ISSN 2515-8260 Volume 08, Issue 02, 2021

PROCESSES WITH INDEPENDENT INCREMENTS THE POISSON PROCESS AND ITS PROPERTIES

MuratovaBaxora

Assistant Karshi branch of the Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, Karshi region, Uzbekistan

Abstract: a Process with independent increments in the theory of random processes contains concepts and theorems that allow us to build probabilistic models in decision-making problems. The Poisson process plays a crucial role in the construction of models. The article analyzes the Poisson process, which is a stochastic process that models many real-world phenomena. The author defines the Poisson process and considers some examples of calculating the corresponding values associated with the Poisson process.

Keywords: Poisson process, random process theory, probability theory, discrete events, modeling, intensity.

INTRODUCTION

The theory of random processes is a branch of the theory of probability associated with the mathematical analysis of random phenomena in which the time factor is present. Formally, a random process can be defined as a set of random variables, indexed by some set. In the theory of stochastic processes, questions of the existence of processes play an important role. The point is that the joint distributions of the random variables Xt cannot be completely arbitrary, but must be consistent in a certain way.

To begin with, let's look at a process with independent increments - a Poisson process.

One of the most important processes observed in nature is the Poisson point process. Therefore, it is important to understand how such processes can be modeled.

Modeling methods differ depending on the type of Poisson point process, i.e. the space in which the process takes place and the homogeneity or heterogeneity of the process.

So, the Poisson process is a model for a number of discrete events, where the average time between events is known, but the exact time of events is random. The Poisson process is one of the most widely used counting processes. It is commonly used in scenarios where events are counted that occur at a certain rate, but completely randomly (without a specific structure).

Let's see what a Poisson sequence might look like (see Figure 1).

ISSN 2515-8260 Volume 08, Issue 02, 2021

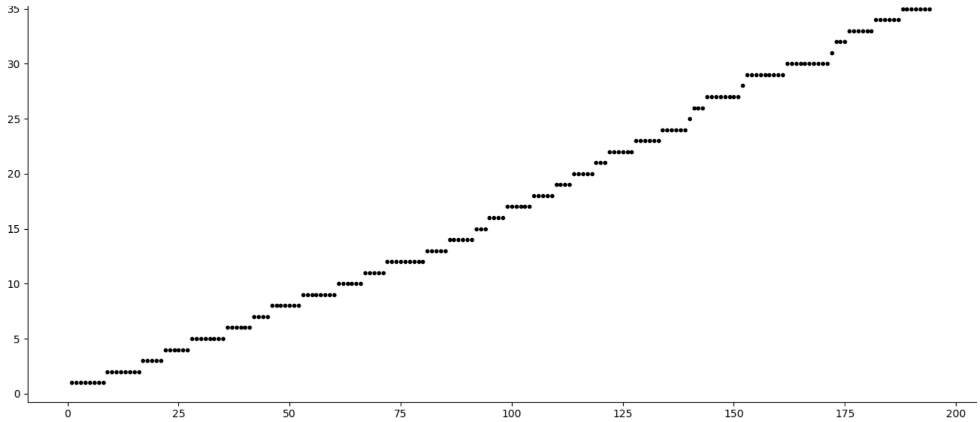

Fig. 1. - An example of a Poisson process

The graph shows the patient's arrival time in hours (starting at some arbitrary hour - 0) at the hospital emergency department. We also know that the average arrival rate is 5 patients per hour. So, such "incoming" data can be very well modeled using the Poisson process.

Poisson processes can be observed in all spheres of life. So, let's say we have a website that our content delivery network (CDN) reports crashes on average once every 60 days, but one failure doesn't affect the likelihood of the next. The important point is that we know the average time between events, but they are located randomly (stochastically). We can have concurrent failures, but we can also overcome failures for years due to the randomness of the process.

The Poisson process meets the following criteria (in fact, many phenomena modeled as Poisson processes do not exactly match them):

1. Events are independent from each other. The occurrence of one event does not affect the likelihood of another event.

2. The average speed (events over a period of time) is constant.

3. Two events cannot happen at the same time.
The last point - the events are not simultaneous - means that we can consider each subinterval of the Poisson process as a Bernoulli process, that is, either success or failure.

Bernoulli'sformulaisconvenientforcalculationsonlywitharelativelysmallnumberoftests. At large values, it is inconvenient to use the этoth formula. Most often in these cases, Poisson's formula is used. By the way, this formula is determined by Poisson's theorem. Note that the theorem, if the probability of an event in each test is constant and small, and the number of independent tests is large enough, then the probability of an event is approximately equal to exactly once:

$$P_n(m) = \frac{\lambda^m}{m!} e^{-\lambda} \qquad (1)$$

где $\lambda = np$.

Thus, the theorem that establishes the property of frequency stability under variable conditions of experiment is called Poisson's theorem and is formulated as follows: if independent experiments are performed and the probability of the occurrence of an event in the ith

ISSN 2515-8260 Volume 08, Issue 02, 2021

experiment is equal, then with an increase the frequency of the event converges in probability to the arithmetic mean of probabilities.

Poisson's theorem is of great fundamental importance for the practical application of probability theory. The fact is that probabilistic methods are often used to study phenomena that, under the same conditions, do not have a chance to repeat themselves many times, but are repeated many times under very different conditions, and the probabilities of the events of interest to us strongly depend on these conditions.

A Poisson process with a rate (or intensity) λ> 0 is a time-continuous stochastic process (N (t), t ≥ 0), taking values in Z + such that:

(i) N(0) = 0;

(ii) the trajectories t 7 → N (t) are straight-line;

(iii) for any 0 ≤ t1 <t2 <· · <tk. the increments N (tn + 1) - N (tn), 1 ≤ n ≤ k - 1, are independent Poisson random variables with means λ (tn + 1 - tn).

Note that properties (i) and (iii) imply that N (t) is Poisson with mean λt.

So, let's give one more example. Customers arrive at the store as a Poisson process at a rate of 20 people per hour. Within half an hour, we calculate the probability that 4 clients will arrive in the first 15 minutes, and 6 in the next 15 minutes.

The two 1/4-hour periods do not overlap, so the receipts in each are independent Poisson random variables with a mean of 20 x 1/4 = 5.

Thus, we get:

$$e^{-5}\frac{5^4}{4!}\cdot e^{-5}\frac{5^6}{6!} = e^{-10}\frac{5^{10}}{4!6!} \approx 0.0256 \qquad (2)$$

Thus, this is an important property of the Poisson process, which gives the probability of a series of events in the interval generated by the Poisson process. The Poisson distribution is defined by the velocity parameter, which is the expected number of events in an interval and the largest number of event probabilities.

REFERENCE

1. Borovkov A. A. probability Theory. 4-e Izd. -M.: Edito-Rial URSS, 2003.
2. Bulinski A.V., Shiryaev A. N. The theory of random processes. -M.: Fizmatlit, 2005.
3. Krylov N. In. Introduction to the theory of random processes. Part 1 and part 2 -M.: Izd-vo MGU, Moscow, 1986.
4. Sevastyanov B. A. the theory of probabilities and mathematical statistics. 2nd ed. - Moscow-Izhevsk: Institute of computer research, 2004.
5. Fellerv. Introduction to probability theory and its applications.In 2 volumes: Mir, 1984.

6.Shiryaev A. N. Probability. 4th edition. -Moscow: mtsnmo, Moscow, 2007.Fayzullayev J.S. Improvement of Economic Efficiency of Development of Railway // Asian Journal of Technology & Management Research (AJTMR) ISSN: 2249 –0892 Vol9 Issue–2, Dec -2019.

http://www.ajtmr.com/papers/Vol9Issue2/Vol9Iss2_P9.pdf
7.Fayzullayev J.S. The economic-mathematical model of the organization activity of the modern transport-logistics system. // Бюллетень науки и практики – Bulletin of Science and Pracrice научный журнал (scientific journal). 2018. №5 (4). С. 345-352. DOI:10.5281/zenodo.1147072. Impact Factor (5) GIF – 0.454; (21) Info Base Index – 1.4; (17) OAJI – 0.350, (43) (UIF) – 0.1502; (4) JCR – 1.021. http://oaji.net/articles/2017/3996-1526585902.pdf
8.FayzullayevJ. (2020). Effectiveness of modern methods in the development of the integration transport systems. *Архив научных исследований*, *1*(3). https://tsue.scienceweb.uz/index.php/archive/article/view/3892
9.Fayzullayev J.S. Effectiveness of modern methods in the development of the integration transport systems // Asian Journal of Technology & Management Research (AJTMR) ISSN: 2249 –0892 Vol9 Issue–2, Dec -2019 62-65. https://saarj.com/wp-content/uploads/SAJMMR-JULY-2020-FULL-JOURNAL.pdf
10.Fayzullayev J.S. Theoretical Aspects of the Transport Logistics System Management // Asian Journal of Technology & Management Research (AJTMR) ISSN: 2249 –0892 Special Issue–2, Sep -2019. http://ajtmr.com/papers/SplIss2/SplIss2_8.pdf
11.FaizullaevJavlonbekSultonovich. (2020). Transportation and logistics system: problems and solutions. *Archive of Conferences*, *10*(1), 50-54. Retrieved from https://www.conferencepublication.com/index.php/aoc/article/view/424
12.Fayzullayev J.S. Problems And Status Of Efficiency Of The Transport And Logistics System In The Railway Network , Archive of Conferences: Vol. 13 No. 1 (2021): GTIMSC-2021, https://www.conferencepublication.com/index.php/aoc/article/view/573