Modified Zagreb Indices of Product of Graphs

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Abstract

In this paper, we obtain the oremson modified Zagrebindex of Cartesian product, Strong product and Tensor product of graphs. Graph slike path and cycle are considered in this work.

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1 Introduction

In this article, we are concerned with simple graphs, that is finite and undirected graphs without loops or multiple edges. Let G be such a graph and V(G) and E(G) be its vertex set and edge set respectively. An edge of G, connecting the vertices u and v will be denoted by uv. The degree d(v) of a vertex vєV(G) is the number of vertex of G adjacent to v. The most elementary constituents of a (molecular) graph are vertices, edges, vertex-degrees, walks and paths[7]. They are the basis of many graph-theoretical invariants referred to as topological index, which have found considerable use in Zagreb index. The Modified first Index is denoted by \( m_1(G) \), and defined as \( m_1(G) = \sum_{v \in V(G)} \frac{1}{d(v)^2} \). These have been conceived in the 1970s and found considerable applications in chemistry[2,5,6]. The Zagreb indices were subject to a large number of mathematical studies, of which we mention only a few nearest [3,4].

The Cartesian Product G□H is a graph such that, the vertex set of G□H is the Cartesian product V(G)×V(H) and two vertices (u,u') and (v,v') are adjacent in G□H if and only if either u=v and u' is adjacent to v' and u is adjacent to v in G. The Strong Product G☒H is the Cartesian product V(G)×V(H) and distinct vertices (u,u') and (v,v') are adjacent in G☒H if and only if either u=v and u' is adjacent to v' or u'=v' and u is adjacent to v or u is adjacent to v and u' is adjacent to v'. The Tensor Product G×H of graphs G and H is a graph such that, the vertex set of G×H is the Cartesian product V(G)×V(H) and distinct vertices (u,u') and (v,v') are adjacent in G×H if and only if u is adjacent to v and u' is adjacent to v'.

All the definitions and notations in graphs and digraphs, which are not mentioned in this paper, one may refer[1].

2 MAIN RESULTS

In this section, we obtain results on modified first Zagreb indices of Cartesian, strong and Tensor product of paths and cycles.

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**Theorem 1:**

*The Modified first Zagreb index $G$ of a Cartesian product of two path $p_n$ and $p_m$ is*

$$mM_1(G) = \frac{9nm + 14n + 14m + 52}{144}$$

**Proof:**

The Cartesian Product of two path $p_n$ and $p_m$ has 4 vertices of degree 2, 2n-4 vertices degree 3, 2m-4 vertices degree and 3(nm-2n-2m+4) vertices of degree 4, then the Modified first Zagreb index $G$ is

$$mM_1(G) = \sum_{v \in V(G)} \frac{1}{d(v)^2}$$

$$= \frac{4}{2^2} + \frac{2n-4}{3^2} + \frac{2m-4}{3^2} + \frac{nm-2n-2m+4}{4^2}$$

$$= \frac{4}{2^2} + \frac{2n-4}{9} + \frac{2m-4}{9} + \frac{nm-2n-2m+4}{16}$$

$$= \frac{144 + (2n-4)(16) + (2m-4)(16) + (nm-2n-2m+4)(9)}{144}$$

$$= \frac{144 + 32n-64 + 32m-64 + 9nm + 8n + 18m + 36}{144}$$

$$= \frac{9nm + 14n + 14m + 52}{144}$$

**Figure 1**

![Figure 1](image1.png)

**Theorem 2:**

*The Modified first Zagreb index $G$ of a strong product of two path $p_n$ and $p_m$ is*

$$mM_1(G) = \frac{225nm + 702n + 702m + 2692}{14400}$$

**Proof:**

The strong product of two path $p_n$ and $p_m$ has 4 vertices of degree 3, 2n-4 vertices of degree 5, 2m-4 vertices of degree 5 and (nm-2n-2m+4) vertices of 8, then the Modified first Zagreb index
The Modified first Zagreb index $G$ of a Tensor product of two path $p_n$ and $p_m$ is

$$mM_1(G) = \frac{nm+6n+6m+36}{16}$$

Proof:

The Tensor product of two path $p_n$ and $p_m$ has 4 vertices of degree 1, 2n-4 vertices of degree 2, 2m-4 vertices of degree 4 and (nm-2n-2m+4) vertices of degree 4, then the Modified first Zagreb index $G$ is

$$mM_1(G) = \sum_{v \in V(G)} \frac{1}{d(v)^2}.$$
Theorem 4:
The Modified first Zagreb index $G$ of a Cartesian product of two cycle $c_n$ and $c_m$ is

$$mM_1(G) = \frac{nm}{16}$$

Proof:
The Cartesian product of two cycle $c_n$ and $c_m$ has $nm$ vertices of degree 4, then the Modified first Zagreb index $G$ is

$$mM_1(G) = \sum_{v \in V(G)} \frac{1}{d(v)^2}$$

$$= \frac{nm}{16}$$

Theorem 5:
The Modified first Zagreb index $G$ of a strong product of two cycle $c_n$ and $c_m$ is

$$mM_1(G) = \frac{225\cdot nm + 350n + 350m + 900}{14400}$$

Proof:
The strong product of two cycle $c_n$ and $c_m$ has 4 vertices of degree 5, $2n-4$ vertices of degree 6, $2m-4$ vertices of degree 6 and $(nm - 2n - 2m + 4)$ vertices of degree 8, then the Modified first Zagreb index $G$ is

$$mM_1(G) = \sum_{v \in V(G)} \frac{1}{d(v)^2}$$

$$= \frac{4 \cdot 2n - 4}{6^2} + \frac{2m - 4}{6^2} + \frac{nm - 2n - 2m + 4}{8^2}$$

$$= \frac{2304 + (2n - 4)\cdot 400 + (2m - 4)\cdot 400 + (nm - 2n - 2m + 4) \cdot 225}{14400}$$

$$= \frac{2304 + 800n - 1600 + 800 - 1600 + 225nm - 450n - 450m + 1800}{14400}$$

$$= \frac{225nm + 350n + 350m + 900}{14400}$$
Theorem 6:

The modified first Zagreb index $G$ of a tensor product of two cycle $c_n$ and $c_m$ is

$$mM_1(G) = \frac{nm+6n+6m+36}{16}$$

Proof:

The Tensor product of two cycle $c_n$ and $c_m$ has 4 vertices of degree 1, 2n-4 vertices of degree 2, 2m-4 vertices of degree 2 and (nm-2n-2m+4) vertices of degree 4, then the modified first Zagreb index $G$ is

$$mM_1(G) = \sum_{v \in V(G)} \frac{1}{d(v)^2}$$

$$= \frac{4}{1^2} + \frac{2n-4}{2^2} + \frac{2m-4}{2^2} + \frac{nm-2n-2m+4}{4^2}$$

$$= \frac{64+(2n-4)+4+(2m-4)+4+(nm-2n-2m+4)}{16}$$

$$= \frac{64+8n-16+8m-16+nm-2n-2m+4}{16}$$

$$= \frac{nm+6n+6m+36}{16}$$

Theorem 7:

The Modified first Zagreb index $G$ of a Cartesian product of path $p_n$ and cycle $c_m$ is

$$mM_1(G) = \frac{9nm+14m}{144}$$

Proof:

The Cartesian product of path $p_n$ and cycle $c_m$ has 2m vertices of degree 3 and (nm-2m) vertices of degree 4, then the Modified first Zagreb index $G$ is

$$mM_1(G) = \sum_{v \in V(G)} \frac{1}{d(v)^2}$$

$$= \frac{2m}{3^2} + \frac{nm-2m}{4^2}$$

$$= \frac{2m}{9} + \frac{nm-2m}{16}$$

$$= \frac{32m+(nm-2m)9}{144}$$

$$= \frac{32m+9m-18m}{144}$$

$$= \frac{9nm-14m}{144}$$

Theorem 8:

The Modified first Zagreb index $G$ of a strong product of path $p_n$ and cycle $c_m$ is

$$mM_1(G) = \frac{225nm+350n+720m+596}{14400}$$

Proof:

The Strong Product of path $p_n$ and cycle $c_m$ has 4 vertices of degree 4,(2n-4) vertices of degree 6,
(2m-4) vertices of degree 5 and (nm-2n-2m+4) vertices of degree 8, then the Modified first Zagreb index \( G \) is

\[
M_1^m(G) = \sum_{v \in V(G)} \frac{1}{d(v)^2}
\]

\[
= \frac{4}{2^2} + \frac{2n-4}{4^2} + \frac{2m-4}{4^2} + \frac{(nm-2n-2m+4)}{8^2}
\]

\[
= \frac{4}{16} + \frac{2n-4}{36} + \frac{2m-4}{25} + \frac{(nm-2n-2m+4)}{64}
\]

\[
= \frac{4(900)+(2n-4)400+(2m-4)576+(nm-2n-2m+4)225}{14400}
\]

\[
= \frac{3600+800n-1600+1152m-2304+225nm-450n+450m+900}{14400}
\]

\[
= \frac{225nm+350n+702m+596}{14400}
\]

**Theorem 9:**

The Modified first Zagreb index \( G \) of a Tensor product of path \( p_n \) and cycle \( c_m \) is

\[
M_1^m(G) = \frac{nm+6n+6m+36}{16}
\]

**Proof:**

The Tensor product of path \( p_n \) and cycle \( c_m \) has 4 vertices of degree 1, 2n-4 vertices of degree 2, 2m-4 vertices of degree 2 and (nm-2n-2m+4) vertices of degree 4, then the Modified first Zagreb index \( G \) is

\[
M_1^m(G) = \sum_{v \in V(G)} \frac{1}{d(v)^2}
\]

\[
= \frac{4}{1^2} + \frac{2n-4}{2^2} + \frac{2m-4}{2^2} + \frac{(nm-2n-2m+4)}{4^2}
\]

\[
= \frac{4}{4} + \frac{2n-4}{4} + \frac{2m-4}{4} + \frac{(nm-2n-2m+4)}{16}
\]

\[
= \frac{64+(2n-4)4+(2m-4)4+(nm-2n-2m+4)}{16}
\]

\[
= \frac{64+8n-16+8m-16+nm-2n-2m+4}{16}
\]

\[
= \frac{nm+6n+6m+36}{16}
\]

3 **CONCLUSION**

In this paper, modified first Zagreb index of product of graphs are obtained. This index can be used as a numerical description in comparison with chemical, physical and biological parameters to study about its relationships.

**REFERENCES**
