The Intutionistic Anti Fuzzy Km Ideal On K-Algebras

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Abstract: This paper mainly aims at introducing anti fuzzy km ideal on k algebras and intutionistic magnified translation. Here, basic concepts of fuzzy K-algebra and its KM ideal on K-algebras, intuitionistic fuzzy and intuitionistic fuzzy magnified translations under homomorphism and epimorphismpropertiesare defined and discussed. The results of anti-fuzzy KM Ideal on K-algebras and Instutionistic anti fuzzy Ideal are analyzed.

Keywords: K-algebras, KM ideal, Fuzzy KM ideal, Anti fuzzy KM ideal on K- algebras, intuitionistic fuzzy KM ideal, intuitionistic anti fuzzy magnified translation of KM ideals.

1. INTRODUCTION

R.Biswas [1] introduced the concept of Anti fuzzy subgroup of a group[1].Dar and Akramis introduced K-Algebra $(G,\odot.e)$ [2], and built on a group (G,.,e) of Algebra with identity e and adjoined of induced binary operation $\odot$ on G. It is non-commutative and non-associative with a right identity of e. It is proved in [2,3] that the statement of K-algebra on an abelian group is equivalent to a p-semisimple BCI-algebra is proved by [2, 3]. For the suitability of study, authors renamed a K-algebra built on a group as K(G) algebra in [2].After the introduction of fuzzy sets, fuzzy set theory is developed by Zadeh[5] and also joined with others in many rulesare found for different scientific applications.Akram et alintroduced the notions of sub algebras and fuzzy (maximal) ideals of K-algebras in [6] is further analyzed by Jun et al. [7].In this Fuzzy ideals of K Algebras are introduced and their properties are verified.The extension ofFuzzy KM an ideal on K-algebras is also hosted and verified [8].An instuitionistic fuzzy magnified translation of km ideals on k-algebras is introduced and the solutions are discussed in[9].In this paper, antifuzzy km ideal on k algebras is introduced and also applied instuitionistic magnified translation in it.

Preliminaries:
In this section, Fuzzy K-algebra, KM idealon K-algebras, intuitionistic fuzzy and intuitionistic fuzzy magnified translations of homomorphism under epimorphismpropertiesare explained. The results of anti-fuzzy KM Ideal on Kalgebras and Instutionistic anti fuzzy Ideal are stated.

Definition 2.1:
Let (G, ·, e) be a group with the identity e such that $x^2 \neq e$ for some $x(\neq e) \in G$.
A K-algebra built on G (briefly, K-algebra) is a structure K = (G, ., ⊙, e) where "⊙" is a binary operation on G which is induced from the operation "·", that satisfies the following:
(k1) $(\forall a, x, y \in G)\ ((a \odot x) \odot (a \odot y) = (a \odot (y^{-1} \odot x^{-1})) \odot a)$,
(k2) $(\forall a, x \in G)\ (a \odot (a \odot x) = (a \odot x^{-1}) \odot a)$,
(k3) $(\forall a \in G)\ (a \odot a = e)$,
(k4) $(\forall a \in G)\ (a \odot e = a)$,
(k5) $(\forall a \in G)\ (e \odot a = a^{-1})$.
If G is abelian, then conditions (k1) and (k2) are replaced by:
(k1') $(\forall a, x, y \in G)\ ((a \odot x) \odot (a \odot y) = y \odot x)$,
(k2') $(\forall a, x \in G)\ (a \odot (a \odot x) = x)$,
respectively. A nonempty subset H of a K-algebra K is called a subalgebra of K if it satisfies:
- $(\forall a, b \in H)\ (a \odot b \in H)$.

Note that every subalgebra of a K-algebra K contains the identity e of the group (G, ·).
Definition 2.2:
A fuzzy set ζ in a k-algebra k is called a fuzzy ideal of k if it satisfies:
(i) $(\forall x \in G)(\zeta(e) \geq \zeta(x))$
(ii) $(\forall x, y \in G)\ (\zeta(y) \geq \min\{\zeta(y \odot x), \zeta((x \odot (x \odot y))\})$
Definition 2.3:
An intuitionistic fuzzy set A = {<x, ζ_A(x), ς(x)>/x∈ X} in X is called an intuitionistic fuzzy ideal of X if it satisfies
(i) $\zeta_A(0) \geq \zeta_A(x)$, $\xi_A(0) \leq \xi_A(x)$
(ii) $\zeta_A(x) \geq \min\{\zeta_A(x*y), \zeta_A(y)\}$
(iii) $\xi_A(x) \leq \max\{\xi_A(x*y), \xi_A(y)\}$ for all x,y∈ X.
Definition 2.4:
Let $(X,\odot,0)$ be a K-algebra, if the following conditions are satisfied, for all x,y,zϵX
(i) $(0) \geq \zeta(x)$.
(ii) $\eta(y \odot z) \geq \min\{\eta(x \odot y), \eta(z \odot x)\}$
then, a fuzzy subset ζin x is called a fuzzy KM-Ideal of X
Definition 2.5:
An intuitionistic fuzzy set A = {<x, ζ_A(x), ξ_A(x)>/x∈ X} in K-algebra X is called an intuitionistic fuzzy KM-ideal of A if
(i) $\zeta_A(0) \geq \zeta_A(x)$ and $\xi_A(0) \geq \xi_A(x)$
(ii) $\zeta_A(y \odot z) \geq \min\{\zeta_A(x \odot y), \zeta_A(z \odot x)\}$
(iii) $\xi_A(y \odot z) \leq \max\{\xi_A(x \odot y), \xi_A(z \odot x)\}$
Definition 2.6:
Let $(X,\odot,0)$ and $(Y,\Delta,0')$ be K-algebras. A mapping $f : X \rightarrow Y$ is called a homomorphism iff $(x \odot y) = f(x)\ \Delta f(y)$, for all $x, y\ \epsilon X$.

Definition 2.7:
Let $f : X \rightarrow X$ be an endomorphism and μ be a fuzzy set in X. We define a new fuzzy set in X by μ_f in X as $\mu_f(x) = \mu(f(x))$ for all x in X.
Definition 2.8:
For any homomorphism f : X → Y the set $\{x \in X / f(x) = 0'\}$ is called the Kernel of f, denoted by Ker(f) and the set $\{f(x)/x \in X\}$ is called the image of f denoted by Im(f).
Definition 2.9:

Let $(X,\odot,0)$ be a K-algebra, if the following conditions are satisfied, for all x,y,zϵX

(iii) $\zeta(0) \leq \zeta(x)$

(iv) $\zeta(y \odot z) \leq \max\{\zeta(x \odot y), \zeta(z \odot x)\}$

then, a fuzzy subset ζin x is called a anti fuzzy KM-Ideal of X

Definition 2.10:

An instuitionistic anti fuzzy set A = {<x, ζ_A(x), ξ_A(x)>/x$\in X$} in K-algebra X is called an instuitionistic anti fuzzy KM-ideal of A if

(i) $\zeta_A(0) \leq \zeta_A(x)$ and $\xi_A(0) \leq \xi_A(x)$

(ii) $\zeta_A(y\odot z) \leq \max\{\zeta_A(x\odot y), \zeta_A(z\odot x)\}$

(iii) $\xi_A(y\odot z) \geq \min\{\xi_A(x\odot y), \xi_A(z\odot x)\}$.

Anti Fuzzy KM Ideals on K Algebras:

InThis topic we discuss about antifuzzy KM ideals,and Homomorphism of K Algebras

Theorem3.1:

Every fuzzy KM-Ideal μ of K-algebra X is order preserving that is y$\leq$ x then $\zeta(y) \leq \zeta(x)$ for all x,yϵ X.

Proof:

Letη be a fuzzy KM-Ideal of K-algebra X and let x,yϵ X such that y$\leq$ x then y $\odot$ x = 0

$$\zeta(y) \leq \max\{\zeta(y \odot x), \zeta(x \odot (x \odot y)\}$$
$$\leq \max\{\zeta(0), \zeta(x)\}$$
$$= \zeta(x)$$

Hence $\zeta(y) \leq \zeta(x)$.

Theorem 3.2:

Let f be an endomorphism of a K – algebra X. If ζ is a anti fuzzy KM- Ideal of X, then so is η_f

Proof:

Let x, y ϵ X

$$\zeta_f(x) = \zeta(f(x))$$
$$\leq \max\{\zeta(f(y \odot x)), \eta(f(x) \odot f(x \odot y))\}$$
$$= \max\{\zeta(f(y \odot x)), \eta(f(x \odot (x \odot y)))\}$$
$$= \max\{\zeta_f((y \odot x)), \eta_f(x \odot (x \odot y))\}.$$

Hence ζ_f is anti fuzzy KM ideal of X.

Theorem 3.3:

Let $(X,\odot,0)$ and $(Y,\Delta,0')$ be K-algebras. A mapping f : X → Y is a anti homomorphism of K-algebra. Then Ker(f) is a KM-ideal.

Proof:

Let (y $\odot$ x) $\odot$ (x$\odot$ (x $\odot$ y)) ϵker(f) & x ϵ ker(f)

Then f((y $\odot$ x) $\odot$ (x$\odot$ (x $\odot$ y))) = 0' and f(x) = 0'

$$0=f((y \odot x) \odot (x\odot (x \odot y)))$$
$$= f(x\odot (x \odot y))\Delta f(y \odot x)$$
$$= (f(x) \Delta f(x \odot y))\Delta(f(y) \Delta f(x))$$
$$= (f(x)\Delta(f(x)\Delta f(y))\Delta(f(y) \Delta f(x))$$
$$=(0'\Delta(f(x)\Delta f(y)))\Delta(f(y) \Delta 0')$$

=f (x⊙ (x ⊙ y))
⟹x⊙ (x ⊙ y)ϵ ker f
Hence Ker(f) is a KM – ideal

Intuitionistic anti Fuzzy KM-Ideal
Theorem 4.1:
Let A be a nonempty subset and intuitionistic anti fuzzy KM ideals of a k-algebra. Then so is A = (x, $\zeta_A, \overline{\zeta_A}$) and A = (x, $\xi_A, \overline{\xi_A}$)
Proof:
We know that,$\zeta_A(0) \leq \zeta_A(x)$. This implies that 1- $\overline{\zeta_A}$ (0) ≤ 1- $\overline{\zeta_A}$ (x)
$\overline{\zeta_A}$ (0) ≥ $\overline{\zeta_A}$ (x) for every a∈ A.
Let us consider ∀ a, b, c ∈ X. Then

ζ_A(y⊙ z) ≤ max{ζ_A(x⊙ y), ζ_A(z⊙ x)}

⟹ 1- $\overline{\zeta_A}$ (y⊙ z) ≤ max {1- $\overline{\zeta_A}$ (x⊙ y), 1- $\overline{\zeta_A}$ (z⊙ x)}
⟹$\overline{\zeta_A}$ (y⊙ z) ≤ 1- {$\overline{\zeta_A}$ (x⊙ y), $\overline{\zeta_A}$ (z⊙ x)}
⟹$\overline{\zeta_A}$ (y⊙ z) ≥ min {$\overline{\zeta_A}$ (x⊙ y), $\overline{\zeta_A}$ (z⊙ x)}
Hence A = (x, $\zeta_A, \overline{\zeta_A}$) is an intuitionistic fuzzy KM ideal of X.

To prove the second part,
Let A be an intuitionistic fuzzy KM ideal of a K-algebra X.
Now$\xi_A(0) \geq \xi_A(x)$
⟹ 1- $\overline{\xi_A}$ (0) ≤ 1- $\overline{\xi_A}$ (x)
⟹$\overline{\xi_A}$ (0) ≤ $\overline{\xi_A}$ (x) ∀x∈ X.
Now consider ∀ a, b, c ∈ A. Then we have
ξ_A(y⊙ z) ≥ min {ξ_A(x⊙ y), ξ_A(z⊙ x)}
⟹ 1- $\overline{\xi_A}$ (y⊙ z) ≥ min { 1- $\overline{\xi_A}$ (x⊙ y), 1- $\overline{\xi_A}$ (z⊙ x)}
⟹$\overline{\xi_A}$ (y⊙ z) ≤ 1- min { $\overline{\xi_A}$ (x⊙ y), $\overline{\xi_A}$ (z⊙ x)}
⟹$\overline{\xi_A}$ (y⊙ z) ≤ max {$\overline{\xi_A}$ (x⊙ y), $\overline{\xi_A}$ (z⊙ x)}
Hence A = (x, $\xi_A, \overline{\xi_A}$) is an instuitionistic fuzzy KM ideal of X.
Definition 4.2:
An ideal A of a KM ideal is said to be closed if 0^*x ∈ X, ∀ x ∈ X.
Theorem4.3:
Let A be a nonempty set and an a intuitionistic anti fuzzy closed KM-ideal of a K-algebra A. Then A = (x, $\zeta_A, \overline{\zeta_A}$) and A = (x, $\xi_A, \overline{\xi_A}$) is also a closed intuitionistic anti fuzzy closed KM-ideal.
Proof:
First we prove A = (x, $\zeta_A, \overline{\zeta_A}$) is a intuitionistic closed anti fuzzy KM-ideal .
For all x ∈ X we have
ζ_A(0*x) ≥ ζ_A(x)⟹ 1- $\overline{\zeta_A}$(0*x) ≥ 1-$\overline{\zeta_A}$(x)⟹$\overline{\zeta_A}$(0*x) ≤ $\overline{\zeta_A}$(x)
Therefore A = (x, $\zeta_A, \overline{\zeta_A}$) is a intuitionistic closed anti fuzzy KM-ideal of A.
To prove the second part,
let x ∈ X, then we have
ξ_A(0*x) ≤ ξ_A(x) ⟹ 1- $\overline{\xi_A}$(0*x) ≤ 1- $\overline{\xi_A}$(x) ⟹$\overline{\xi_A}$(0*x) ≥ $\overline{\xi_A}$(x) ∀ a, b, c ∈A.
Hence A = (x, $\xi_A, \overline{\xi_A}$) is an intuitionistic anti fuzzy closed KM-ideal of A.
Theorem 4.4:
Let an intuitionistic anti fuzzy set A= (ζ_A, ξ_A) in X be an intuitionistic anti fuzzy ideal of X. If the inequality
(x⊙ y) ≥ z holds in X, then

ζ_A(y⊙ z) ≤ max{ζ_A(x⊙ y), ζ_A(z⊙ x)}

ξ_A(y⊙ z) ≥ min {ξ_A(x⊙ y), ξ_A(z⊙ x)}

Proof:

Let x, y, z ∈ X be such that x⊙ y ≥ z.

Then (x⊙ y) ⊙ z = 0 and thus

ζ_A(y⊙ z) ≤ max{ζ_A(x⊙ y), ζ_A(z⊙ x)}

≤ max {max{ζ_A((x⊙ y) ⊙ z), ζ_A(z⊙ x)}, ζ_A(x⊙ y)}

= max {max{ζ_A(0), ζ_A(z⊙ x), ζ_A(x⊙ y)}

= max {ζ_A(x⊙ y), ζ_A(z⊙ x)}

ξ_A(y⊙ z) ≥ min {ξ_A(x⊙ y), ξ_A(z⊙ x)}

≥min {min{ξ_A((x⊙ y) ⊙ z), ξ_A(z⊙ x), ξ_A(x⊙ y)}

=min {min{ξ_A(0), ξ_A(z⊙ x)}, ξ_A(x⊙ y)}

=min {ξ_A(z⊙ x), ξ_A(x⊙ y)}

=min {ξ_A(x⊙ y), ξ_A(z⊙ x)}

Hence the proof.

Theorem 4.5:

An intuitionistic anti fuzzy set A= (ζ_A, ξ_A) is an intuitionistic anti fuzzy KM ideal of X if and only if the fuzzy set ζ_A and $\overline{\xi_A}$ are fuzzy KM ideal of X.

Proof:

Let A= (ζ_A, ξ_A) be an intuitionistic fuzzy ideal of X.

Clearly, $\zeta_A$is a fuzzy KM ideal of X.

For every x, y, z ∈X we have

$\overline{\xi_A}$ (0) = 1- ξ_A(0) ≤ 1- ξ_A(x) = $\overline{\xi_A}$ (x).

$\overline{\xi_A}$ (y⊙ z) = 1- ξ_A(x) ≤ 1- min{ξ_A(x⊙ y),ξ_A(z⊙ x) }

= max {1- ξ_A(x⊙ y), 1-ξ_A(z⊙ x)}

= max{ $\overline{\xi_A}$ (x⊙ y), $\overline{\xi_A}$ (z⊙ x)}

Hence $\overline{\xi_A}$ is a fuzzy KM ideal of X.

Conversely, assume that ζ_A and $\overline{\xi_A}$ are fuzzy KM ideal of X. For every x, y, z ∈X we get

ζ_A(0) ≤ ζ_A(x), 1- ξ_A(0) = $\overline{\xi_A}$ (0) ≤ $\overline{\xi_A}$ (x) = 1- ξ_A(x)

That is,

ξ_A(0) ≥ ξ_A(x), ζ_A(y⊙ z) ≤ max{ζ_A(x⊙ y), ζ_A(z⊙ x)}

1 − ξ_A(y⊙ z) = $\overline{\xi_A}$ (y⊙ z) ≤ max{ $\overline{\xi_A}$ (x⊙ y), $\overline{\xi_A}$ (z⊙ x)}

≥ min {1- ξ_A(x⊙ y), 1-ξ_A(z⊙ x)}

= 1- min{ξ_A(x⊙ y),ξ_A(z⊙ x) }

That is, ξ_A(y⊙ z) ≤min{ξ_A(x⊙ y),ξ_A(z⊙ x) }

Hence A= (ζ_A, ξ_A) is an intuitionistic fuzzy ideal of X.

2. CONCLUSION

In this paper, Fuzzy km ideal on k algebras, Homomorphism and intutionistic anti fuzzy are discussed. The results of anti-fuzzy KM Ideal on K-algebras and Instutionistic anti fuzzy Ideal are analyzed.

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