMERICAL ILLUSTRATION

The following tables are the numerical values of the mean and variance of time to replacement for the three cases. By fixing the thresholds C_1=200 and C_2=300 and varying the other parameters, numerical values of mean and variance of time to replacement are studied.

CASE - I

$\frac{1}{\lambda}$	$\frac{1}{\alpha_1}$	$\frac{1}{\alpha_2}$	E(R)	V(R)
0.033	0.016	0.014	2220.7	3750183.2
0.040	0.016	0.014	1832.1	2552468.5
0.050	0.016	0.014	1465.7	1633579.8
0.066	0.016	0.014	1110.4	937545.8
0.10	0.016	0.014	732.84	408394.9
0.028	0.014	0.014	2497.8	5351321.1
0.028	0.012	0.014	2437.5	5484858.6
0.028	0.011	0.014	2420.1	5534372.4
0.028	0.01	0.014	2408.0	5572599.2
0.028	0.009	0.014	2399.6	5601243.1
0.028	0.016	0.012	1658.7	1650667.6
0.028	0.016	0.011	1384.5	1070572.7
0.028	0.016	0.01	1199.1	815539.7
0.028	0.016	0.009	1076.8	721806.0
0.028	0.016	0.008	998.21	701294.5

CASE-II

P	$\frac{1}{\lambda}$	$\frac{1}{\alpha_1}$	$\frac{1}{\alpha_2}$	E(R)	V(R)
0.2	0.028	0.016	0.014	13086.4	130227984.7
0.3	0.028	0.016	0.014	8724.3	57879104.3
0.4	0.028	0.016	0.014	6543.2	32556996.1
0.5	0.028	0.016	0.014	5234.6	20836477.5
0.6	0.028	0.016	0.014	4362.2	14469776.0
0.1	0.033	0.016	0.014	22207.3	375018328.8
0.1	0.040	0.016	0.014	18321.0	255246850
0.1	0.050	0.016	0.014	14656.8	163357984
0.1	0.066	0.016	0.014	11103.6	93754582.2
0.1	0.10	0.16	0.014	7328.4	40839496

0.1	0.028	0.014	0.014	24978.3	535132116.5
0.1	0.028	0.012	0.014	24374.9	548485868.3
0.1	0.028	0.011	0.014	24200.7	553437246.3
0.1	0.028	0.01	0.014	24079.7	557259925.9
0.1	0.028	0.009	0.014	23996.2	560124310.6
0.1	0.028	0.016	0.012	16586.9	165066763.1
0.1	0.028	0.016	0.011	13845.0	107057278.9
0.1	0.028	0.016	0.01	11990.8	81553975.7
0.1	0.028	0.016	0.009	10767.9	72180608.9
0.1	0.028	0.016	0.008	9982.1	70129459.5

CASE-III

$\frac{1}{\lambda_1}$	$\frac{1}{\lambda_2}$	$\frac{1}{\alpha_1}$	$\frac{1}{\alpha_2}$	**E(R)**	**V(R)**
0.033	0.033	0.016	0.014	1110.4	937545.8
0.040	0.033	0.016	0.014	1003.9	766363.2
0.050	0.033	0.016	0.014	882.9	59281.8
0.066	0.033	0.016	0.014	740.2	416687
0.10	0.033	0.016	0.014	551	230875
0.028	0.040	0.016	0.014	1077.7	883207
0.028	0.050	0.016	0.014	939.5	671260.6
0.028	0.066	0.016	0.014	779.6	462194.3
0.028	0.10	0.016	0.014	572.5	249264.5
0.028	0.20	0.016	0.014	321.4	78561.6
0.028	0.033	0.014	0.014	1146.5	1127502.2
0.028	0.033	0.012	0.014	1118.9	1155638
0.028	0.033	0.011	0.014	1110.9	1166070.4
0.028	0.033	0.01	0.014	1105.3	1174124.6
0.028	0.033	0.009	0.014	1101.5	1180159.7
0.028	0.033	0.016	0.012	761.3	347789.1
0.028	0.033	0.016	0.011	635.5	225565.4
0.028	0.033	0.016	0.01	550.4	171831
0.028	0.033	0.016	0.009	494.2	152081.6
0.028	0.033	0.016	0.008	458.2	147760

NUMERICAL RESULTS:

In all the three cases, if the average magnitude of the Electricutions and its damages decreases, then the magnitude of the Electricutions and its damages increases, which leads to the reduction in the mean time to replacement.

For the Cases I and II, if the average inter Electricutions time increases, then the inter-Electricutions time decreases, which makes the mean time to replacement declines and in Case – III, if the average inter external Electricutions and inter internal Electricutions increases, then the inter external Electricutions time and inter internal Electricutions time decreases. This reduces the mean time to replacement.

Perceiving the results for the three cases from the tables, the numerical results coincides with the realistic surveillance.

CONCLUSION:

The idea of considering the damages to the system due to the magnitude of Electricutions is a convincing one. The general observation of hike in damages leads to the reduction of time to replacement and the time between the shock (inter-shock times) elongates, the replacement time of a system extended. This observation coincides with realistic scrutiny. The concept of considering the independence in the inter-shock times and the damages can be dropped in the future to study the dependence nature of shock and its damages.

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