

# A Deteriorating Inventory Model of EOQ with Beta Distributed in quadratic time function Demand and fully Backlogged with Shortages

S.Meenakshi Sundaram<sup>1</sup>, Dr.T.Harikrishnan<sup>2</sup>,Mr.V.Sivan<sup>3</sup>

<sup>1,3</sup>Department of Mathematics

<sup>1,3</sup>Assistant Professor (SS)

<sup>1,3</sup>Rajalakshmi Institute of Technology

Chennai, Tamilnadu, India.

<sup>2</sup>Department of Mathematics

<sup>2</sup>Assistant Professor

<sup>2</sup>R.M.K College of Engineering and Technology

Puduvoyal, Tamilnadu, India.

<sup>1</sup>sms\_msc@yahoo.com

<sup>2</sup>mokshihari2009@gmail.com

<sup>3</sup>shivan.ve@gmail.com

## **Abstract:**

*A deteriorating inventory model of EOQ has probabilistic and shortages. Shortages can be accepted, but it is balanced by fully backlogged. Demand considered as quadratic time function and holding cost is parabolic. Demand items considered as a quadratic with highest power coefficient has negative sign, which pursues the Beta passion. From these inferences, a mathematical model has been framed and worked out using numerical solution with the help of computation table and represented using graph. Here the main aim determines the time period should be optimal, highest order quantity of very least overall average cost.*

## **Key Words:**

*Deterioration, Beta distribution, fully backlog, quadratic demand, parabolic holding cost.*

## **1. Introduction**

In the modern world demand of some businesses cannot be predicted over a period or using previous sales, it always unpredictable in that time, the seller cannot expect his sales and profit. He can use the trail and error method only to hold the stock. Here his decisions may play vital role. Based on these conditions there will be systematic rules to follow demand of that business. This demand may follow the distributions like quadratic functions of changeable time, Deterioration has the important thing to consider here, it makes the very minimum. So we assume as probabilistic deterioration like Beta distribution.

Palanivel, M., Uthayakumar, R. presented a paper of deteriorating items with variable production cost, time dependent holding cost and partial backlogging under inflation [1]. Sarkar, M., Sarkar, B. have proposed a model with probabilistic deterioration in a production system [2]. Pavan Kumar and D. Dutta A have investigated a topic in deteriorating Inventory Model with Uniformly Distributed Random Demand and Completely Backlogged Shortages [3]. Pavan Kumar, P.S. Keerthika. have proposed a paper of Time-Linked Holding Cost, Salvage Value and Probabilistic Deterioration following Various Distributions, In this study we addressed the optimal values of inventory management with time-related keeping costs and salvage value. The decay of things after two distributions is a continuous random number uniform distribution and triangular distribution. Shortage is allowed, and is partially backlogged. The demand-rate

depends on time. An expression for mean total cost is derived as a quantity economic order. [4]. Dutta, D., Kumar, P. have formulated a partial backlogging inventory model for deteriorating items with time-varying demand and holding cost [5]. Chetansinh R. Vaghela and Nita H. Shah gave the result of Dynamic Pricing, Advertisement Investment and Replenishment Model for Deteriorating Items [6]. Pavan Kumar has established a formation of time-varying linear demand, parabolic holding cost with salvage value for an inventory planning problem [7].

Here this article, we have to develop the EOQ deterioration stock model with consistently circulated irregular demand and overall combined deficiencies. This article has formed in the structure of : In Sect. 2, the symbols and inferences proposed model are given. In Sect. 3, the mathematical model is defined. In Sect. 4, To list numerical examples. In Sect. 5, sensitivity analysis tables and graphs. In Sect. 6, The effect in the parameters with overall average cost, In the last, we conclude in Sect. 7.

## 2. Symbols and inferences

### 2.1. Symbols

$C_{\beta o}$	Ordering cost /order
$C_{\beta s}$	Deficiency cost / unit time
$C_{\beta p}$	Purchase cost / unit time
$D_{\beta}(t)$	Scale demand rate of the customer; $a_1 + a_2t - a_3t^2$
$I_{\beta H}$	Parabolic Holding cost / unit time; $u + vt^2, u > 0, v > 0$
$I_{\beta D}$	Decayed units
$I_{\beta S}$	Deficiency units
$\theta_{\beta}$	Deterioration, $0 < \theta_{\beta} < 1$
$T_{\beta}$	The cycle length.
$t_{\beta}$	Begining of the shortage time.
$E(t)$	Expectation of random variable $t$ .
$I_{\beta 1}(t)$	Inventory level $0 \leq t \leq t_{\beta}$
$I_{\beta 2}(t)$	Inventory level $t_{\beta} \leq t \leq T_{\beta}$
$Q_{\beta}$	Order quantity
$\frac{\alpha}{\alpha + \beta}$	Expected value of Beta distribution, $\alpha > 0, \beta > 0$
$AT_{\beta}C(t_{\beta}, T_{\beta})$	Over all average expences of the cost / unit time

### 2.2. Inferences

- (i) Single item to be considered for this inventory.
- (ii)  $D_{\beta}(t)$  is quadratic demand with highest power of  $t$  coefficient is negative.
- (iii) Using full backlogging the shortages managed.
- (iv) Using lead time is zero and infinite replenishment rate.
- (v)  $\theta_{\beta}$  is taken as expected value of beta distribution.

## 3. Mathematical Model

The equation using mathematical model can be formed as follows

The amount  $Q_{\beta}$  bought in purchased at the starting of each period with completing of backorders. The inventory level continuously diminish due to joint operations of demand, decaying in the period

$[0, t_\beta]$ , implies to be 0 at  $t = t_\beta$ , the shortages may happen in the period  $[t_\beta, T_\beta]$  it will be fully backlogged, using on hand stock  $I_\beta(t)$  in time t. The formation of diff. eqn. can be composed of the form.

$$\frac{dI_{\beta 1}(t)}{dt} + \theta_\beta I_{\beta 1}(t) = -(a_1 + a_2 t - a_3 t^2) \quad \text{for } 0 \leq t \leq t_\beta \quad (1)$$

$$\frac{dI_{\beta 2}(t)}{dt} = -(a_1 + a_2 t - a_3 t^2) \quad \text{for } t_\beta \leq t \leq T_\beta \quad (2)$$

With boundary conditions  $I_{\beta 1}(t_\beta) = 0$ ,  $I_{\beta 1}(0) = Q_\beta$  &  $I_{\beta 2}(t_\beta) = 0$  (3)

The solution of equation (1) with the B.C  $I_{\beta 1}(t_\beta) = 0$  we get

$$I_{\beta 1}(t) = - \left[ \frac{a_1 + a_2 t - a_3 t^2}{\theta_\beta} - \frac{a_2 - 2a_3 t}{\theta_\beta^2} - \frac{2a_3}{\theta_\beta^3} \right] + \left[ \frac{a_1 + a_2 t_\beta - a_3 t_\beta^2}{\theta_\beta} - \frac{a_2 - 2a_3 t_\beta}{\theta_\beta^2} - \frac{2a_3}{\theta_\beta^3} \right] e^{\theta_\beta(t_\beta - t)} \quad (4) \quad \text{The}$$

solution of equation (3) with the B.C  $I_{\beta 2}(t_\beta) = 0$  we get

$$I_{\beta 2}(t) = - \left[ a_1 t + \frac{a_2 t^2}{2} - \frac{a_3 t^3}{3} \right] + \left[ a_1 t_\beta + \frac{a_2 t_\beta^2}{2} - \frac{a_3 t_\beta^3}{3} \right] \quad (5)$$

$$Q_\beta = I_{\beta 1}(0) \Rightarrow Q_\beta = - \left[ \frac{a_1}{\theta_\beta} - \frac{a_2}{\theta_\beta^2} - \frac{2a_3}{\theta_\beta^3} \right] + \left[ \frac{a_1 + a_2 t_\beta - a_3 t_\beta^2}{\theta_\beta} - \frac{a_2 - 2a_3 t_\beta}{\theta_\beta^2} - \frac{2a_3}{\theta_\beta^3} \right] e^{\theta_\beta t_\beta} \quad (6)$$

$$\text{Total demand } (B) = \int_0^{t_\beta} D_\beta(t) dt$$

$$B = a_1 t_\beta + \frac{a_2 t_\beta^2}{2} - \frac{a_3 t_\beta^3}{3} \quad (7)$$

$$\text{Decayed units } (I_{\beta D}) = Q_\beta - B$$

$$I_{\beta D} = - \left[ \frac{a_1}{\theta_\beta} - \frac{a_2}{\theta_\beta^2} - \frac{2a_3}{\theta_\beta^3} \right] - \left[ a_1 t_\beta + \frac{a_2 t_\beta^2}{2} - \frac{a_3 t_\beta^3}{3} \right] + \left[ \frac{a_1 + a_2 t_\beta - a_3 t_\beta^2}{\theta_\beta} - \frac{a_2 - 2a_3 t_\beta}{\theta_\beta^2} - \frac{2a_3}{\theta_\beta^3} \right] e^{\theta_\beta t_\beta} \quad (8) \quad \text{Holding}$$

$$\text{cost } (I_{\beta H}) = \int_0^{t_\beta} (u + vt^2) I_{\beta 1}(t) dt$$

$$= \frac{1}{\theta_\beta^6} \left\{ \begin{aligned} & \left( (a_1 \theta_\beta^2 - a_2 \theta_\beta - 2a_3) (-2v - u \theta_\beta^2) + (-a_1 u \theta_\beta^5 - 2a_1 v \theta_\beta^3) t_\beta + \left( -\frac{a_2 u \theta_\beta^5}{2} - a_1 v \theta_\beta^4 - a_2 v \theta_\beta^3 \right) t_\beta^2 \right) \\ & + \left( \frac{a_3 u \theta_\beta^5 - a_1 v \theta_\beta^5 - 2a_2 v \theta_\beta^4 + 2a_3 v \theta_\beta^3}{3} \right) t_\beta^3 - \left( \frac{a_2 v \theta_\beta^5 - 2a_3 v \theta_\beta^4}{4} \right) t_\beta^4 + \left( \frac{a_3 v \theta_\beta^5}{5} \right) t_\beta^5 \\ & + (u \theta_\beta^2 + 2v) \left[ (a_1 \theta_\beta^2 - a_2 \theta_\beta - 2a_3) + (a_2 \theta_\beta^2 + 2a_3 \theta_\beta) t_\beta - a_3 \theta_\beta^2 t_\beta^2 \right] e^{\theta_\beta t_\beta} \end{aligned} \right\} \quad (9) \quad \text{All out}$$

usual deficiency number units ( $I_{\beta S}$ ) in the period  $[0, T_\beta]$ ,

$$I_{\beta S} = \int_{t_\beta}^{T_\beta} I_{\beta 2}(t) dt$$

$$I_{\beta S} = \left( \frac{6a_1T_{\beta}^2 + 2a_2T_{\beta}^3 - a_3T_{\beta}^4}{12} \right) - a_1T_{\beta}t_{\beta} + \left( \frac{a_1 - a_2T_{\beta}}{2} \right) t_{\beta}^2 + \left( \frac{a_2 + a_3T_{\beta}}{3} \right) t_{\beta}^3 - \frac{a_3}{4} t_{\beta}^4 \quad (10)$$

$$AT_{\beta}C(t_{\beta}, T_{\beta}) = \frac{1}{T_{\beta}} (\text{Ordering cost} + \text{Holding cost} + \text{Deteriorating cost} + \text{Deficiency cost})$$

$$(i.e) AT_{\beta}C = \frac{1}{T_{\beta}} (c_{\beta o} + I_{\beta H} + c_{\beta p} I_{\beta D} + c_{\beta s} I_{\beta S})$$

$$AT_{\beta}C = \left\{ \begin{aligned} & \frac{1}{T_{\beta}} \left[ c_{\beta o} + \frac{(a_1\theta_{\beta}^2 - a_2\theta_{\beta} - 2a_3)(-2v - u\theta_{\beta}^2)}{\theta_{\beta}^6} - c_{\beta p} \left( \frac{a_1\theta_{\beta}^2 - a_2\theta_{\beta} - 2a_3}{\theta_{\beta}^3} \right) \right] + \frac{a_1c_{\beta s}}{2} T_{\beta} \\ & + \frac{a_2c_{\beta s}}{6} T_{\beta}^2 - \frac{a_3c_{\beta s}}{12} T_{\beta}^3 + \left( -a_1c_{\beta p} - \frac{a_1u}{\theta_{\beta}} - \frac{2a_1v}{\theta_{\beta}^3} \right) \left( \frac{t_{\beta}}{T_{\beta}} \right) + \left( -\frac{a_2c_{\beta p}}{2} - \frac{a_2u}{2\theta_{\beta}} - \frac{a_1v}{\theta_{\beta}^2} - \frac{a_2v}{\theta_{\beta}^3} + \frac{a_2c_{\beta s}}{2} \right) \left( \frac{t_{\beta}^2}{T_{\beta}} \right) \\ & + \left( \frac{c_{\beta p}a_3\theta_{\beta}^3 + a_3u\theta_{\beta}^2 - a_1v\theta_{\beta}^2 - 2a_2v\theta_{\beta} + 2a_3v + a_2c_{\beta s}\theta_{\beta}^3}{3\theta_{\beta}^3} \right) \left( \frac{t_{\beta}^3}{T_{\beta}} \right) + \left( \frac{-a_2v\theta_{\beta} + 2a_3v - a_3c_{\beta s}\theta_{\beta}^2}{4\theta_{\beta}^2} \right) \left( \frac{t_{\beta}^4}{T_{\beta}} \right) \\ & + \left( \frac{a_3v}{5\theta_{\beta}} \right) \left( \frac{t_{\beta}^5}{T_{\beta}} \right) + \left[ \frac{a_1c_{\beta p}}{\theta_{\beta}} - \frac{a_2c_{\beta p}}{\theta_{\beta}^2} + \left( \frac{u\theta_{\beta}^2 + 2v}{\theta_{\beta}^6} \right) (a_1\theta_{\beta}^2 - a_2\theta_{\beta} - 2a_3) \right] \left( \frac{e^{\theta_{\beta}t_{\beta}}}{T_{\beta}} \right) - c_{\beta s}a_1t_{\beta} - \frac{c_{\beta s}a_2}{2} t_{\beta}^2 \\ & + \frac{c_{\beta s}a_3}{3} t_{\beta}^3 + \left[ \frac{a_2c_{\beta p}}{\theta_{\beta}} + \frac{2a_3c_{\beta p}}{\theta_{\beta}^2} + \left( \frac{u\theta_{\beta}^2 + 2v}{\theta_{\beta}^5} \right) (a_2\theta_{\beta} + 2a_3) \right] \left( \frac{t_{\beta}e^{\theta_{\beta}t_{\beta}}}{T_{\beta}} \right) - \left[ \frac{a_3c_{\beta p}}{\theta_{\beta}} + a_3 \left( \frac{u\theta_{\beta}^2 + 2v}{\theta_{\beta}^4} \right) \right] \left( \frac{t_{\beta}^2e^{\theta_{\beta}t_{\beta}}}{T_{\beta}} \right) \end{aligned} \right\} \quad (11)$$

$$\text{Where } \theta_{\beta} = \frac{\alpha}{\alpha + \beta}$$

$$\frac{\partial(AT_{\beta}C)}{\partial t_{\beta}} = \left\{ \begin{aligned} & \left( -a_1c_{\beta p} - \frac{a_1u}{\theta_{\beta}} - \frac{2a_1v}{\theta_{\beta}^3} \right) \left( \frac{1}{T_{\beta}} \right) + \left( -a_2c_{\beta p} - \frac{a_2u}{\theta_{\beta}} - \frac{2a_1v}{\theta_{\beta}^2} - \frac{2a_2v}{\theta_{\beta}^3} + a_2c_{\beta s} \right) \left( \frac{t_{\beta}}{T_{\beta}} \right) \\ & + \left( \frac{c_{\beta p}a_3\theta_{\beta}^3 + a_3u\theta_{\beta}^2 - a_1v\theta_{\beta}^2 - 2a_2v\theta_{\beta} + 2a_3v + a_2c_{\beta s}\theta_{\beta}^3}{\theta_{\beta}^3} \right) \left( \frac{t_{\beta}^2}{T_{\beta}} \right) \\ & + \left( \frac{-a_2v\theta_{\beta} + 2a_3v - a_3c_{\beta s}\theta_{\beta}^2}{\theta_{\beta}^2} \right) \left( \frac{t_{\beta}^3}{T_{\beta}} \right) + \left( \frac{a_3v}{\theta_{\beta}} \right) \left( \frac{t_{\beta}^4}{T_{\beta}} \right) \\ & + \left[ \frac{a_1c_{\beta p}}{\theta_{\beta}} - \frac{a_2c_{\beta p}}{\theta_{\beta}^2} + \left( \frac{u\theta_{\beta}^2 + 2v}{\theta_{\beta}^6} \right) (a_1\theta_{\beta}^2 - a_2\theta_{\beta} - 2a_3) \right] \left( \frac{\theta_{\beta}e^{\theta_{\beta}t_{\beta}}}{T_{\beta}} \right) - c_{\beta s}a_1 - c_{\beta s}a_2t_{\beta} \\ & + c_{\beta s}a_3t_{\beta}^2 + \left[ \frac{a_2c_{\beta p}}{\theta_{\beta}} + \frac{2a_3c_{\beta p}}{\theta_{\beta}^2} + \left( \frac{u\theta_{\beta}^2 + 2v}{\theta_{\beta}^5} \right) (a_2\theta_{\beta} + 2a_3) \right] \left( \frac{e^{\theta_{\beta}t_{\beta}}}{T_{\beta}} \right) \\ & + \left[ \frac{a_2c_{\beta p}}{\theta_{\beta}} + \frac{2a_3c_{\beta p}}{\theta_{\beta}^2} + \left( \frac{u\theta_{\beta}^2 + 2v}{\theta_{\beta}^5} \right) (a_2\theta_{\beta} + 2a_3) \right] \left( \frac{\theta_{\beta}t_{\beta}e^{\theta_{\beta}t_{\beta}}}{T_{\beta}} \right) \\ & - \left[ \frac{a_3c_{\beta p}}{\theta_{\beta}} + a_3 \left( \frac{u\theta_{\beta}^2 + 2v}{\theta_{\beta}^4} \right) \right] \left( \frac{2t_{\beta}e^{\theta_{\beta}t_{\beta}}}{T_{\beta}} \right) - \left[ \frac{a_3c_{\beta p}}{\theta_{\beta}} + a_3 \left( \frac{u\theta_{\beta}^2 + 2v}{\theta_{\beta}^4} \right) \right] \left( \frac{\theta_{\beta}t_{\beta}^2e^{\theta_{\beta}t_{\beta}}}{T_{\beta}} \right) \end{aligned} \right\} \quad (12)$$

$$\frac{\partial(AT_\beta C)}{\partial T_\beta} = \left\{ \begin{aligned} & \frac{-1}{T_\beta^2} \left[ c_{\beta o} + \frac{(a_1\theta_\beta^2 - a_2\theta_\beta - 2a_3)(-2v - u\theta_\beta^2)}{\theta_\beta^6} - c_{\beta p} \left( \frac{a_1\theta_\beta^2 - a_2\theta_\beta - 2a_3}{\theta_\beta^3} \right) \right] + \frac{a_1c_{\beta s}}{2} \\ & + \frac{a_2c_{\beta s}}{3}T_\beta - \frac{a_3c_{\beta s}}{4}T_\beta^2 - \left( -a_1c_{\beta p} - \frac{a_1u}{\theta_\beta} - \frac{2a_1v}{\theta_\beta^3} \right) \left( \frac{t_\beta}{T_\beta^2} \right) - \left( -\frac{a_2c_{\beta p}}{2} - \frac{a_2u}{2\theta_\beta} - \frac{a_1v}{\theta_\beta^2} - \frac{a_2v}{\theta_\beta^3} + \frac{a_2c_{\beta s}}{2} \right) \left( \frac{t_\beta^2}{T_\beta^2} \right) \\ & - \left( \frac{c_{\beta p}a_3\theta_\beta^3 + a_3u\theta_\beta^2 - a_1v\theta_\beta^2 - 2a_2v\theta_\beta + 2a_3v + a_2c_{\beta s}\theta_\beta^3}{3\theta_\beta^3} \right) \left( \frac{t_\beta^3}{T_\beta^2} \right) - \left( \frac{-a_2v\theta_\beta + 2a_3v - a_3c_{\beta s}\theta_\beta^2}{4\theta_\beta^2} \right) \left( \frac{t_\beta^4}{T_\beta^2} \right) \\ & - \left( \frac{a_3v}{5\theta_\beta} \right) \left( \frac{t_\beta^5}{T_\beta^2} \right) - \left[ \frac{a_1c_{\beta p}}{\theta_\beta} - \frac{a_2c_{\beta p}}{\theta_\beta^2} + \left( \frac{u\theta_\beta^2 + 2v}{\theta_\beta^6} \right) (a_1\theta_\beta^2 - a_2\theta_\beta - 2a_3) \right] \left( \frac{e^{\theta_\beta t_\beta}}{T_\beta^2} \right) \\ & - \left[ \frac{a_2c_{\beta p}}{\theta_\beta} + \frac{2a_3c_{\beta p}}{\theta_\beta^2} + \left( \frac{u\theta_\beta^2 + 2v}{\theta_\beta^5} \right) (a_2\theta_\beta + 2a_3) \right] \left( \frac{t_\beta e^{\theta_\beta t_\beta}}{T_\beta^2} \right) + \left[ \frac{a_3c_{\beta p}}{\theta_\beta} + a_3 \left( \frac{u\theta_\beta^2 + 2v}{\theta_\beta^4} \right) \right] \left( \frac{t_\beta^2 e^{\theta_\beta t_\beta}}{T_\beta^2} \right) \end{aligned} \right\} \quad (13) \text{ The}$$

necessary condition for least value of  $AT_\beta C(t_\beta, T_\beta)$  are  $\frac{\partial(AT_\beta C(t_\beta, T_\beta))}{\partial t_\beta} = 0$  &

$\frac{\partial(AT_\beta C(t_\beta, T_\beta))}{\partial T_\beta} = 0$ . The sufficient condition for least of  $AT_\beta C(t_\beta, T_\beta)$ ,  $t_\beta > 0$ ,  $T_\beta > 0$ .

$$\begin{vmatrix} \frac{\partial^2(AT_\beta C)}{\partial t_\beta^2} & \frac{\partial^2(AT_\beta C)}{\partial t_\beta \partial T_\beta} \\ \frac{\partial^2(AT_\beta C)}{\partial T_\beta \partial t_\beta} & \frac{\partial^2(AT_\beta C)}{\partial T_\beta^2} \end{vmatrix} > 0$$

The equations 12 and 13 are nonlinear, so to solve using the any technique of computer based software and obtain optimal order cycle time  $t_\beta, T_\beta$  is  $(t_\beta^*, T_\beta^*)$ , the calculation of second order differentiation of  $AT_\beta C(t_\beta, T_\beta)$ , is not simple. The entire mathematical model process verified by using computer software and with assist of a graph which measured the optimal progress, represented by table.

#### 4. Simple numerical problems

**Example 1:** Using the data:  $a_1 = 230$  units,  $a_2 = 25$  units,  $a_3 = 10$  units,  $\alpha = 0.06$  units,

$\beta = 0.16$  units,  $u = 0.25$  units,  $v = 0.625$  units,  $c_{\beta o} = Rs.150$ ,  $c_{\beta p} = Rs.25$ ,  $c_{\beta s} = Rs.15$ . We have the optimum solution is  $t_\beta^* = 1.2557$ ,  $T_\beta^* = 1.9914$ ,  $AT_\beta C = 1356.3236$  and quantity  $Q_\beta = 360.7272$ .

**Example 2:** Using the data:  $a_1 = 150$  units,  $a_2 = 50$  units,  $a_3 = 5$  units,  $\alpha = 0.08$  units,

$\beta = 0.2$  units,  $u = 5$  units,  $v = 3$  units,  $c_{\beta o} = Rs.250$ ,  $c_{\beta p} = Rs.50$ ,  $c_{\beta s} = Rs.30$ . We have the optimum solution is  $t_\beta^* = 1.5231$ ,  $T_\beta^* = 2.1886$ ,  $AT_\beta C = 4154.3162$  and quantity  $Q_\beta = 355.9618$ .

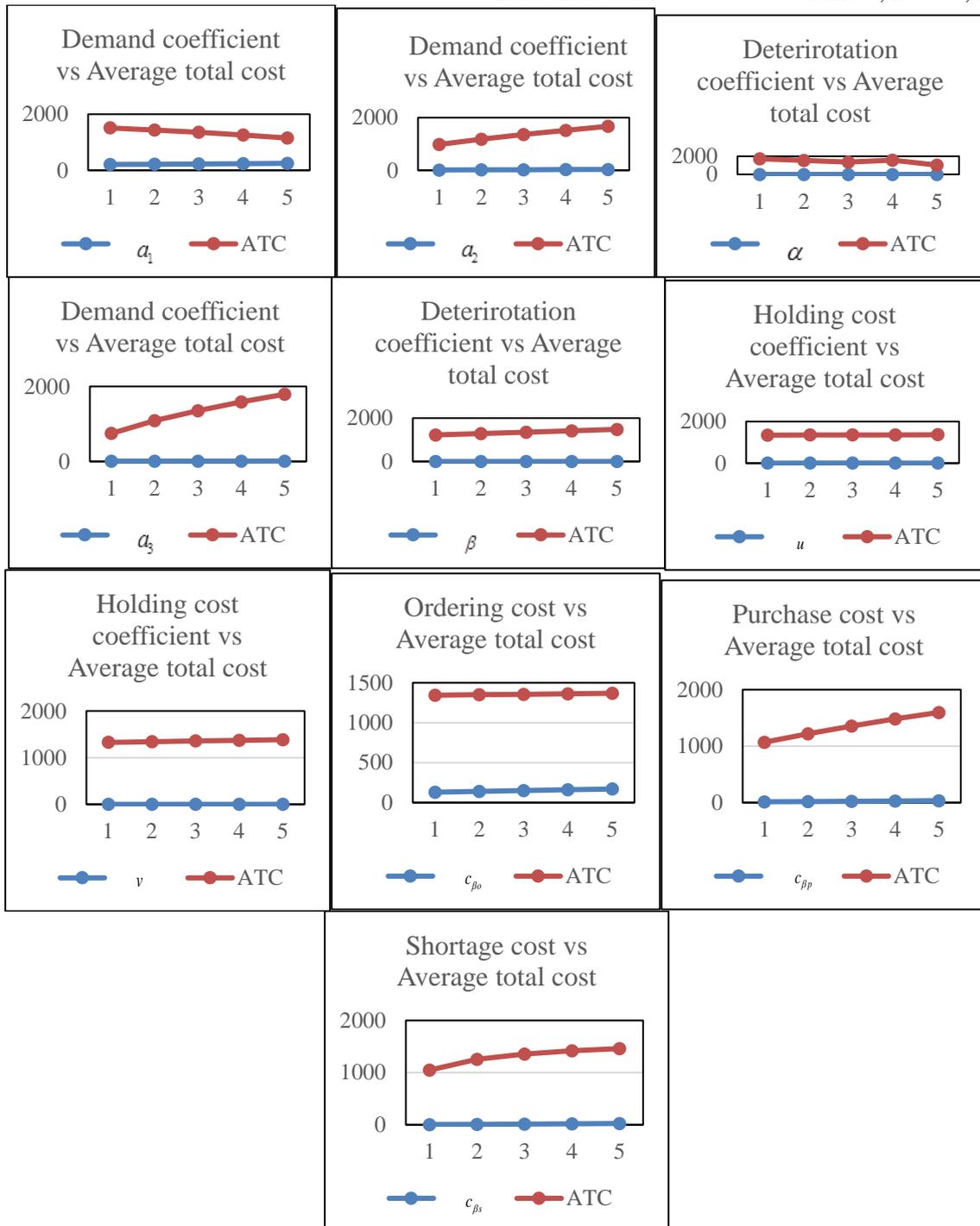
#### 5. Sensitivity analysis and representations using graphs

**5.1. To demonstrate the modifications in the parameters using the table is shown.**

Parameter	Variation	$t_\beta$	$T_\beta$	$Q_\beta$	$AT_\beta C(t_\beta, T_\beta)$
$a_1$	210	1.4904	2.4078	407.708	1510.4223
	220	1.3745	2.1999	385.8031	1438.73
	230	1.2557	1.9914	360.7272	1356.3236
	240	1.1311	1.7772	331.6088	1261.1305
	250	0.9966	1.5509	297.1636	1150.1715

$a_2$	15	0.9412	1.4591	251.3127	982.8191
	20	1.1199	1.7581	310.7506	1179.9097
	25	1.2557	1.9914	360.7272	1356.3236
	30	1.3635	2.1805	404.549	1519.2165
	35	1.4513	2.3372	443.9478	1672.6379
$a_3$	8	0.6473	0.9848	167.9035	748.3133
	9	1.0108	1.5745	279.2374	1088.3983
	10	1.2557	1.9914	360.7272	1356.3236
	11	1.4563	2.3462	430.6162	1585.6783
	12	1.6373	2.6782	495.4457	1791.5916
$\alpha$	0.056	1.6283	2.6045	490.2877	1721.4419
	0.058	1.4349	2.2843	422.0784	1531.6624
	0.06	1.2557	1.9914	360.7272	1356.3236
	0.062	1.4349	2.2843	426.3981	1579.2534
	0.064	0.91	1.4334	248.3141	1025.0579
$\beta$	0.156	1.1204	1.7719	315.7149	1224.9584
	0.158	1.1892	1.8833	338.4497	1291.5795
	0.16	1.2557	1.9914	360.7272	1356.3236
	0.162	1.3202	2.0964	382.5716	1419.2599
	0.164	1.3854	2.203	404.9314	1483.1027
$u$	0.15	1.2627	1.9934	363.1294	1346.7802
	0.2	1.2592	1.9923	361.9224	1351.5595
	0.25	1.2557	1.9914	360.7272	1356.3236
	0.3	1.2523	1.9904	359.5438	1361.0727
	0.35	1.2489	1.9894	358.372	1365.807
$v$	0.575	1.2331	1.9499	352.9323	1327.3262
	0.6	1.2445	1.9708	356.8632	1341.8882
	0.625	1.2557	1.9914	360.7272	1356.3236
	0.65	1.2667	2.0117	364.5261	1370.6365
	0.675	1.2774	2.0317	368.2613	1384.8306
$c_{\beta o}$	130	1.2537	1.9878	360.0206	1343.8857
	140	1.2547	1.9896	360.374	1350.1096
	150	1.2557	1.9914	360.7272	1356.3236
	160	1.2568	1.9931	361.0801	1362.5278
	170	1.2578	1.9949	361.4327	1368.7223
$c_{\beta p}$	15	1.5058	2.0973	450.1638	1069.3513
	20	1.3627	2.0289	398.2617	1220.4623
	25	1.2557	1.9914	360.7272	1356.3236
	30	1.1721	1.9726	332.111	1480.9035
	35	1.1043	1.9659	309.4198	1596.6969
$c_{\beta s}$	5	1.0009	2.6723	275.6232	1048.0805
	10	1.1705	2.1812	331.5782	1253.8526
	15	1.2557	1.9914	360.7272	1356.3236
	20	1.3079	1.8887	378.8856	1418.2983
	25	1.3432	1.824	391.3519	1459.9386

### 5.2. Graph of parameters with average total cost



### 6. Observations using table values

#### The Impact of all parameters with time and order quantity with average overall cost

Here the investigations are using tabular values, we can observe the following progress.

- 1) The raising in  $a_1$  results in decline in time  $t_\beta$  &  $T_\beta$ , thereby the order amount  $Q_\beta$  and average total cost  $AT_\beta C$  has also been diminishing.
- 2) The growth in  $a_2$  leads to the raising in time  $t_\beta$  &  $T_\beta$ , the order amount  $Q_\beta$  and Avg. tot.cost  $AT_\beta C$  is also raising.
- 3) The raise in  $a_3$  enable the time  $t_\beta$  &  $T_\beta$  to grow marginally, here the order amount  $Q_\beta$  is also mounting, and Avg. tot.cost  $AT_\beta C$  get equally mounted.

- 4) The deterioration coefficients  $\alpha$  started raising whereas the time  $t_\beta$  &  $T_\beta$  gets diminishing, here both the order amount  $Q_\beta$  and Avg. tot.cost  $AT_\beta C$  are declined.
- 5) The deterioration coefficients  $\beta$  started raising whereas the time  $t_\beta$  &  $T_\beta$  gets raising, here both the order amount  $Q_\beta$  and Avg. tot.cost  $AT_\beta C$  are mounting.
- 6) The augmentation of holding Coefficients  $u$  leads to the decline in the time  $t_\beta$  &  $T_\beta$ , here the order amount  $Q_\beta$  gets reduced whereas the Avg. tot.cost  $AT_\beta C$  is growing.
- 7) The holding Coefficients  $v$  raising as well as the time  $t_\beta$  &  $T_\beta$ , grows, and besides growth of the order amount  $Q_\beta$ , Avg. tot.cost  $AT_\beta C$  also growing.
- 8) Both the ordering cost  $C_{\beta o}$  and the time  $t_\beta$  &  $T_\beta$  are raising, also the order amount  $Q_\beta$  as well as the Avg. tot.cost  $AT_\beta C$  will be raising.
- 9) When the purchase cost  $C_{\beta p}$  grows up, the time  $t_\beta$  &  $T_\beta$  drops down, then the order amount  $Q_\beta$  will be declined and Avg. tot.cost  $AT_\beta C$  will be increased.
- 10) If the shortages cost  $C_{\beta s}$  grows up and the time  $t_\beta$  &  $T_\beta$  also grows up along with the raising of order amount  $Q_\beta$  and Avg. tot.cost  $AT_\beta C$ .

## 7. Conclusion

A model is developed in deteriorating inventory of EOQ of probabilistic with shortages. Shortages can be accepted, it is balanced by fully backlogged. The holding cost considered parabolic equations and reduced the cost. Demand items considered as a scaling of quadratic with highest power coefficient has negative sign, which pursues the Beta passion. The solution of the mathematical model has been worked out using numerical solution with the help of computation table, graph. The main aim of this paper using less time, more quantity and gaining overall average cost. The decisions makers can utilize this model to run their business and it can be further developed using fuzzy demand of variable deterioration linear equation, different types of probabilistic deterioration like Weibull distributions, Gamma distributions, using inflation rate and promotion technology etc.

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