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A Study On Fuzzy Queue With N Policy By Triangular Fuzzy Numbers

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ABSTRACT
Fuzzy queue tries to reduce the length of the queues in many practical situation. In this paper we studied about the fuzzy queue with N-policy. Hence we discuss about the characteristics of N- policy fuzzy queue under vague data. Hence we used triangular fuzzy numbers as input parameters and used markov chain approach to find the expected waiting time in the queue and the expected number of customers in the system under fuzzy environment.

Keywords: Fuzzy queue, N- policy, waiting time, Number of customers.

1. INTRODUCTION

In the literature of queuing theory many researchers like Tadj et al [15][16], Lee et al[11][12][13], Choudhury et al [7][8], Arumuganathan et al[6] studied well about the N-policy queue. In fuzzy environment also N-policy queue was widely studied by researchers like Tsung – yin wang et al[18], Ritha et al[4] and so on. When the arrival rate and service rate are known exactly then we can easily calculate the performance of the operating policy. Fuzzy queue allows us to determine the performance even when arrival rate and service rate are vague. Here we have proposed a method to estimate the performance of the operating sysyem with unclear data under fuzzy environment and also we have solved an example as an evident of the proposed method. This journal is ordered as follows: Kin second part, we discuss about the basic definitions. Third part explains a theorem which is essential to find the expected waiting time in the queue and the expected number of customers in the system. Fourth part illustrates an example. Finally we have concluded this work.

2. PRELIMINARIES

Definition 2.1

Afuzzy set $\tilde{A}$ is defined on the set of real numbers R is called a **fuzzy number** if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ has the following conditions:

(a) $\tilde{A}$ is convex, which means that there exists $x_1, x_2 \in R$ and $\lambda \in [0,1]$, such that

$$\mu_{\tilde{A}}\left(\lambda x_1 + (1-\lambda) x_2\right) \geq \min\left\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right\}$$

(b) $\tilde{A}$ is normal, which means that there exists an $x \in R$ such that $\mu_{\tilde{A}}(x) = \tilde{1}$

(c) $\tilde{A}$ is piecewise continuous.

Definition 2.2

A fuzzy number $\tilde{A}$ is defined on the set of real numbers R is said to be a **triangular fuzzy number** if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ which satisfy the following conditions:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & \text{for} \quad a_1 \le x \le a_2 \\ 1 & \text{for} \quad x = a_2 \\ \dfrac{a_3-x}{a_3-a_2} & \text{for} \quad a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.3

Let the two triangular fuzzy numbers be $\tilde{M} \approx (a_1, a_2, a_3)$ and $\tilde{N} \approx (b_1, b_2, b_3)$ and then the **arithmetic operations on triangular fuzzy numbers** be given as follows:

(A)Addition

$$\tilde{M} + \tilde{N} \approx (a_1, a_2, a_3) + (b_1, b_2, b_3)$$
$$\tilde{M} + \tilde{N} \approx (m_1, \alpha_1, \beta_1) + (m_2, \alpha_2, \beta_2)$$
$$\tilde{M} + \tilde{N} \approx \left(m_1 + m_2, \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\}\right)$$

(B)Subtraction

$$\tilde{M} - \tilde{N} \approx (a_1, a_2, a_3) - (b_1, b_2, b_3)$$
$$\tilde{M} - \tilde{N} \approx (m_1, \alpha_1, \beta_1) - (m_2, \alpha_2, \beta_2)$$
$$\tilde{M} - \tilde{N} \approx \left(m_1 - m_2, \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\}\right)$$

***(**C) Multiplication*

$$\tilde{M}.\tilde{N} \approx (a_1, a_2, a_3).(b_1, b_2, b_3)$$
$$\tilde{M}.\tilde{N} \approx (m_1, \alpha_1, \beta_1).(m_2, \alpha_2, \beta_2)$$
$$\tilde{M}.\tilde{N} \approx \left(m_1.m_2, \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\}\right)$$

(D) Division

$$\frac{\tilde{M}}{\tilde{N}} \approx \frac{(a_1, a_2, a_3)}{(b_1, b_2, b_3)}$$

$$\frac{\tilde{M}}{\tilde{N}} \approx \frac{(m_1, \alpha_1, \beta_1)}{(m_2, \alpha_2, \beta_2)}$$

$$\frac{\tilde{M}}{\tilde{N}} \approx \left(\frac{m_1}{m_2}, \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\} \right)$$

3. EXPECTED NUMBER OF CUSTOMERS IN THE SYSTEM AND THEIR EXPECTED WAITING TIME IN THE QUEUE:
N-POLICY FM/FM/1 QUEUE:

We consider N-policy FM/FM/1 queue with infinite capacity. The arrival rate of the customer express as poisson distribution with parameters $\tilde{\lambda}$ and their service rate express as exponential distribution with parameters $\tilde{\mu}$. The fuzzy queue is considered with single server and first – come, first – serve discipline. Let $\tilde{T}_q$ and $\tilde{N}_s$ be the waiting time of the customers in the queue and the expected number of customers in the system respectively.

By markov chain , it can be derived as follows:

$$\tilde{T}_q = \frac{N-1}{2\tilde{\lambda}} + \frac{\tilde{\lambda}}{\tilde{\mu}\left(\tilde{\mu}-\tilde{\lambda}\right)}$$

$$\tilde{N}_s = \frac{N-1}{2} + \frac{\tilde{\lambda}}{\left(\tilde{\mu}-\tilde{\lambda}\right)}$$

Theorem: 3.1

If $FM/FM/1$ is a simple fuzzy queue, with N-policy fuzzy queuing model with infinite capacity whose arrival rate and service rate are respectively positive triangular fuzzy numbers $\tilde{\lambda} = (a_1, a_2, a_3)$ and $\tilde{\mu} = (b_1, b_2, b_3)$ with $a_3 < b_1$; the threshold value be N, then the expected number of customers in the system is denoted by $\tilde{N}_s$ and is given by,

$$\tilde{N}_S = \frac{N-1}{2}\left\{ \frac{m_1}{m_2 - m_1}, \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\} \right\}$$

Proof

In fuzzy queueing theory, parameters like the number of customers, arrival rate, service rate , the threshold value are taken as non-negative values. Let $\tilde{N}_s$ be the expected number of customers in the system at the stable state. Then $\tilde{N}_s$ can be computed by replacing λ and μ by $\tilde{\lambda}$ and $\tilde{\mu}$ respectively.

Let $\tilde{N}_s$ be the expected number of customers in the system and is given by

$$\tilde{N}_s = \frac{N-1}{2} + \frac{\tilde{\lambda}}{(\tilde{\mu}-\tilde{\lambda})}$$

The arrival rate and the service rate are taken by two triangular numbers as $\tilde{\lambda}$ and $\tilde{\mu}$ has the form of

$$\tilde{\lambda} = (m_1, \alpha_1, \beta_1) \text{ and}$$
$$\tilde{\mu} = (m_2, \alpha_2, \beta_2)$$

Where

m_1 & m_2 are the middle number,
α_1 & α_2 are the left spread,
β_1 & β_2 are the right spread.

Then the value of $\tilde{N}_S$ can be computed using the following formula,

$$\tilde{N}_s = \frac{N-1}{2} + \frac{\tilde{\lambda}}{(\tilde{\mu}-\tilde{\lambda})}$$

$$\tilde{N}_S = \frac{N-1}{2} + \frac{\tilde{\lambda}}{(\tilde{\mu}-\tilde{\lambda})}$$

$$\tilde{N}_S = \frac{N-1}{2} + \frac{(m_1, \alpha_1, \beta_1)}{(m_2, \alpha_2, \beta_2) - (m_1, \alpha_1, \beta_1)}$$

$$\tilde{N}_S = \frac{N-1}{2} + \frac{(m_1, \alpha_1, \beta_1)}{\left(m_2 - m_1, \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\}\right)}$$

$$\tilde{N} = \frac{N-1}{2} + \left\{\frac{m_1}{m_2 - m_1}, \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\}\right\}$$

Theorem 3.2

If $FM/FM/1$ is a simple fuzzy queue, with N-policy fuzzy queuing model with infinite capacity whose arrival rate and service rate are respectively positive triangular fuzzy numbers $\tilde{\lambda} = (a_1, a_2, a_3)$ and $\tilde{\mu} = (b_1, b_2, b_3)$ with $a_3 < b_1$; the threshold value be N, then the expected waiting time of customers in the queue is denoted by $\tilde{T}_q$ and is given by,

$$\tilde{T}_q = \frac{N-1}{2(m_1, \alpha_1, \beta_1)} + \left\{\frac{m_1}{m_2(m_2 - m_1)}, \max\{\alpha_1, \alpha_2\}, \max\{\beta_1, \beta_2\}\right\}$$

Proof

In fuzzy queueing theory, parameters like the number of customers, arrival rate, service rate , the threshold value are taken as non-negative values. Let $\tilde{T}_q$ be the expected waiting time of customers in the fuzzy queue at the stable state. Then $\tilde{T}_q$ can be computed by replacing λ and μ by $\tilde{\lambda}$ and $\tilde{\mu}$ respectively.

Let $\tilde{T}_q$ be the expected waiting time of customers in the queue and is given by

$$\tilde{T}_q = \frac{N-1}{2\tilde{\lambda}} + \frac{\tilde{\lambda}}{\tilde{\mu}\left(\tilde{\mu}-\tilde{\lambda}\right)}$$

The arrival rate and the service rate are taken by two triangular numbers as $\tilde{\lambda}$ and $\tilde{\mu}$ has the form of

$$\tilde{\lambda} = \left(m_1, \alpha_1, \beta_1\right) \quad \text{and}$$
$$\tilde{\mu} = \left(m_2, \alpha_2, \beta_2\right)$$

Where

m_1 & m_2 are the middle number,

α_1 & α_2 are the left spread,

β_1 & β_2 are the right spread.

Then the value of $\tilde{T}_q$ can be computed using the following formula,

$$\tilde{T}_q = \frac{N-1}{2\tilde{\lambda}} + \frac{\tilde{\lambda}}{\tilde{\mu}\left(\tilde{\mu}-\tilde{\lambda}\right)}$$

$$\tilde{T}_q = \frac{N-1}{2\left(m_1,\alpha_1,\beta_1\right)} + \frac{\left(m_1,\alpha_1,\beta_1\right)}{\left(m_2,\alpha_2,\beta_2\right)\left\{\left(m_2,\alpha_2,\beta_2\right)-\left(m_1,\alpha_1,\beta_1\right)\right\}}$$

$$\tilde{T}_q = \frac{N-1}{2\left(m_1,\alpha_1,\beta_1\right)} + \frac{\left(m_1,\alpha_1,\beta_1\right)}{\left(m_2,\alpha_2,\beta_2\right)\left(m_2-m_1,\max\left\{\alpha_1,\alpha_2\right\},\max\left\{\beta_1,\beta_2\right\}\right)}$$

$$\tilde{T}_q = \frac{N-1}{2\left(m_1,\alpha_1,\beta_1\right)} + \frac{\left(m_1,\alpha_1,\beta_1\right)}{\left\{m_2.\left(m_2-m_1\right),\max\left\{\alpha_1,\alpha_2\right\},\max\left\{\beta_1,\beta_2\right\}\right\}}$$

$$\tilde{T}_q = \frac{N-1}{2\left(m_1,\alpha_1,\beta_1\right)} + \left\{\frac{m_1}{m_2.\left(m_2-m_1\right)},\max\left\{\alpha_1,\alpha_2\right\},\max\left\{\beta_1,\beta_2\right\}\right\}$$

4. NUMERICAL DESCRIPTION

The following numerical description was inspired by Tsung-yin wang, Dong yub yang, Meng – Julie[18] and later it was solved by W.Ritha and B.Sreelekha menon[4].

Example:4.1

Let us discuss on printed ciruit Board Assembly. When the number of printed circuit Board reaches a threshold value N , the reflow machine starts its operation to save cost. The expert would like to find the performance of the system, the expected number of customers in the system $\tilde{N}_S$ and also the expected waiting time in the queue $\tilde{T}_q$ The arrival rate at a machine by means of poisson distribution is given by $\tilde{\lambda} = (1,2,3)$ per hour and the service rate of the reflow machine experiences the exponential distribution at the rate of $\tilde{\mu} = (11,12,13)$ per hour and the threshold value N = 3.

i) Find out the expected number of customers in the system?
ii) Find out the expected waiting time of customers in the queue?

Solution

Let the triangular fuzzy numbers be $\tilde{\lambda} = (1,2,3)$ and $\tilde{\mu} = (11,12,13)$.

i) The expected number of customers in the system and the expected waiting time of customers in the queue is given by,

$$\tilde{N}_s = \frac{N-1}{2} + \frac{\tilde{\lambda}}{(\tilde{\mu} - \tilde{\lambda})}$$

$$\tilde{T}_q = \frac{N-1}{2\tilde{\lambda}} + \frac{\tilde{\lambda}}{\tilde{\mu}(\tilde{\mu} - \tilde{\lambda})}$$

ii) Determine the triangular fuzzy numbers be $\tilde{\lambda} = (1,2,3)$ and $\tilde{\mu} = (11,12,13)$ as of the form (m,α,β),

$\tilde{\lambda} = (2,1,1)$

$\tilde{\mu} = (12,1,1)$

iii) The number of customers in the system are,

$$\tilde{N}_s = \frac{N-1}{2} + \frac{\tilde{\lambda}}{(\tilde{\mu} - \tilde{\lambda})}$$

Given N = 3,
Hence,

$$\tilde{N}_s = \frac{N-1}{2} + \frac{\tilde{\lambda}}{(\tilde{\mu} - \tilde{\lambda})}$$

$$\tilde{N}_s = \frac{3-1}{2} + \frac{(2,1,1)}{(12,1,1)-(2,1,1)}$$

$$\tilde{N}_s = \frac{2}{2} + \frac{(2,1,1)}{(10,1,1)}$$

$$\tilde{N}_s = 1 + (0.2,1,1)$$

$$\tilde{N}_s = (1.2,1,1)$$

$$\tilde{N}_s = (0.2,1.2,2.2)$$

iv) The waiting time of customers in the queue are,

$$\tilde{T}_q = \frac{N-1}{2\tilde{\lambda}} + \frac{\tilde{\lambda}}{\tilde{\mu}(\tilde{\mu} - \tilde{\lambda})}$$

Given N = 3

Hence,

$$\tilde{T}_q = \frac{3-1}{2(2,1,1)} + \frac{(2,1,1)}{(12,1,1)\{(12,1,1)-(2,1,1)\}}$$

$$\tilde{T}_q = \frac{2}{2(2,1,1)} + \frac{(2,1,1)}{(12,1,1)(10,1,1)}$$

$$\tilde{T}_q = \frac{1}{(2,1,1)} + \frac{(2,1,1)}{(120,1,1)}$$

$$\tilde{T}_q = \frac{1}{(2,1,1)} + (0.0166,1,1)$$

$$\tilde{T}_q = (0.5,1,1) + (0.0166,1,1)$$

$$\tilde{T}_q = (0.5166,1,1)$$

$$\tilde{T}_q = (0.484, 0.516, 1.516)$$

Therefore the mean value of the number of customers in the system is exactly 1.2; that is the much more possible value for the expected number of customers in the system. Similarly the expected waiting time of the customers in the queue is most probably 0.515 (≈30 minutes)

5. CONCLUSION

Here we have followed the new technique, to solve fuzzy queue with N- policy. Based on this approach, some characteristics such as expected number of customers in the system and the expected waiting time of customers in the queue can be calculated successfully, and results are the same than those obtained by other methods. Hence we have concluded that our new approach also have the potential to extract the result in a successful way.

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