INVERSE DOMINATION IN VARIOUS SPECIAL GRAPHS

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Abstract: In this paper, the idea of minimal inverse dominating set (IDS) in a graph G(V,E) is presented. A set $I \subseteq (V - D)$ ofG (V,E) is anIDSof G , if I is the dominating set of the sub graph $< V - D >$, D is a minimal dominating set ofthe G. A minimal IDS in a graph G is anIDSthat contains no IDSas a proper subset. A minimum cardinality among all the IDSis an ID number of G(V,E), and it is denoted by $\gamma_{ID}(G)$. Further theIDS and ID number of various special graphs like Bidiakis cube,Durer graph,Golomb graph and etc. have been discussed.

Keywords: Graphs, Domination, inverse domination, inverse domination number.

INTRODUCTION

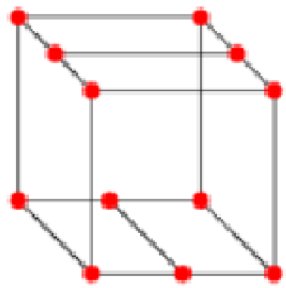
A set S of vertices of G is a dominating set of G if every vertices of G is dominated by at least one vertex of S. Equivalently: a set S of vertices of G is a dominating set if every vertex in $V - S$ is adjacent to at least one vertex in S.

A set $I \subseteq (V - D)$ of G (V,E) is an IDSof G , if I is the dominating set of the sub graph $< V - D >$, D is a minimal dominating set of the G. A minimal IDSin a graph G is anIDSthat contains no IDSas a proper subset. A minimum cardinality among all the IDSis an ID number of G(V,E), and it is denoted by $\gamma_{ID}(G)$.

1. MAIN RESULTS

In this section**,** theconcept of IDS and IDnumber of various special graphs like Bidiakis cube,Durer graph,Golomb graph and etc. is discussed.

Bidiakis Cube: Bidiakis cubeis a 3-regular graph with 12 vertices and 18 edges .



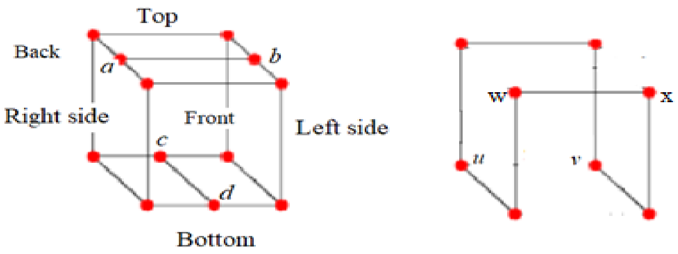
The 12-node graph containing of a cube in which two opposite surfaces (say, top and bottom) be necessary edges drawn through them which join the midpoints of opposite sides of the faces in such a way that the orientation of the edges added on top and bottom are perpendicular to each other.

Theorem 2.1 Let $G(V,E)$ is a Bidiakis Cube. Then the minimal IDS $I=\{u,v,w,x\}$ where $u,v,w\&x$ are the two diagonal vertices in an opposite surfaces (say right and left, top and bottom)

Proof: Let $G(V,E)$ is a Bidiakis Cube. Let $u,v,w\&x$ containg the vetices in two opposite faces have edges drawn across them which connect the centres of opposite sides of the faces. Let $D=\{a,b,c,d\}$, it dominates the corner vertices of the Bidiakis Cube. Therefore $D=\{a,b,c,d\}$ is a dominating set of Bidiakis Cube. In a sub graph $<V-D>$, the set $I=\{u,v,w,x\}$ where $u,v,w\&x$ are the two diagonal vertices in an opposite surfaces (say right and left, top and bottom). The set $I=\{u,v,w,x\}$ dominates every vertex in $<V-D>$. Therefore $I=\{u,v,w,x\}$ is an IDSof a Bidiakis Cube $G(V,E)$.

Assume $I=\{u,v,w,x\}$ where $u,v,w\&x$ are the two diagonal vertices in an opposite surfaces (say right and left, top and bottom) is not a minimal IDSof Bidiakis Cube. There exist a set $I'=(I-\{v_i\})$ is a minimalIDSof Bidiakis Cube. If $v_i \in I$ is a diagonal vertices of the any side of Bidiakis Cube, this implies $I'=(I-\{v_i\})$ is not dominated the another another diogonal vertices in the same side. This is contradict to our assumtion $I'=(I-\{v_i\})$ is a minimal IDSof Bidiakis Cube.Therefore $I'=(I-\{v_i\})$ is not a dominating set of $<V-D>$. Hence $I=\{u,v,w,x\}$ where $u,v,w\&x$ are the two diagonal vertices in an opposite surfaces (say right and left, top and bottom) is a minimal IDS of Bidiakis Cube.

Illustration:

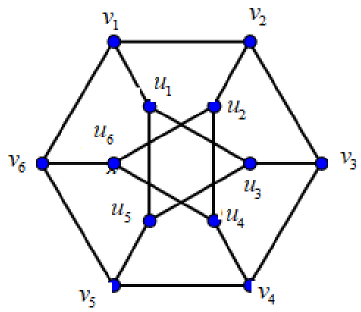


$G(V,E)$: Bidiakis Cube $\quad$ $<V-D>$

Figure 2.1

The graph $G(V,E)$ is the Bidiakis Cube the minimal dominating set $D=\{a,b,c,d\}$ and minimal IDSis $I=\{u,v,w,x\}$and ID number of Bidiakis Cube is $\gamma_{ID}(G)=4$.

Durer graph: The Durer graph is an undirected cubic graph with 12 vertices and 18 edges.



$G(V,E)$: *Durer graph*

Remarks:

1. Durer graph is a 3 regular graph.
2. In the Durer graph 6 vertices forms a outer Hexagon and remaining 6 vertices formsinner 2 triangle region.
3. The 6 vertices of the inner triangles are adjacent to only one vertex on the Hexagon.

Theorem 2.2 Let $G(V,E)$ is a Durer graph. Then the minimal IDS $I=\{v_{(i+2)},v_{(i+4)},u_{(i+3)}\}or\{v_{(i+2)},v_{(i+4)},u_{(i+5)}\}$.

Proof: Let $G(V,E)$ is a Durer graph and, $D=\{u_i,u_{(i+_6 1)},v_i,v_{(i+_6 3)}\}$ is a dominating set of Durer graph. The subset $\{u_i,u_{i+1}\}$ dominates remaining inner vertices of Durer graph and the subset $\{v_i,v_{i+3}\}$ remaining outer vertices of Durer graph. Therfore $D=\{u_i,u_{(i+_6 1)},v_i,v_{(i+_6 3)}\}$ is a minimal dominating set of Durer graph $G(V,E)$ .The sub graph $<V-D>$, there is a path $v_{i+1}v_{i+2}u_{i+2}u_{i+4}v_{i+4}v_{i+5}u_{i+5}u_{i+3}$. Assume $I=\{v_{(i+2)},v_{(i+4)},u_{(i+3)}\}or\{v_{(i+2)},v_{(i+4)},u_{(i+5)}\}$ be a dominating set of thesub graph $<V-D>$. Since $<V-D>$ is a path of 8 vertices.This implies $I=\{v_{(i+2)},v_{(i+4)},u_{(i+3)}\}or\{v_{(i+2)},v_{(i+4)},u_{(i+5)}\}$ is anIDSof Durer graph.

Assume $I=\{v_{(i+2)},v_{(i+4)},u_{(i+3)}\}or\{v_{(i+2)},v_{(i+4)},u_{(i+5)}\}$ is not a minimal IDS of Durer graph.There exist a set $I'=(I-\{x\})$ is anIDSof Durer graph.The subgraph $<V-D>$ is path of 8 vertices such that $I'=(I-\{x\})$ is not an dominating set of $<V-D>$. Hence $I=\{v_{(i+2)},v_{(i+4)},u_{(i+3)}\}or\{v_{(i+2)},v_{(i+4)},u_{(i+5)}\}$ is a minimal IDSof Durer graph.

Illustration:

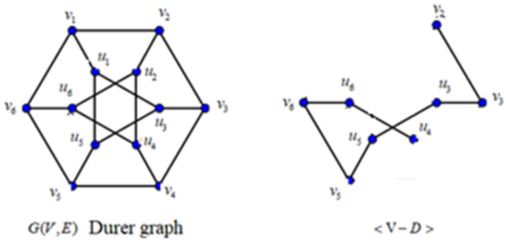
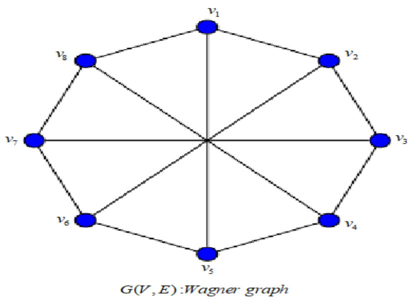


Figure 2.2

The graph $G(V,E)$ is the Durer graph the minimal dominating set $D=\{u_1,u_2,v_1,v_4\}$ and minimal IDSis $I=\{v_3,v_5,u_4\}$ and ID number of Durer graph is $\gamma_{ID}(G)=3$.

Wagner graph:the Wagner graph is a 3- regular with 8 vertices and 12 edges



Theorem 2.3 Let $G(V,E)$ is a Wagner graph. Then the minimal IDS $I=\{v_{i(+_8)6}, v_{i(+_8)6_8}\}$.

Proof: Let $G(V,E)$ is a Wagner graph . Let $D=\{v_i, v_{(i+_8 2)}, v_{(i+_8 4)}\}$ is a dominating set of Wagner graph . The remaining vertices is ajacent to vertices in $D=\{v_i, v_{(i+_8 2)}, v_{(i+_8 4)}\}$ .Therfore $D=\{v_i, v_{(i+_8 2)}, v_{(i+_8 4)}\}$ is a minimal dominating set of Wagner graph $G(V,E)$ .The sub graph $<V-D>$, there is a path $v_{(i+_8 1)}v_{(i+_8 5)}v_{(i+_8 6)}v_{(i+_8 7)}v_{(i+_8 3)}$. Let $I=\{v_{i(+_8)6}, v_{i(+_8)6_8}\}$ is the dominating set of $<V-D>$. since $<V-D>$ is a path of five vertices.This implies $I=\{v_{i(+_8)6}, v_{i(+_8)6_8}\}$ is an IDSof Wagner graph.

Assume $I=\{v_{i(+_8)6}, v_{i(+_8)6_8}\}$ is not a minimal IDSof a Wagner graph.There exist a set $I'=(I-\{x\})$ is IDSof Wagner graph.. The sub graph $<V-D>$, there is a path of five vertices.Hence $I=\{v_{i(+_8)6}, v_{i(+_8)6_8}\}$ is a minimal IDSset of Wagner graph.

Illustration:

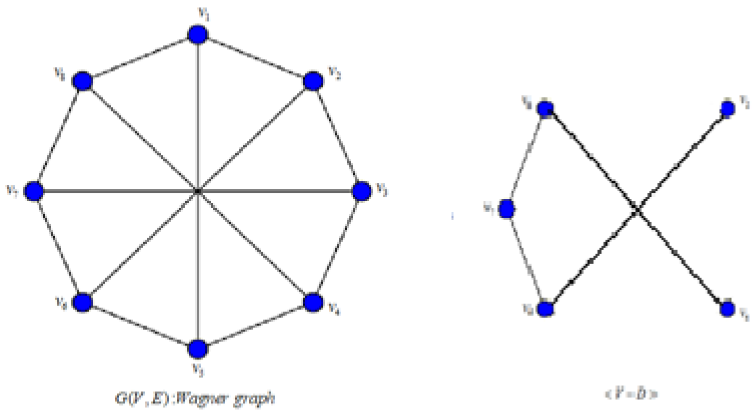
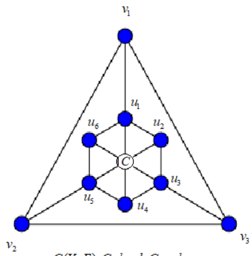


Figure 2. 3

The graph $G(V,E)$ is the Wagner graph, the minimal dominating set $D=\{v_1, v_3, v_5\}$ minimal IDSis $I=\{v_6, v_8\}$ and ID number of Wagner graph is $\gamma_{ID}(G)=2$.

Golomb Graph: In Golomb graph is a graph with 10 vertex and 18 edges.



$G(V,E)$:Golomb Graph

Remarks:

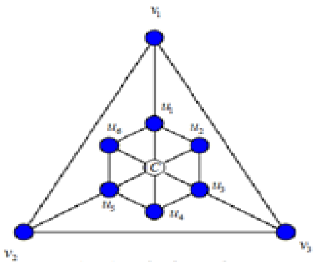
1. In the Golomb graph, 6 vertices forms a inner Hexagon ,3 vertices forms the outer triangle and the remaining vertex C lies in the Centre of the Hexagon and triangle.
2. The 3 vertices of the triangle is adjacent to only one vertices on the Hexagon.

Theorem2.4 Let $G(V,E)$ is a Golombgraph . Then the minimal IDS $I=\{u_1,u_3,u_5\}$,where u_1,u_3,u_5 are the vertices in the Hexagon adjacent to vertices of the triangle.

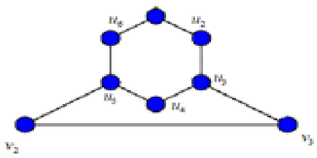
Proof: Let $G(V,E)$ is a Golomb graph. Let $D=\{v_i,\text{C}\}$ here v_i is a vertex lies in the triangle is a dominating set of Golomb graph. The center C is all other vertices in the Hexagon and v_i is adjacent to remaining vertices in the triangle. Therefore $D=\{v_i,\text{C}\}$ is a minimal dominating set of Golomb graph $G(V,E)$ .Let $I=\{u_1,u_3,u_5\}$, where u_1,u_3,u_5 the vertices in the Hexagon are adjacent to vertices of the triangle. The vertices u_1,u_3,u_5 is adjacent to remaining vertices in $<V-D>$ This implies $I=\{u_1,u_3,u_5\}$ is anIDS of Golomb graph.

Assume $I=\{u_1,u_3,u_5\}$ is not aIDSof Golomb graph.There exist a set $I'=(I-\{u\})$, where $u=\{u_1\} or \{u_3\} or \{u_5\}$ is IDSof Golombgraph. There exist a vertices ina triangle is not dominated by $I'=(I-\{u\})$ .Hence $I=\{u_1,u_3,u_5\}$ is a minimal IDSof Golomb graph. Hence $S=\{u_i,u_j,u_k\}$ is a minimal IDSof Golomb graph.

Illustration:



$G(V,E)$:Golomb Graph

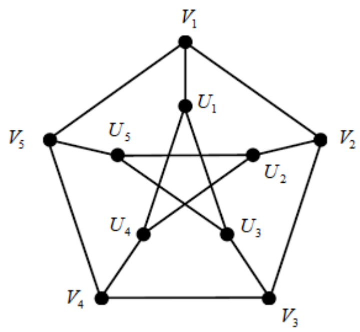


$<V-D>$

Figure 2. 4

The graph $G(V,E)$ is the Golomb graph, the minimal dominating set $D=\{\mathrm{v}_1,\mathrm{C}\}$ minimal IDSis $I=\{u_1,u_3,u_5\}$ and ID number of Wagner graph is $\gamma_{ID}(G)=3$.

Petersen graph :The Petersen graph is an undirected graph with 10 vertices and 15 edges.



G(V,E):Petersen graph

Remarks:

1. Petersen graph is 3 regular graph.
2. Petersen graph consist outer and inner region with 5 vertices each. The outer region form a pentagon and inner region forms a star.
3. One vertices of the pentagon adjacent with at most one vertices in the inner region

Theorem 2.5. Let $G(V,E)$ is a Petersengraph. Then the minimal IDS $I=\{u_{(i+_5 2)}, v_{(i+_5,1)}, v_{(i+_5,3)}\}$.

Proof: Let $G(V,E)$ is a Petersen graph and, $D=\{u_i, u_{(i+_5 4)}, v_{(i+_5 2)}\}$ is a minimal dominating set of Petersen graph . The vertices $u_i, u_{(i+_5 4)}$ dominates remaining vertices in inner region and also dominates $v_i, v_{(i+_5 4)}$ in inner region. The vertex $v_{(i+_5 2)}$ dominates $v_{(i+_5 1)}$ & $v_{(i+_5 3)}$ Therfore $D=\{u_i, u_{(i+_5 4)}, v_{(i+_5 2)}\}$ is a minimal dominating set of Petersengraph $G(V,E)$ .The sub graph $<V-D>$, there is a cycle $v_i v_{(i+_4 1)} u_{(i+_4 1)} u_{(i+_5 3)} v_{(i+_5 3)} v_{(i+_5 4)} v_i$ and the isolated vertex $u_{(i+_5 2)}$ in $<V-D>$. Therefore the vertices in the cycle are dominated by the vertices $v_{(i+_5,1)}, v_{(i+_5,3)}$ and the isolated vertex $u_{(i+_5 2)}$ dominating itself. This implies the set $I=\{u_{(i+_5 2)}, v_{(i+_5,1)}, v_{(i+_5,3)}\}$ is the IDSof a Petersengraph $G(V,E)$.

Assume $I=\{u_{(i+_5 2)}, v_{(i+_5,1)}, v_{(i+_5,3)}\}$ is not a minimal IDSof Petersengraph.There exist a set $I'=(I-\{x\})$ is an IDSof aPetersengraph.. If x belongs to the cycle in $<V-D>$ there exist a vertex in the cycleis not dominated. If $x=u_{(i+_5 2)}$ is an obvious case.Hence $I=\{u_{(i+_5 2)}, v_{(i+_5,1)}, v_{(i+_5,3)}\}$ is a minimal IDSof Petersengraph.

Illustration:

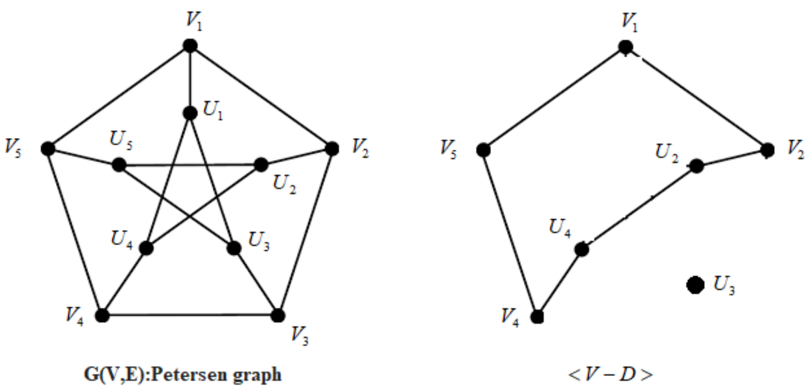
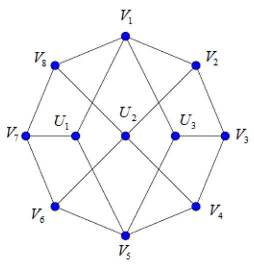


Figure 2.5

The graph $G(V,E)$ is the Petersengraph, the minimal dominating set is $D=\{U_1,U_5,V_3\}$ and minimal IDSis $I=\{u_2,v_1,v_3\}$ ID number of Petersengraph is $\gamma_{ID}(G)=3$.

Herschel graph: The Herschel graph is the smallest non Hamiltonian polyhedral graph. It is the unique such graph on 11 nodes and 18 edges.



$G(V,E)$:**Herschel graph**

Remarks:

1. Herschel graph consist outer and inner region with 11 and 3 vertices respectively.
2. The outer region form aOctagon and inner vertices not adjacent with each other vertices.

Theorem 2.6 Let $G(V,E)$ is a Herschelgraph. Then the minimal IDS $S=\{u_i,u_j,u_k,v_i,v_{(i+_8 4)}\}$,where u_i,u_j,u_k are the vertices inside the octagon and non-adjacent vertices and $v_i,v_{(i+_8 4)}$ are vertices on the octagon.

Proof: Let $G(V,E)$ is a Herschelgraph and, $D=\{u_i,u_j,u_k\}$ where u_i,u_j,u_k are the vertices inside the octagon and non-adjacent vertices. The vertices in D are adjacent to the vertices in octagon. Therefore $D=\{u_i,u_j,u_k\}$ is a dominating set of Herschelgraph $G(V,E)$. The set $(D-\{\mathrm{U}_i\})$ is not dominated a vertex Herschelgraph $G(V,E)$. Since the vertex V_i is only adjacent to U_i. Therefore $D=\{u_i,u_j,u_k\}$ is a minimal dominating set of Herschelgraph $G(V,E)$.The sub graph $<V-D>$, forms a cycle with 8 vertices. Let $I=\{v_i,v_{(i+_8 3)},v_{(i+_8 6)}\}$is the dominating set of the subgraph $<V-D>$, since sub graph $<V-D>$, forms a cycle with 8 vertices.This implies $I=\{v_i,v_{(i+_8 3)},v_{(i+_8 6)}\}$ is anIDSof Herschelgraph.

Assume $I=\{v_i,v_{(i+_8 3)},v_{(i+_8 6)}\}$is not a minimal IDSof Herschelgraph.There exist a set $I'=(I-\{x\})$is IDSof Herschelgraph. If x belongs to the cycle in $<V-D>$there exist a vertex in the cycle of 8 vertices is not dominated. This is contradict to our assumption $I'=(I-\{x\})$is inverse dominating set of Herschelgraph.Hence $I=\{v_i,v_{(i+_8 3)},v_{(i+_8 6)}\}$is a minimal IDSof Herschelgraph.

Illustration:

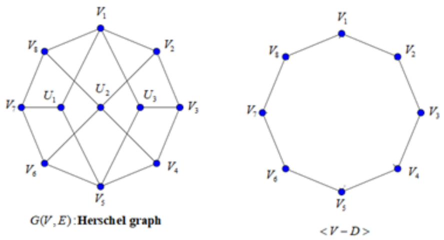
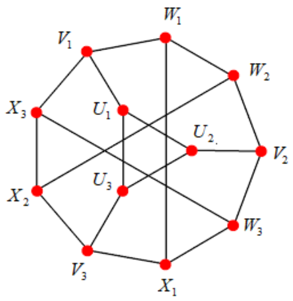


Figure 2.6

The graph $G(V,E)$ is the Herschelgraph, the minimal dominating set $D=\{\mathrm{U}_1,\mathrm{U}_2,\mathrm{U}_3\}$, the minimal IDSis $I=\{v_1,v_4,v_7\}$and ID number of Herschelgraph is $\gamma_{ID}(G)=3$.

Tietze's graph: Tietze's graph $G(V,E)$ is an undirected cubic graph with 12 vertices and 18 edges.



$G(V,E)$: Tietze's graph

Remarks:

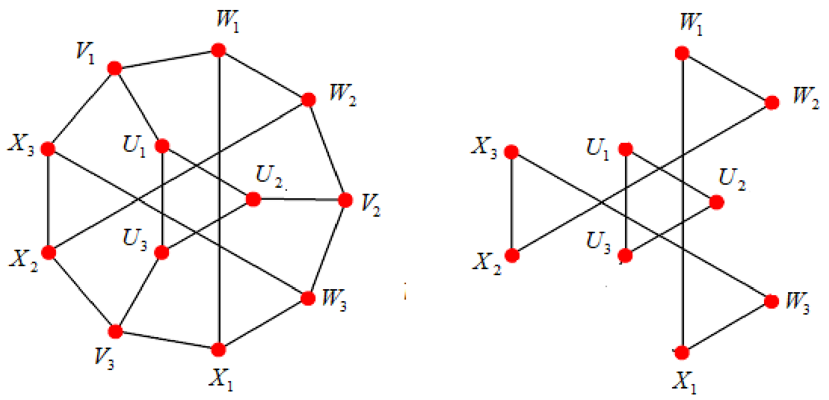
1. Tietze'sgraph consist outer and inner region with 9 vertices and 3 vertices respectively. The outer region form a nonagon and inner region forms a triangle.
2. The 3 vertices of the triangle adjacent to only one vertices of the nonagon.

Theorem 2.7 Let $G(V,E)$ is a Tietze'sgraph. Then the minimal IDS $I=\{W_2,W_4,U_i\}$, i = 1,2,3 and U_i 's are vertices of the inner triangle.

Proof: Let $G(V,E)$ is a Tietze'sgraph and, $D=\{V_i,V_j,V_k\}$ is a minimal dominating set of Tietze'sgraph . The vertices U_i, W_i & X_i are dominated by the vertex V_i. Similarly The vertices sets $\{U_j, W_j$ & $X_j\}$ and $\{U_k, W_k$ & $X_k\}$ are dominated by the vertex V_j & V_k respectively. Therfore $D=\{V_i,V_j,V_k\}$ is a dominating set of Tietze'sgraph. The set $(D-\{V_i\})$ is not dominated a vertex Tietze'sgraph $G(V,E)$ .Since the vertex $N(U_i)=\{V_i\}$, $N(U_j)=\{V_j\}$ and $N(U_k)=\{V_k\}$. Therefore $D=\{V_i,V_j,V_k\}$ is a minimal dominating set of Tietze'sgraph $G(V,E)$ .Let $I=\{W_2,W_4,U_i\}$, i = 1,2,3 and U_i 's are vertices of the inner triangle is an IDSof Tietze'sgraph $G(V,E)$. Since the sub graph $<V-D>$, contains 2cycle, first one the inner trinangle andthe 2nd cycle consist of 6 vertices $\{X_1W_1W_2X_2X_3W_3\}$. This implies $I=\{W_2,W_4,U_i\}$ is anIDSof Tietze'sgraph .

Assume $I=\{W_2,W_4,U_i\}$ is not a minimal IDSof Tietze'sgraph.There exist a set $I'=(I-\{x\})$ is an IDSof Tietze'sgraph.. If x belongs cycle consist of 6 vertices $\{X_1W_1W_2X_2X_3W_3\}$ there exist a vertex in the cycle $\{X_1W_1W_2X_2X_3W_3\}$ is not dominated by $I'=(I-\{x\})$. If x belongs trinangle there exist a vertex in the cycle trinangle is not dominated by $I'=(I-\{x\})$ This is contradict to our assumtion. Hence $I=\{W_2,W_4,U_i\}$ is a minimal IDSof Tietze'sgraph.

Illustration:



$G(V,E):Tietze's\ graph$ $\quad\quad$ $<V-D>$

Figure 2.7

The graph $G(V,E)$ is the Tietze'sgraph, the minimal dominating set $D=\{V_1,V_2,V_3\}$ and $I=\{W_2,W_4,U_1\}$ the minimal IDS of $G(V,E)$. TheID number of Tietze'sgraph is $\gamma_{ID}(G)=3$.

Conclusion:

The ideaof inverse domination has been extendedto inverse split dominating set and connected dominating set in special graphs. Theinverse split dominating number and connected dominating numberof various special graphs like Bidiakis cube,Durer graph,Golomb graph and etc. have also been discussed.

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