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Fibonacci Triple Sequence

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Abstract: In this paper a we have established some new generalised identities on one of the schemes of multiplicative Triple Fibonacci sequence.

Keywords: Multiplicative Triple Fibonacci sequence

1.1 Introduction

Sequence and series are eternal parts of mathematics. Many mathematicians have generalised many properties on well-known Fibonacci sequence, but the concept of Fibonacci triple sequence is less known to us. It was first introduced by Jin-Zai Lee & Jia-Sheng Lee [1] in 1987. There are different schemes possible for Fibonacci triple sequence, in this paper we have established some new results of multiplicative triple Fibonacci Sequences of the one of the schemes [2-4].

1.2 Multiplicative Triple Fibonacci sequence

The Multiplicative Triple Fibonacci sequence is defined by the recurrence relation

$$\alpha_{n+2} = \gamma_{n+1}\gamma_n\,, \qquad \beta_{n+2} = \alpha_{n+1}\alpha_n, \qquad \gamma_{n+2} = \beta_{n+1}\beta_n \tag{1.2.1}$$

for all integer $n \geq 0$, with initial conditions

$$\alpha_0 = a,\ \ \alpha_1 = d,\ \ \beta_0 = b,\ \ \beta_1 = e,\ \ \gamma_0 = c,\ \ \gamma_1 = f$$

Where $a,\ d,\ b,\ e,\ c$ and f are real numbers

Theorem 1 If α_n and γ_n are define by equation (1.2.1) then (for $n > 1$)

$$\alpha_{n+8} = \prod_{i=n}^{n+4} \gamma_i \left(\prod_{j=n+1}^{n+3} \gamma_j \right)^3 \gamma_{n+2}^2 \tag{1.2.2}$$

Proof: Theorem can be proved by mathematical induction method on n

For $n = 1$ by equations (1.2.1) and (1.2.2)

$$\prod_{i=2}^{6} \gamma_i \left(\prod_{j=3}^{5} \gamma_j \right)^3 \gamma_4^2 = \gamma_2\, \gamma_3\gamma_4\gamma_5\gamma_6(\gamma_3\gamma_4\gamma_5)^3\gamma_4^2$$

by using equation (1.1) repeatedly we have

$$\prod_{i=2}^{6}\gamma_i\left(\prod_{j=3}^{5}\gamma_j\right)^3\gamma_4^2=\alpha_{10}$$

which proves for $n=1$

Suppose the theorem is true for $n=k$, so by equation (1.2.2)

$$\alpha_{k+8}=\prod_{i=k}^{k+4}\gamma_i\left(\prod_{j=k+1}^{k+3}\gamma_j\right)^3\gamma_{k+2}^2 \qquad (1.2.3)$$

Now to prove for $n=k+1$, by using equation (1.2.1) and (1.2.2)

$$\prod_{i=k+1}^{(k+1)+4}\gamma_i\left(\prod_{j=(k+1)+1}^{(k+1)+3}\gamma_j\right)^3\gamma_{(k+1)+2}^2=\gamma_{k+1}\,\gamma_{k+2}\gamma_{k+3}\gamma_{k+4}\gamma_{k+5}(\gamma_{k+2}\gamma_{k+3}\gamma_{k+4})^3\gamma_{k+3}^2$$

by using equation (1.2.1) repeatedly we have

$$\prod_{i=k+1}^{(k+1)+4}\gamma_i\left(\prod_{j=(k+1)+1}^{(k+1)+3}\gamma_j\right)^3\gamma_{(k+1)+2}^2=\alpha_{(m+1)+8}$$

which proves the theorem.

Theorem 2 If α_n and β_n are define by equation (1.2.1) then (for $n>1$)

$$\beta_{n+8}=\prod_{i=n}^{n+4}\alpha_i\left(\prod_{j=n+1}^{n+3}\alpha_j\right)^3\alpha_{n+2}^2 \qquad (1.2.4)$$

Proof: Theorem can be proved by mathematical induction method on n

For $n=1$ by equations (1.2.1) and (1.2.4)

$$\prod_{i=2}^{6}\alpha_i\left(\prod_{j=3}^{5}\alpha_j\right)^3\alpha_4^2=\alpha_2\,\alpha_3\alpha_4\alpha_5\alpha_6(\alpha_3\alpha_4\alpha_5)^3\alpha_4^2$$

by using equation (1.1) repeatedly we have

$$\prod_{i=2}^{6}\alpha_i\left(\prod_{j=3}^{5}\alpha_j\right)^3\alpha_4^2=\beta_{10}$$

which proves for $n=1$

Suppose the theorem is true for $n=k$, so by equation (1.2.4)

$$\beta_{k+8}=\prod_{i=k}^{k+4}\alpha_i\left(\prod_{j=k+1}^{k+3}\alpha_j\right)^3\alpha_{k+2}^2 \qquad (1.2.5)$$

Now to prove for $n = k + 1$, by using equation (1.2.1) and (1.2.5)

$$\prod_{i=k+1}^{(k+1)+4} \alpha_i \left(\prod_{j=(k+1)+1}^{(k+1)+3} \alpha_j \right)^3 \alpha_{(k+1)+2}^2 = \alpha_{k+1}\, \alpha_{k+2}\alpha_{k+3}\alpha_{k+4}\alpha_{k+5}(\alpha_{k+2}\alpha_{k+3}\alpha_{k+4})^3\alpha_{k+3}^2$$

by using equation (1.2.1) repeatedly we have

$$\prod_{i=k+1}^{(k+1)+4} \alpha_i \left(\prod_{j=(k+1)+1}^{(k+1)+3} \alpha_j \right)^3 \alpha_{(k+1)+2}^2 = \beta_{(m+1)+8}$$

which proves the theorem.

Theorem 3 If β_n and γ_n are define by equation (1.2.1) then (for $n > 1$)

$$\gamma_{n+8} = \prod_{i=n}^{n+4} \beta_i \left(\prod_{j=n+1}^{n+3} \beta_j \right)^3 \beta_{n+2}^2 \tag{1.2.6}$$

Proof: Theorem can be proved by mathematical induction method on n

For $n = 1$ by equations (1.2.1) and (1.2.6)

$$\prod_{i=2}^{6} \beta_i \left(\prod_{j=3}^{5} \beta_j \right)^3 \beta_4^2 = \beta_2\, \beta_3\beta_4\beta_5\beta_6(\beta_3\beta_4\beta_5)^3\beta_4^2$$

by using equation (1.2.1) repeatedly we have

$$\prod_{i=2}^{6} \beta_i \left(\prod_{j=3}^{5} \beta_j \right)^3 \beta_4^2 = \gamma_{10}$$

which proves for $n = 1$

Suppose the theorem is true for $n = k$, so by equation (1.2.6)

$$\gamma_{k+8} = \prod_{i=k}^{k+4} \beta_i \left(\prod_{j=k+1}^{k+3} \beta_j \right)^3 \beta_{k+2}^2 \tag{1.2.7}$$

Now to prove for $n = k + 1$, by using equation (1.2.1) and (1.2.6)

$$\prod_{i=k+1}^{(k+1)+4} \beta_i \left(\prod_{j=(k+1)+1}^{(k+1)+3} \beta_j \right)^3 \beta_{(k+1)+2}^2 = \beta_{k+1}\, \beta_{k+2}\beta_{k+3}\beta_{k+4}\beta_{k+5}(\beta_{k+2}\beta_{k+3}\beta_{k+4})^3\beta_{k+3}^2$$

by using equation (1.2.1) repeatedly we have

$$\prod_{i=k+1}^{(k+1)+4} \gamma_i \left(\prod_{j=(k+1)+1}^{(k+1)+3} \gamma_j \right)^3 \gamma_{(k+1)+2}^2 = \gamma_{(m+1)+8}$$

which proves the theorem.

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