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A FIXED POINT THEOREM FOR JSC-CONTRACTION TYPE MAPPINGS ON GROUP THEORY

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Abstract

A fixed point theorem is established for a new class of JSC-contraction type mappings. In this paper,we introduced a new method of proofs that allow to prove fixed point theorem for JSC-contraction on group theory. These theorems generalizes the results of [7].

Keywords: Complete metric spaces, JSC contraction

Mathematics Subject Classification: Primary 47H10; Secondary 54H25

1. INTRODUCTION

Fixed point theory has become one of the most interesting areas of research in the last fifty years for instance research about optimization problem, control theory, differential equations and etc. The fixed point theorem generally known as the Banach contraction mapping principle, appeared in explicit from Banach's thesis in 1992. It has became a very popular tool in solving many problems in mathematical Analysis.

The study of metric spaces express the most important role to many fields both in pure and applied science such as biology, medicine, physics and computer science ([12,16]). Inspired from the impact of this natural idea to functional analysis, several researches have been extended and generalized this principle for different kind of contraction in various spaces such as S-metric, pseudo metric, fuzzy metric, JS-quasi contraction, Banach contraction principle.

Here, we introduced the notion of fixed point theorem established a fixed point theorem for JSC-contraction type mappings on group theory.

The main purpose of this paper is to show that the results concerned in metric spaces with JSC-contraction in [14] are sequences of theorem 2.1.18. Before going to the main results. Let us recall the basic definitions and theorems. The concepts of ciric contraction JS-contraction have been introduced, respectively, by ciric [5] andHusain et al. [6] as follows [13].

2. Preliminaries:

Definition 2.1.1[10]

Let (X,d) be a metric space, A mapping T:$X \to X$ is said to be a JSC-contraction if there exist $\psi \in \Psi$ and non-negative number q,r,s,t with q+r+s+2t< 1 such that $\psi\big(d(Tx,Ty)\big) \leq q\psi\big(d(x,y)\big) + r\psi\big(d(x,Tx)\big) + s\psi\big(d(y,Ty)\big) + t\big(d(x,Ty) + d(y,Tx)\big)\ \forall\, x,y \in X$ (3)

Where Ψ is the set of all functions ψ: $[0,+\infty) \to [0,+\infty)$ satisfying conditions:

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(ψ_1) ψ is non-decreasing and ψ(t) =0 if and only if t=0;
(ψ_2) For each sequence $\{t_n\} \subset (0,+\infty), \lim_{n\to\infty} \psi(t_n) = 0$ *if and only if* $\lim_{n\to\infty} t_n = 0$;
(ψ_3) There exist $r \in (0,1)$ and $l \in (0,+\infty)$ such that $\lim_{t\to 0^+} \frac{\psi(t)}{t^r} = l$;
(ψ_4) $\psi(a+b) \le \psi(a) + \psi(b)$ *for all* $a,b > 0$.
For convenience, we denote by ψ_1, the set of all non-decreasing functions $\psi: (0,+\infty) \to (1,+\infty)$ satisfying (ψ_2) *and* (ψ_3) and by ψ_2, the set of all functions $\psi: (0,+\infty) \to (1,+\infty)$ satisfying (ψ_1),(ψ_2) *and* (ψ_4).

Remark 2.1.2 [10]

(1) If $f(t)=\sqrt{t}, t \ge 0$. Then $f \in \Psi \cap \Psi_1 \cap \Psi_2$.

(2) If $g(t) = t$ for $t \ge 0$,then g Ψ_2, *but* $g \ne \Psi \cup \Psi_1$. since $\frac{l}{t^r} = 0$ for each r$\in (0,1)$, that is, (ψ_3) is not satisfied.

(3) Clearly $\Psi \subseteq \Psi_1$ *and* $\Psi \subseteq \Psi_2$.

Results 2.1.3 [10]

Let (X,d) be a complete metric space, and T:$X \to X$ be JSC contraction. Assume that there exist $\psi \in \Psi_1$ and k$\in (0,1)$, such that$\forall\ x, y \in X$,

$$d(Tx,Ty) \neq 0 \Longrightarrow \psi\big(d(Tx,Ty)\big) \le k\psi\big(d(x,y)\big). \quad (4)$$

Then T has a unique fixed point in X.

3. A FIXED POINT THEOREM FOR JSC-CONTRACTION TYPE MAPPINGS ON GROUP THEORY

3.1 Introduction

Let Ψ is the set of all functions ψ: $[0,+\infty) \to [0,+\infty)$ satisfying the following conditions:
(ψ_1) ψ is non-decreasing;
(ψ_2) Foreach sequence $\{t_n\} \subset (0,+\infty)$,we have$\lim_{n\to\infty} \psi(t_n) = 0$if and only if $\lim_{n\to\infty} t_n = 0$;
(ψ_3) There exists $r \in (0,1)$ and $l \in (0,+\infty)$ such that $\lim_{t\to 0^+} \frac{\psi(t)}{t^r} = l$;
In [10], we introduced the class of JSC-contraction mapping as follows.
In [4], the following generalization of banach contraction principle was established.

Theorem3.1.1

Let (X,d) be a complete metric space, and let $T: X \to X$ be JSC-contraction. Then T has a unique fixed point.

Proof:

Observe that banach contraction principle follows from theorem 3.1.2 by taking $\psi(t) = \sqrt{t}$. For other related results, we refer the reader to [15,20]. In this paper, a fixed point theorem for a new class of JSC-contraction type mappings is presented.

3.2 Main Results

In this section, a new fixed point theorem is established for a new class of JSC-contraction type mappings. The obtained result is an extension of theorem 3.1.2. At first let us introduces some notations. Let M be a non-empty set, and let $T: M \to M$ be a given mapping. We denote by Fix (T) the set of all fixed point of T,
i.e)Fix (T) =$\{x \in M; x = Tx\}$

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Suppose that M is a group with respect to a certain operation +. For $x \in M$ and $N \subset M$.We denoted by x+N the subset of M defined by

$$x + N = \{x + y; y \in N\}$$

We denote $\mathbb{N}$ by $\mathbb{N} = \{0,1,2, \dots \dots \dots .\}$

we denote $\mathbb{N}^+$ by $\mathbb{N}^+$=$\{1,2,3, \dots \dots \dots .\}$

our fixed point theorem is stated as follows:

Theorem 3.2.1

Let E be a group with respect to a certain operation +. Let X be a subset of E endowed with a certain metric d such that (X, d) is complete. Let $X_0 \subset X$be a closed subset of X. such that $X_0$is a subgroup of E. let $T: X \to X$ be a given mapping satisfying,

$$(x, y) \in X \times X, \qquad x - y \in X_0, d(Tx, Ty) \neq 0$$
$$\Rightarrow \psi\big(d(Tx, Ty)\big) \leq k\psi\big(d(x, y)\big). \qquad \longrightarrow (1)$$

There$k \in (0,1)$ is a constant and $\psi \in \Psi$.

Suppose that the operation mapping $\pm(x, y) = x \pm y$, $(x, y) \in X \times X$ is continuous w.r.to the metric d. moreover, suppose that

$$x - Tx \in X_0\,, \quad x \in X. \qquad \longrightarrow (2)$$

Then we have,

(i) For every $x \in X$, the picard sequence $\{T^n x\}$ converges to a fixed point of T.

(ii) For every $x \in X$, $(x + X_0) \cap Fix\ (T) = \{\lim_{n\to\infty} T^n\, x\}$.

Proof:

Let $x \in X$ be an arbitrary point in X. If for some$p \in \mathbb{N}$, we have $T^p x = T^{P+1}x$, Then $T^p x$ will be fixed point of T. So without loss of generality,

We can suppose that $d(T^n x, T^{n+1}x) > 0\ \forall\ n \in \mathbb{N}$.

From (2), we have $x - Tx \in X_{0.}$

Using (1), we obtained $\psi\big(d(Tx, T^2x)\big) \leq k\psi\big(d(x, Tx)\big)$.

Again, using (2), we obtain $Tx - T^2x = Tx - T(Tx) \in X_{0.}$

Which implies from (1) that,

$$\psi\big(d(T^2x, T^3x)\big) \leq k\left(\psi\big(d(Tx, T^2x)\big)\right) \leq k^2\psi\big(d(x, Tx)\big)$$

Therefore, by induction we obtain

$$T^n x - T^{n+1}x \in X_0, \quad n \in \mathbb{N} \longrightarrow (3)$$

And $\psi\big(d(T^n x, T^{n+1}x)\big) \leq K^n\left(\psi\big(d(x, Tx)\big)\right),\ n \in \mathbb{N}$

Thus, we have

$$0 \leq \psi\big(d(T^n x, T^{n+1}x)\big) \leq K^n\left(\psi\big(d(x, Tx)\big)\right), \qquad n \in \mathbb{N} \longrightarrow (4)$$

Passing to the limit as $n \to \infty$ in (4), we obtain,

$$\lim_{n\to\infty} \psi\big(d(T^n x, T^{n+1}x)\big) = 0$$

Which implies from (ψ_2)that $\lim_{n\to\infty} d(T^n x, T^{n+1}x) = 0$. From condition$(\psi_3)$, There exist $r \in (0,1)$ and $l \in [0, \infty]$ such that $\lim_{n\to\infty} \frac{\psi\left(d(T^n x, T^{n+1}x)\right)}{[d(T^n x, T^{n+1}x)]^r} = l$.

Suppose that $l < \infty$. In this case, let $B = \frac{l}{2} > 0$.

From the definition of the limit, there exists $n_0 \in \mathbb{N}$,

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Such that $\left|\frac{\psi\left(d(T^n x,T^{n+1}x)\right)}{[d(T^n x,T^{n+1}x)]^r}-l\right| \le B, n \in n_0$

This implies that,

$$\frac{\psi\left(d(T^n x,T^{n+1}x)\right)}{[d(T^n x,T^{n+1}x)]^r} \ge l-B=B,\ \ n \in n_0$$

Then, $n[d(T^n x,T^{n+1}x)]^r \le An\left[\psi\left(d(T^n x,T^{n+1}x)\right)\right],\ n \ge n_0$ where $A = {}^{1}/_{B}$

Suppose now that $l=\infty$. let $B>0$ be an arbitrary positive number. From the definition of the limit, there exist $n_0 \in \mathbb{N}$ such that

$$\frac{\psi\left(d(T^n x,T^{n+1}x)\right)}{[d(T^n x,T^{n+1}x)]^r} \ge B,\ \ n \ge n_0$$

This implies that,

$$n[d(T^n x,T^{n+1}x)]^r \le An\left[\psi\left(d(T^n x,T^{n+1}x)\right)\right], n \ge n_0 \text{where } A = {}^{1}/_{B}$$

Thus, in all cases, there exist $A>0$ and $n_0 \in \mathbb{N}$ such that

$$n[d(T^n x,T^{n+1}x)]^r \le An\left[\psi\left(d(T^n x,T^{n+1}x)\right)\right], \qquad n \ge n_0$$

Using (4), we obtain,

$$n[d(T^n x,T^{n+1}x)]^r \le An\ \left[K^n\left(\psi\left(d(x,Tx)\right)\right)\right], \qquad n \ge n_0\,.$$

Letting $n \to \infty$ is the above inequality,

We obtain,

$$\lim_{n\to\infty} n\,[d(T^n x,T^{n+1}x)]^r = 0.$$

Thus, There exist $n_1 \in \mathbb{N}$ such that

$$d(T^n x,T^{n+1}x)\ \le \frac{1}{n^{\frac{1}{r}}}\ \ ,n \ge n_1 \qquad \to (6)$$

Using (6), we have

$$d(T^n x,T^{n+m}x) \le d(T^n x,T^{n+1}x)+d(T^{n+1}x,T^{n+2}x)+\cdots\cdots+ d(T^{n+m-1}x,T^{n+m}x)$$
$$\le \frac{1}{n^{1/r}}+\frac{1}{(n+1)^{1/r}}+\cdots\cdots+\frac{1}{(n+m-1)^{1/r}}$$
$$\le \sum_{i=n}^{\infty}\frac{1}{i^{1/r}}$$

Which implies that the Picard sequence $\{T^n x\}$ is Cauchy in the complete metric space (X, d).$\left(Since\ r \in (0,1)\right)$. Then there is some $\omega \in X$

Such that, $\lim_{n\to\infty} d(T^n x,\omega)=0 \qquad \to (7)$

On the other hand, observe that for $n,p \ge 1$.

$$T^n x - T^{n+p} = (T^n x - T^{n+1}x)+(T^{n+1}x - T^{n+2}x)+\cdots\cdots + (T^{n+p-1}x - T^{n+p}x)$$

Therefore, by (3) and using the fact that $(X_0,+)$ is a group, we deduce that,

$$T^p x - T^{n+p}x\ \in X_{0\ ,}\ n,p \ge 1.$$

Passing to the limit as $p \to \infty$, using (7), the continuity of the operation mapping $\pm$, and the closure of $X_{0\ ,}$

We obtain that $T^n x,\omega \in X_{0\ ,}\ n \in \mathbb{N} \qquad \to (8)$

W.L.G, we may suppose that $d(T^n x,T\omega)>0$ for all $n \in \mathbb{N}$.

Therefore, using (8) & (1), we have

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$$0 \leq \psi\big(d(T^{n+1}x, T\omega)\big) \leq k\left(\psi\big(d(T^n x, \omega)\big)\right), n \in \mathbb{N}.$$

As $n \to \infty$. Using (7) and(ψ_2), we deduce that

$$\lim_{n\to\infty} d(T^{n+1}x, \omega) = 0 \qquad \to (9)$$

Next by (7), (9) &the uniqueness of the limit yield $\omega = T\omega$.

That is, ω is a fixed point of T.

Then (1) is proved,

$$0 \leq \psi\big(d(z, T^{n+1}x)\big) = \psi\big(d(Tz, T^{n+1}x)\big)$$
$$\leq k\big[\psi\big(d(z, T^n x)\big)\big]$$
$$\vdots$$
$$\leq k^{n+1}\big[\psi\big(d(z, x)\big)\big], n \in \mathbb{N}.$$

Passing to the limit as $n \to \infty$ and Using(ψ_2), We deduce that

$$\lim_{n\to\infty} d(T^n x, z) = 0.$$

Which yields $z \in \{\lim_{n\to\infty} T^n x\}$.

Then we proved that,

$$(x + X_0) \cap Fix\,(T) \subset \left\{\lim_{n\to\infty} T^n x\right\}.$$

The proof is completed.

Corollary 3.2.1

Let E be a group with respect to a certain operation +. Let X be a subset of E endowed with a certain metric d such that (X, d) is complete. Let $X_0 \subset X$ be a closed suset of X. such that $X_0$is a subgroup of E. let $T: X \to X$ be a given mapping satisfying,

$$(x, y) \in X \times X, \qquad x - y \in X_0, d(Tx, Ty) \neq 0$$
$$\Rightarrow \psi\big(d(Tx, Ty)\big) \leq \big[\psi\big(d(x, y)\big)\big]^k \qquad \longrightarrow (11)$$

Where$k \in (0,1)$ is a constant and$\psi \in \Psi$.

Suppose that the operation mapping $\pm(x, y) = x \pm y$, $(x, y) \in X \times X$ is continuous with respect to the metric d. moreover, suppose that $x - Tx \in X_0$, $x \in X$.

Then we have,

(i) For every $x \in X$, the picard sequence $\{T^n x\}$ converges to a fixed point of T.

(ii) For every $x \in X$, $(x + X_0) \cap Fix\,(T) = \{\lim_{n\to\infty} T^n x\}$.

Example 3.2.2

Let X be the set defined by X=$\{\tau_n : n \in \mathbb{N}\}$,

Where,$\tau_n = \frac{n(n+1)}{2}$, for all $n \in \mathbb{N}$.

We endow X with the metric d given by $d(x, y) = |x - y|$for all $x, y \in X$. It is not difficult to show that (X, d) is a complete metric space. Let $T: X \to X$ be the mapping defined by

$T\tau_1 = \tau_1$,$T\tau_n = \tau_{n-1}$, for all $n \geq 2$.

Clearly, the Banach contraction is not satisfied. In fact, we can check easily that

$$\lim_{n\to\infty} \frac{d(T\tau_n, T\tau_1)}{d(\tau_n, \tau_1)} = 1.$$

Now, consider the function $\psi: (0,1) \to (1, \infty)$ defined by

$\psi(t) = \sqrt{te^t}$.

It is not difficult to show that $\Psi \in \psi$

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.we shall prove that T satisfies the condition (1) that is,

$$d(T\tau_n, T\tau_m) \neq 0 \Rightarrow \sqrt{d(T\tau_n, T\tau_m)e^{d(T\tau_n,T\tau_m)}} \leq kd(\tau_n, \tau_m)e^{d(\tau_n,\tau_m)}$$

For some $k \in (0,1)$.the above condition is equivalent to

$d(T\tau_n, T\tau_m) \neq 0 \Rightarrow \sqrt{d(T\tau_n, T\tau_m)e^{d(T\tau_n,T\tau_m)}} \leq d(\tau_n, \tau_m)e^{d(\tau_n,\tau_m)}$.

So, we have to check that

$$d(T\tau_n, T\tau_m) \neq 0 \Rightarrow \frac{d(T\tau_n,T\tau_m)e^{d(T\tau_n,T\tau_m)-d(\tau_n,\tau_m)}}{d(\tau_n,\tau_m)} \leq k^2 , \qquad \rightarrow (11)$$

For some $k \in (0,1)$.we consider two cases.

Case (i) n=1 and m> 2. In this case, we have

$$\frac{d(T\tau_n, T\tau_m)e^{d(T\tau_n,T\tau_m)-d(\tau_n,\tau_m)}}{d(\tau_n, \tau_m)}$$

$$= \frac{\left[\frac{1.(1+1)}{2} - \frac{(m-1)(m-1+1)}{2}\right] e^{\left[\left(\frac{1.(1+1)}{2}-\frac{(m-1)(m-1+1)}{2}\right)-\left[\frac{1.(1+1)}{2}-\frac{m(m+1)}{2}\right]\right]}}{\left[\frac{1.(1+1)}{2} - \frac{m(m+1)}{2}\right]}$$

$$= \frac{\left[\frac{1.2}{2}-\frac{(m-1)m}{2}\right]e^{\left[\frac{1.2}{2}-\frac{(m-1)m}{2}\right]-\left[\frac{1.2}{2}-\frac{m(m+1)}{2}\right]}}{\left[\frac{1.2}{2}-\frac{m(m+1)}{2}\right]}$$

$$= \frac{\left[1-\frac{m^2-m}{2}\right]e^{\left[1-\frac{m^2-m}{2}\right]-\left[1-\frac{m^2+m}{2}\right]}}{\left[1-\frac{m^2+m}{2}\right]}$$

$$= \frac{\left[\frac{2-m^2+m}{2}\right]}{\left[\frac{2-m^2-m}{2}\right]} e^{\left[\frac{2-m^2-m}{2}\right]-\left[\frac{2-m^2+m}{2}\right]}$$

$$= \frac{m^2-m-2}{m^2+m-2} e^{\left[\frac{2-m^2-m-2+m^2-m}{2}\right]}$$

$$= \frac{m^2-m-2}{m^2+m-2} e^{\left[{-2m}/{2}\right]}$$

$$= \frac{m^2-m-2}{m^2+m-2} e^{-m}$$

$$\leq e^{-1}.$$

Case (ii) $m > n > 1$. In this case,

We have,

$$\frac{d(T\tau_m, T\tau_n)e^{d(T\tau_m,T\tau_n)-d(\tau_m,\tau_n)}}{d(\tau_m, \tau_n)}$$

$$= \frac{\left[\frac{(m-1)(m-1+1)}{2} - \frac{(n-1)(n-1+1)}{2}\right] e^{\left[\left(\frac{(m-1)(m-1+1)}{2}\right)-\left(\frac{(n-1)(n-1+1)}{2}\right)\right]}}{\left[\frac{m(m+1)}{2} - \frac{n(n+1)}{2}\right]}$$

$$= \frac{\left[\frac{m(m-1)}{2}-\frac{(n-1)n}{2}\right]e^{\left[\left(\frac{m(m-1)}{2}\right)-\left(-\frac{(n-1)n}{2}\right)\right]}}{\left[\frac{m^2+m}{2}-\frac{n^2+n}{2}\right]}$$

$$= \frac{\left[\frac{m^2-m-n^2+n}{2}\right]}{\left[\frac{m^2-m-n^2-n}{2}\right]} e^{\left[\frac{m^2-m-n^2+n-m^2-m+n^2+n}{2}\right]}$$

$= \frac{m^2-m-n^2+n}{m^2+m-n^2-n} e^{\left[\frac{2(n-m)}{2}\right]}$

$= \frac{m^2-m-n^2+n}{m^2+m-n^2-n} e^{n-m}$

$= \frac{(m^2-n^2)-(m-n)}{(m^2-n^2)-(m-n)} e^{n-m}$

$= \frac{(m-n)(m+n)-(m-n)}{(m-n)(m+n)-(m-n)} e^{n-m}$

$= \frac{(m+n-1)(m-n)}{(m+n+1)(m-n)} e^{n-m}$

$= \frac{(m+n-1)}{(m+n+1)} e^{n-m} \quad \leq e^{-1}$

Thus the inequality (11) is satisfied with$k = e^{1/2}$. Theorem 3.2.1
(Or corollary 3.2.1) implies that T has a unique fixed point.
$\therefore \tau_1$is the unique fixed point of T.

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