Bipolar valued fuzzy B-ideals on B-Algebra

C.Yamini, S.Kailasavalli ,Dr. P. Hema, Dr. S. Sangeetha

[1]Assistant Professor, Department of Mathematics
PSNA College of Engineering and Technology, Dindigul, Tamilnadu.
e-mail - yaminichandran@gmail.com

[2]Associate Professor, Department of Mathematics
PSNA College of Engineering and Technology, Dindigul, Tamilnadu.
e-mail - skvalli2k5@gmail.com

[3] Assistant Professor, Department of Mathematics
RMK College of Engineering and Technology, Puduvoyal-601206
e-mail - hemaraghav74@gmail.com

[4] Assistant Professor, Department of Mathematics
Dhanalakshmi Srinivasan College of Arts and Science for women,
Autonomus, Perambalur- 621 212
e-mail - sangeethasankar2016@gmail.com

Abstract:

In this paper, B-ideals and fuzzy B-ideals and bipolar valued fuzzy setare discussed and bipolar valued fuzzy B-ideal on B-Algebra is introduced and related topics are discussed.

Keywords:

B-algebras, B-ideals, Fuzzy B-ideals, Cartesian product, Bipolar valued fuzzy set, Bipolar valued fuzzy B-ideals on B-Algebra

1.Introduction

In 1965, Zadeh [14] introduced the notion of a fuzzy subset of a set. Since then it has become a vigorous area of research in different domains. There have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets. In 1994,W.R. Zhang [15] introduced bipolar fuzzy set and relations and Lee [6] introduced the operation in bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval $[0, 1]$ to $[-1, 1]$. In a bipolar-valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree $(0, 1]$ indicates that elements somewhat satisfy the property, and the membership degree $[-1, 0)$ indicates that elements somewhat satisfy the implicit counter-property. Bipolar-valued fuzzy sets and intuitionistic fuzzy sets look similar each other.

In this note,the notion of Bipolar-valued fuzzy B-ideals on B-algebrasare introduced and studied their properties with Cartesian product of B-ideals and strongest fuzzy relation in detail.

2. Preliminaries

In this section we give some basic definitions and preliminariesof B algebras , B-ideals , fuzzy B-ideals , bipolar valued fuzzy set and bipolar valued fuzzy B-ideals on B-algebras .

Definition 2.1. A B- algebra is a non-empty set P with a constant 0 and a binary operation "*" satisfying axioms:

(i) p * p = 0

(ii) p * 0 = p

(iii) (p*q) * r = p * (r * (0*q)), for all p, q, r∈P.

For brevity we also call P a B-algebra. In P we can define a binary relation "≤ " by p ≤ q if and only if p * q = 0.

Definition 2.2. A non-empty subset I of a B-algebra P is called a subalgebra of Pif p * q∈ I for any p, q∈ I.

Example 2.3. Let P be the set of all real numbers except for a negative integer –n. Define a binary operation * on P by p * q =$\frac{n(p-q)}{n+q}$. Then (p, *, 0) in a B-algebra.

Definition 2.4. Let η be a fuzzy set in a B-algebra. Then η is called a fuzzy subalgebra of P if $\eta(\ p * q\ ) \geq \min\{\eta(p), \eta(q)\}$ for all p, q∈P.

Definition 2.5. A nonempty subset I of a B- algebra P is called a B –Ideal of P if it satisfies for p ,q, r ∈ P .

(i) 0 ∈ I

(ii) (p * q) ∈ I and (r * p) ∈ I imply (q * r) ∈ I

Definition 2.6. Let (p, *, 0) be a B-algebra, a fuzzy subset η in P is called a fuzzy B-Ideal of P if it satisfies the following conditions: for all p, q, r∈P.

(i)$\eta(0) \geq \eta(x)$

(ii) $\eta(q * r) \geq \min\{\eta(\ p * q), \eta(r * p)\}$.

Example 2.7. Let P= { 0, 1, 2, 3 } be a set with the following table

*	0	1	2	3
0	0	2	1	3
1	1	0	3	2
2	2	3	0	1
3	3	1	2	0

Then (p,*,0) is a B- algebra and also satisfies the condition of B-ideals.

Definition 2.8. A bipolar valued fuzzy set $(B_iFS)\eta$ in P is defined as an object of the form η = {<p, η^+(p), η^-(p) >/ p∈P}, where η^+: P→ [0, 1] and η^-: P→ [–1, 0]. The positive membership degree η^+(p) denotes the satisfaction degree of an element p to the property corresponding to a bipolar valued fuzzy set η and the negative membership degree η^-(p) denotes the satisfaction degree of an element p to some implicit counter-property corresponding to a bipolar valued fuzzy set η.

Example 2.9. η = { <p, 0.7, –0.4>, <q, 0.5, –0.8>, <r, 0.4, –0.5> } is a B_iFS of P = { p, q, r }.

Definition 2.10. A bipolar valued fuzzy set $(B_iFS)\eta$ in B-Algebra P is said to be a bipolar valued fuzzy B-ideal (B_iFI_b) of P if for every p, q, r ϵ P

1. $\eta_B^+(0) \geq \eta_B^+(p)$ and $\eta_B^+(0) \leq \eta_B^+(p)$
2. $\eta_B^+(q * r) \geq \min\{ \eta_B^+(p * q), \eta_B^+(r * p)\}$ and
   $\eta_B^-(q * r) \leq \max\{ \eta_B^-(p * q), \eta_B^-(r * p)\}$

Example 2.11. For the B Algebra P={0,1,2,3}as in example 2.7, the $B_iFS\ \eta$ = {<p, η^+(p), η^-(p) >/ p∈P} defined by

$\eta_B^+(p) = \begin{cases} 0.7\ ;\ p \neq 2 \\ 0.2\ ;\ p \neq 2 \end{cases}$ and $\eta_B^-(p) = \begin{cases} 0.7\ ;\ p \neq 2 \\ 0.2\ ;\ p \neq 2 \end{cases}$ is a bipolar valued fuzzy B-ideal of P.

Theorem 2.12. Every B_iFI_b of B-Algebra P with q ≤ p, for every p, q∈P is

(i) Order reversing then $\eta_B^+(q) \geq \eta_B^+(p)$

(ii) Order preserving then $\eta_B^-(q) \leq \eta_B^-(p)$

Proof. Let p, q ϵ P such that q≤ p then q ∗ p = 0.

Thus, $\eta_B^+(q) = \eta_B^+(0 * q) \geq \min\{\eta_B^+(p * 0), \eta_B^+(q * p)\}$

$= \min\{ \eta_B^+(p), \eta_B^+(0) \}$

$= \eta_B^+(p)$

Hence $\eta_B^+(q) \geq \eta_B^+(p)$.

And $\eta_B^-(q) = \eta_B^-(0 * q) \leq \max\{\eta_B^-(p * 0), \eta_B^-(q * p)\}$

$= \max\{ \eta_B^-(p), \eta_B^-(0)\}$

$= \eta_B^-(p)$

Hence $\eta_B^-(q) \leq \eta_B^-(p)$.

Theorem 2.13. The intersection of any two B_iFI_bs of P is also a B_iFI_b.

Proof. Let η = {< p, η^+(p), η^-(p) >/ p∈P} and δ = {< p, δ^+(p), δ^-(p) >/ p∈P}

Let W = $\eta \cap \delta$ and W={< p, W^+(p), W^-(p) >/ p∈P}

Then $W_B^+(0) = \min\{\eta_B^+(0), \delta_B^+(0)\}$

$\geq \min\{\eta_B^+(p), \delta_B^+(p)\}$

$= W_B^+(p)$

And $W_B^-(0) = \max\{\eta_B^-(0), \delta_B^-(0)\}$

$\leq \max\{\eta_B^-(p), \delta_B^-(p)\}$

$= W_B^-(p)$

Also $W_B^+(q * r) = \min\{\eta_B^+(q * r), \delta_B^+(q * r)\}$

$\geq \min\{\min\{\eta_B^+(p * q), \eta_B^+(r * p)\}, \min\{\delta_B^+(p * q), \delta_B^+(r * p)\}\}$

$= \min\{\min\{\eta_B^+(p * q), \delta_B^+(p * q)\}, \min\{\eta_B^+(r * p), \delta_B^+(r * p)\}\}$

$= \min\{\{W_B^+(p * q), W_B^+(r * p)\}$

And also $W_B^-(q * r) = \max\{\eta_B^-(q * r), \delta_B^-(q * r)\}$

$\leq \max\{\max\{\eta_B^-(p * q), \eta_B^-(r * p)\}, \max\{\delta_B^-(p * q), \delta_B^-(r * p)\}\}$

$= \max\{\max\{\eta_B^-(p * q), \delta_B^-(p * q)\}, \max\{\eta_B^-(r * p), \delta_B^-(r * p)\}\}$

$= \max\{\{W_B^-(p * q), W_B^-(r * p)\}$

Hence W = $\eta \cap \delta$ is also a B_iFI_b.

Theorem 2.14. The intersection of a family of B_iFI_bs of P is also a B_iFI_b.

Proof. The proof follows from the theorem.2.13.

3.Cartesian Product

Definition 3.1. Let η and δ be the B_iFSs in P and Q respectively. The cartesian product η x δ: Px Q → [0,1] is defined by (ηx δ)={(p,q), $(\eta \text{ x } \delta)_B^+(p,q)$, $(\eta \text{ x } \delta)_B^-(p,q)$ / ∀ p ∈P and ∀q∈Q } where $(\eta \text{ x } \delta)_B^+(p, q) = \min\{\eta_B^+(p), \delta_B^+(q)\}$ and $(\eta \text{ x } \delta)_B^-(p, q) = \max\{\eta_B^-(p), \delta_B^-(q)\}$, ∀p ∈P , q ∈Q.

Definition 3.2. Let η be the B_iFS in a set P, the strongest bipolar valued fuzzy relation on P, that is the strongest bipolar valued fuzzy relation on η is J = { 〈 (p, q), $J_B^+(p, q)$, $J_B^-(p, q)$ 〉/ p, q ∈P }given by $J_B^+(p, q) = \min\{\eta_B^+(p), \eta_B^+(q)\}$ and $J_B^-(p, q) = \min\{\eta_B^-(p), \eta_B^-(q)\}$,∀p, q ∈ P.

Theorem 3.3. IF η and δ are the B_iFI_bs of P and Q respectively, then η x δ is a B_iFI_b of P x Q.

Proof. For any (p, q) ∈P x Q, we have

$(\eta \text{ x } \delta)_B^+(0, 0) = \min\{\eta_B^+(0), \delta_B^+(0)\}$

$\geq \min\{\eta_B^+(p), \delta_B^+(q)\}$

$= (\eta \text{ x } \delta)_B^+(p, q)$

And $(\eta \text{ x } \delta)_B^-(0, 0) = \max\{\eta_B^-(0), \delta_B^-(0)\}$

$\leq \max\{\eta_B^-(p), \delta_B^-(q)\}$

$= (\eta \text{ x } \delta)_B^-(p, q)$

Also, let $(p_1, p_2), (q_1, q_2), (r_1, r_2) \in$ P x Q.

$(\eta \text{ x } \delta)_B^+((q_1, q_2) * (r_1, r_2)) = (\eta \text{ x } \delta)_B^+(q_1 * r_1, q_2 * r_2)$

$= \min\{\eta_B^+(q_1 * r_1), \delta_B^+(q_2 * r_2)\}$

$\geq$min {min{ $\eta_B^+(p_1* q_1),\eta_B^+(r_1* p_1)$}, min{ $\delta_B^+(p_2* q_2),\delta_B^+(r_2* p_2)$}}

$\geq$min {min{ $\eta_B^+(p_1* q_1),\delta_B^+(p_2* q_2)$},min{ $\eta_B^+(r_1* p_1),\delta_B^+(r_2* p_2)$}}

= min{$(\eta \text{ x } \delta)_B^+((p_1* q_1),(p_2* q_2))$}{$(\eta \text{ x } \delta)_B^+((r_1* p_1),(r_2* p_2))$}}

And

$(\eta \text{ x } \delta)_B^-((q_1, q_2) * (r_1, r_2)) = (\eta \text{ x } \delta)_B^-(q_1* r_1, q_2 * r_2)$

= max{ $\eta_B^-(q_1* r_1),\delta_B^-(q_2 * r_2)$ }

$\leq$max {max{ $\eta_B^-(p_1* q_1), \eta_B^-(r_1* p_1)$}, min{ $\delta_B^-(p_2* q_2),\delta_B^-(r_2* p_2)$}}

$\leq$max {max{ $\eta_B^-(p_1* q_1),\delta_B^-(p_2* q_2)$},min{ $\eta_B^-(r_1* p_1),\delta_B^-(r_2* p_2)$}}

= max{$(\eta \text{ x } \delta)_B^-((p_1* q_1),(p_2* q_2))$}{$(\eta \text{ x } \delta)_B^-((r_1* p_1),(r_2* p_2))$}}

Therefore η x δ is a B_iFI_b of P x Q.

Theorem 3.4. For any given B_iFS of a B-algebra P, let Jbe the strongest bipolar valued fuzzy relation on P. If η is a $B_iFI_b$ofPxP, then $J_B^+(P, P) \leq J_B^+(0, 0), \forall p \in P$ and $J_B^-(P, P) \geq J_B^-(0, 0), \forall p \in P$.

Proof. Here J is the strongest bipolar valued fuzzy relation on P x P, then

$J_B^+(P, P) =$ min { $\eta_B^+(p)$, $\eta_B^+(p)$ }

$\leq$ min{ $\eta_B^+(0)$, $\eta_B^+(0)$ }

= $J_B^+(0, 0), \forall p \in P$.

$\Rightarrow J_B^+(P, P) \leq J_B^+(0, 0), \forall p \in P$

In the same way,

$J_B^-(P, P)$ = max {$\eta_B^-(p), \eta_B^-(p)$}

$\geq$ max{ $\eta_B^-(0), \eta_B^-(0)$}

= $J_B^-(0, 0), \forall p \in P$.

$\Rightarrow J_B^-(P, P) \geq J_B^-(0, 0), \forall p \in P$.

Theorem 3.5. Let η be the B_iFS in a B-Algebra P and Jbe the strongest bipolar valued fuzzy relation on P. If η is a B_iFI_b of P iff J is a B_iFI_b of P x P.

Proof. Suppose η is a B_iFI_b of P.

Then $J_B^+(0, 0) =$ min { $\eta_B^+(0)$, $\eta_B^+(0)$ }

$\geq$ min { $\eta_B^+(p)$, $\eta_B^+(q)$ }

= $J_B^+(p, q)$, $\forall p, q \in P$.

And $J_B^-(0, 0)$ = max { $\eta_B^-(0)$, $\eta_B^-(0)$ }

$\leq$ max{ $\eta_B^-(p)$, $\eta_B^-(q)$ }

= $J_B^-(p, q)$, $\forall$ p, q $\in$ P.

Also for any $(p_1, p_2), (q_1, q_2), (r_1, r_2) \in$ P x P

$J_B^+(q_1* r_1, q_2 * r_2) = \min\{ \eta_B^+(q_1* r_1),\eta_B^+(q_2 * r_2) \}$

$\geq\min\{\min\{ \eta_B^+(p_1* q_1), \eta_B^+(r_1* p_1)\}, \min\{ \eta_B^+(p_2* q_2),\eta_B^+(r_2* p_2)\}\}$

$\geq\min\{\min\{ \eta_B^+(p_1* q_1),\eta_B^+(p_2* q_2)\},\min\{ \eta_B^+(r_1* p_1),\eta_B^+(r_2* p_2)\}\}$

$= \min\{(J_B^+((p_1* q_1),(p_2* q_2))\}\{J_B^+((r_1* p_1),(r_2* p_2))\}\}$

And also

$J_B^-(q_1* r_1, q_2 * r_2) = \max\{ \eta_B^-(q_1* r_1),\eta_B^-(q_2 * r_2)\}$

$\leq\max\{\max\{ \eta_B^-(p_1* q_1), \eta_B^-(r_1* p_1)\}, \min\{ \eta_B^-(p_2* q_2),\eta_B^-(r_2* p_2)\}\}$

$\leq\max\{\max\{ \eta_B^-(p_1* q_1),\eta_B^-(p_2* q_2)\},\min\{ \eta_B^-(r_1* p_1),\eta_B^-(r_2* p_2)\}\}$

$= \max\{J_B^-((p_1* q_1),(p_2* q_2))\}\{J_B^-((r_1* p_1),(r_2* p_2))\}\}$

Therefore,J is a B_iFI_b of P x P.

Conversely, suppose that J is a B_iFI_b of P x P, then

$J_B^+(P, P) \leq J_B^+(0, 0)$ where (0,0) is the zero element of P x P.

Which means that

$\min\{ \eta_B^+(p) , \eta_B^+(q) \}\leq \min\{ \eta_B^+(0) , \eta_B^+(0) \}$

$\Rightarrow\eta_B^+(p) \leq \eta_B^+(0), \forall p \in P$ and also $\eta_B^-(p) \geq \eta_B^-(0), \forall p \in P$.

Now, let $(p_1, p_2), (q_1, q_2), (r_1, r_2) \in P \times P$

Then,

$\min\{ \eta_B^+(q_1* r_1),\eta_B^+(q_2 * r_2) \} = J_B^+(q_1* r_1, q_2 * r_2)$

$\geq\min\{J_B^+(p_1 , p_2)*(q_1 , q_2)\}, J_B^+(r_1, r_2) * (p_1, p_2)\}\}$

$= \min\{J_B^+((p_1* q_1),(p_2* q_2)), J_B^+((r_1* p_1),(r_2* p_2))\}$

$= \min\{\min\{ \eta_B^+(p_1* q_1),\eta_B^+(p_2* q_2)\},\min\{ \eta_B^+(r_1* p_1),\eta_B^+(r_2* p_2)\}\}$

In particular, if we take $p_2=q_2=r_2=0$,then

$\eta_B^+(q_1* r_1) \geq\min\{ \eta_B^+(p_1* q_1),\eta_B^+(p_2* q_2)\}$

Also,$\max\{\eta_B^-(q_1* r_1),\eta_B^-(q_2 * r_2)\} = J_B^-(q_1* r_1, q_2 * r_2)$

$\leq\max\{J_B^-(p_1, p_2)*(q_1, q_2)\}, J_B^-(r_1, r_2) * (p_1, p_2)\}\}$

$= \max\{J_B^-((p_1* q_1),(p_2* q_2)), J_B^-((r_1* p_1),(r_2* p_2))\}$

$= \max\{\max\{ \eta_B^-(p_1* q_1),\eta_B^-(p_2* q_2)\},\max\{ \eta_B^-(r_1* p_1),\eta_B^-(r_2* p_2)\}\}$

In particular, if we take $p_2=q_2=r_2=0$,then

$\eta_B^-(q_1* r_1) \leq\max\{ \eta_B^-(p_1* q_1),\eta_B^-(p_2* q_2)\}$

This proves η is a B_iFI_b of P.

Theorem 3.6. Let η and δ be the B_iFSs in a B-Algebra P such that $\eta \times \delta$ is a B_iFI_b of P x P then$\forall p \in P$,

(i) Either $\eta_B^+(0) \geq \eta_B^+(p)$ or $\delta_B^+(0) \geq \delta_B^+(p)$ and $\eta_B^-(0) \leq \eta_B^-(p)$ or $\delta_B^-(0) \leq \delta_B^-(p)$.

(ii) If $\eta_B^+(0) \geq \eta_B^+(p)$ then either $\delta_B^+(0) \geq \eta_B^+(p)$ or $\delta_B^+(0) \geq \delta_B^+(p)$ and $\eta_B^-(0) \leq \eta_B^-(p)$ then either $\delta_B^-(0) \leq \eta_B^-(p)$ or $\delta_B^-(0) \leq \delta_B^-(p)$.

(iii) If $\delta_B^+(0) \geq \delta_B^+(p)$ then either $\eta_B^+(0) \geq \eta_B^+(p)$ or $\eta_B^+(0) \geq \delta_B^+(p)$ and $\delta_B^-(0) \leq \delta_B^-(p)$ then either $\eta_B^-(0) \leq \eta_B^-(p)$ or $\eta_B^-(0) \leq \delta_B^-(p)$.

Proof. Let η x δ is a B_iFI_b of P x P.

Therefore,

$(\eta \text{ x } \delta)_B^+(0, 0) \geq (\eta \text{ x } \delta)_B^+(p, q)$, $\forall$ (p, q) $\in$ P x P

$(\eta \text{ x } \delta)_B^+((q_1, q_2) * (r_1, r_2)) \geq \min \{(\eta \text{ x } \delta)_B^+((p_1, p_2) * (q_1, q_2))(\eta \text{ x } \delta)_B^+((r_1, r_2) * (p_1, p_2))\}$

And $(\eta \text{ x } \delta)_B^-((q_1, q_2) * (r_1, r_2)) \leq \max \{(\eta \text{ x } \delta)_B^-((p_1, p_2) * (q_1, q_2))(\eta \text{ x } \delta)_B^-((r_1, r_2) * (p_1, p_2))\}$

For all $(p_1, p_2), (q_1, q_2), (r_1, r_2) \in$ P x P

(i) Suppose that $\eta_B^+(0) < \eta_B^+(p)$ and $\delta_B^+(0) < \delta_B^+(p)$ for some p, q $\in$ P.

$(\eta \text{ x } \delta)_B^+(p, q) = \min \{ \eta_B^+(p) , \delta_B^+(q) \}$

$> \min \{\eta_B^+(0), \delta_B^+(0) \}$

$= (\eta \text{ x } \delta)_B^+(0, 0)$ which is a contradiction.

Therefore either $\eta_B^+(0) \geq \eta_B^+(p)$ or $\delta_B^+(0) \geq \delta_B^+(p)$, $\forall p \in P$.

And, suppose $\eta_B^-(0) > \eta_B^-(p)$ or $\delta_B^-(0) > \delta_B^-(p)$.

$(\eta \text{ x } \delta)_B^-(p, q) = \max\{ \eta_B^-(p), \delta_B^-(q)\}$

$< \max\{\eta_B^-(0), \delta_B^-(0) \}$

$= (\eta \text{ x } \delta)_B^-(0, 0)$ which is a contradiction

Therefore either $\eta_B^-(0) \leq \eta_B^-(p)$ or $\delta_B^-(0) \leq \delta_B^-(p)$, $\forall p \in P$.

(ii) Assume that there exists p, q $\in$ P such that $\delta_B^+(0) < \eta_B^+(p)$ or $\delta_B^+(0) < \delta_B^+(p)$

Then $(\eta \text{ x } \delta)_B^+(0, 0) = \min \{ \eta_B^+(0) , \delta_B^+(0) \}$

$= \delta_B^+(0)$

and hence $(\eta \text{ x } \delta)_B^+(p, q) = \min \{ \eta_B^+(p) , \delta_B^+(q) \}$

$> \delta_B^+(0)$

$= (\eta \text{ x } \delta)_B^+(0, 0)$ which is a contradiction.

Hence if $\eta_B^+(0) \geq \eta_B^+(p)$ then either $\delta_B^+(0) \geq \eta_B^+(p)$ or $\delta_B^+(0) \geq \delta_B^+(p)$.

Also assume that $\delta_B^-(0) > \eta_B^-(p)$ or $\delta_B^-(0) > \delta_B^-(p)$

$(\eta \text{ x } \delta)_B^-(0, 0) = \max \{ \eta_B^-(0) , \delta_B^-(0) \}$

$= \delta_B^-(0)$

and hence $(\eta \text{ x } \delta)_B^-(p, q) = \max \{ \eta_B^-(p) , \delta_B^-(q) \}$

$> \delta_B^-(0)$

$= (\eta \text{ x } \delta)_B^-(0, 0)$ which is a contradiction.

Hence if $\eta_B^-(0) \leq \eta_B^-(p)$ then either $\delta_B^-(0) \leq \eta_B^-(p)$ or $\delta_B^-(0) \leq \delta_B^-(p)$.

Similarly we can prove that if $\delta_B^+(0) \geq \delta_B^+(p)$ then either $\eta_B^+(0) \geq \eta_B^+(p)$ or $\eta_B^+(0) \geq \delta_B^+(p)$ and $\delta_B^-(0) \leq \delta_B^-(p)$ then either $\eta_B^-(0) \leq \eta_B^-(p)$ or $\eta_B^-(0) \leq \delta_B^-(p)$ which yields (iii).

Theorem 3.7. Let η and δ be the B_iFSs in a B-Algebra P such that η x δ is a B_iFI_b of P x P then $\forall$ p $\in$P. Then η or δ is a B_iFI_b of P.

Proof. By theorem (3.6 (i)), without loss of generality we assume that $\eta_B^+(p) \leq \eta_B^+(0)$ and $\eta_B^-(0) \leq \eta_B^-(p)$ for all p $\in$P. From the theorem (3.6 (ii)) it follows that for all p $\in$P, either $\delta_B^+(0) \geq \eta_B^+(p)$ or $\delta_B^+(0) \geq \delta_B^+(p)$ and $\eta_B^-(0) \leq \eta_B^-(p)$ then either $\delta_B^-(0) \leq \eta_B^-(p)$ or $\delta_B^-(0) \leq \delta_B^-(p)$.

If $\delta_B^+(0) \geq \eta_B^+(p)$ for all p $\in$P

Then $(\eta \text{x } \delta)_B^+(0, p) = \min \{\delta_B^+(0) , \eta_B^+(p) \}$

$= \eta_B^+(p)$

Let (p, q)$\in$ P x P.Since η x δ is a B_iFI_b of P,

we get $(\eta \text{ x } \delta)_B^+(0, 0) \geq (\eta \text{ x } \delta)_B^+(p, q)$ and $(\eta \text{ x } \delta)_B^-(0, 0) \leq (\eta \text{ x } \delta)_B^-(p, q)$

Let $(p_1, p_2), (q_1, q_2), (r_1, r_2) \in$ P x P

Using B-ideal,

$(\eta \text{ x } \delta)_B^+(q_1 * r_1, q_2 * r_2) = \min \{ \eta_B^+(q_1 * r_1),\delta_B^+(q_2 * r_2) \}$

$\geq \min \{\min\{ \eta_B^+(p_1 * q_1), \eta_B^+(r_1 * p_1)\}, \min\{ \delta_B^+(p_2 * q_2),\delta_B^+(r_2 * p_2)\}\}$

$\geq \min \{\min\{ \eta_B^+(p_1 * q_1),\delta_B^+(p_2 * q_2)\},\min\{ \eta_B^+(r_1 * p_1),\delta_B^+(r_2 * p_2)\}\}$

$= \min\{(\eta \text{ x } \delta)_B^+((p_1 * q_1),(p_2 * q_2))\}\{(\eta \text{ x } \delta)_B^+((r_1 * p_1),(r_2 * p_2))\}\}$

In particular, if we take $p_1=q_1=r_1=0$, then

$\delta_B^+(q_2 * r_2) = (\eta \text{ x } \delta)_B^+ (0, q_2 * r_2)$

$\geq \min \{\min\{ \eta_B^+(0),\delta_B^+(p_2 * q_2)\},\min\{ \eta_B^+(0),\delta_B^+(r_2 * p_2)\}\}$

$= \min\{\delta_B^+(p_2 * q_2)\},\delta_B^+(r_2 * p_2)\}\}$

And

$(\eta \text{ x } \delta)_B^-(q_1 * r_1, q_2 * r_2) = \max \{ \eta_B^-(q_1 * r_1),\delta_B^-(q_2 * r_2) \}$

$\leq \max \{\max\{ \eta_B^-(p_1 * q_1), \eta_B^-(r_1 * p_1)\}, \min\{ \delta_B^-(p_2 * q_2),\delta_B^-(r_2 * p_2)\}\}$

$\leq \max \{\max\{ \eta_B^-(p_1 * q_1),\delta_B^-(p_2 * q_2)\},\min\{ \eta_B^-(r_1 * p_1),\delta_B^-(r_2 * p_2)\}\}$

$= \max\{(\eta \text{ x } \delta)_B^-((p_1 * q_1),(p_2 * q_2))\}\{(\eta \text{ x } \delta)_B^-((r_1 * p_1),(r_2 * p_2))\}\}$

In particular, if we take $p_1=q_1=r_1=0$, then

$\delta_B^-(q_2 * r_2) = (\eta \text{ x } \delta)_B^-(0, q_2 * r_2)$

$\leq \max \{\max\{ \eta_B^-(0),\delta_B^-(p_2 * q_2)\}, \max\{ \eta_B^-(0),\delta_B^-(r_2 * p_2)\}\}$

$= \max\{\delta_B^-(p_2 * q_2)\},\delta_B^-(r_2 * p_2)\}\}$

This proves that δ is a B_iFI_b of P.

Similarly, second part can be proved.

This completes the proof.

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