# OPTIMIZING HEXAGONAL FUZZY NUMBER EOQ MODEL USING NEAREST INTERVAL APPROXIMATION METHOD

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## ABSTRACT

In this paper discussed about an EOQ model with shortage. Consider the parameters holding cost, ordering cost, shortage cost, and demand. In real life situation the parameters are not always fixed. So we meet an uncertainty. To face these uncertainties we approach fuzzy parameters. The parameters holding cost, ordering cost, shortage cost and demand are fuzzy numbers. The fuzzy numbers are modified into an interval numbers by nearest interval approximation method. In order to minimize the interval objective function. So we changed to the multi objective function. Then the multi objective function solved by fuzzy optimization technique. To determine the optimum value which is the minimize total cost. Finally, numerical examples are discussed with the proposed model.

Keywords : Hexagonal fuzzy number, Fuzzy interval numbers, Nearest interval approximation method

## **INTRODUCTION**

Operation research is a scientific approach to problem solving for executive decision making which requires the formulation of mathematical, economic and statistical models for decision and control problems to deal with situations arising out of risk and uncertainty. The concept of operations research arose during World War II by military planners. After the war, the techniques used in their operations research were applied to addressing problems in business, the government and society.

Inventory is essential to provide flexibility in operating a system or organization. Inventory model was first developed by Harris in 1915.it is to any kind of resources that has economic value and it is maintained to fulfil the present and future needs of an organization. An inventory can be classified into raw materials inventory, work in process inventory, and finished goods inventory.

Fuzzy set theory considered as a tool to determine the approximate solutions of real problems in an efficient or affordable way. The concept fuzzy was introduced by zadeh in1965. It provides machinery for carrying out approximate reasoning processes when available information is uncertain, incomplete, imprecise or vague. It is the method of representing an uncertainty. Fuzzy rule systems have found a wide range of application in many fields of science and technology. For example, in artificial intelligence, computer

science, control engineering, decision theory, expert systems, logic, management science, operation research, pattern recognition and robotics.

Real situations are very often not crisp are deterministic. And they cannot be described precisely. In inventory control system it may allow some flexibility in the cost parameter values in order to treat the uncertainties which is always fit the real situations. Since we want to satisfy our requirements for such contradictions, the fuzzy set theory meets these requirements to some extent.

Hence, discussed about an inventory model with shortage in hexagonal fuzzy number using nearest interval approximation method. The parameters holding cost, ordering cost, shortage cost and demand are hexagonal fuzzy number. Nearest interval approximation method defuzzify the cost parameters into intervals. To minimize the interval objective functions are changed into multi objective function which is solved by fuzzy optimization techniques and found the optimal values. The proposed method illustrated with numerical examples.

#### DEFINITION

An Ordered pair of real numbers is defined by an **interval numbers** as  $B=[b_L, b_R]=\{x: b_L \le x \le b_R, x \in R\}$ . Where  $b_L$  and  $b_R$  are the interval numbers B of the left and right boundaries' respectively. Then the interval B with a center  $b_C$  and half width  $b_W$  are defined as

$$B=[b_C, b_W] = \{x: b_C - b_W \le x \le b_C + b_W\}.$$

Where  $b_L = \frac{b_L + b_R}{2}$  and  $b_R = \frac{b_R - b_L}{2}$ 

A General non linear objective function are defined as interval numbers

Minimize 
$$\tilde{B}(\mathbf{x}) = \frac{\sum_{i=1}^{n} [c_{L_{i'}} c_{R_i}] \prod_{j=1}^{k} y_j^{q_i}}{\sum_{i=1}^{n} [d_{L_{i'}} d_{R_i}] \prod_{j=1}^{k} y_j^{p_j}} \to (1)$$

Subject  $y_j \ge 0$ , j=1,2,...,n and  $x \in S$  subset of R.

 $c_{L_i}, c_{R_i}$  and  $d_{L_i}, d_{R_i}$  are left and right boundaries of the interval numbers. And  $y_j$  is a variable. And y has a feasible region S,  $0 < c_{L_i} < c_{R_i}$ ,  $0 < d_{L_i} < d_{R_i}$  and  $q_i$  and  $p_j$  are non negative numbers.

A multi objective non linear problem are formulated as follows,

Now, 
$$\tilde{B}(\mathbf{x}) = [B_L(\mathbf{x}), B_R(\mathbf{x})]$$

Where

$$\widetilde{B_{R}}(\mathbf{x}) = \frac{\sum_{i=1}^{n} [c_{R_{i}}] \prod_{j=1}^{k} y_{j}^{r_{j}}}{\sum_{i=1}^{n} [c_{R_{i}}] \prod_{j=1}^{k} y_{j}^{q_{i}}}$$

 $\widetilde{B}_{L}(\mathbf{x}) = \frac{\sum_{i=1}^{n} \left[ c_{L_{i}} \right] \prod_{j=1}^{k} y_{j}^{q_{i}}}{\sum_{i=1}^{n} \left[ c_{L_{i}} \right] \prod_{j=1}^{k} y_{j}^{q_{j}}}$ 

And then the objective function of mid point is,

 $\widetilde{z_{C}}(\mathbf{x}) = \frac{1}{2} [z_{L}(\mathbf{x}), z_{R}(\mathbf{x})]$ 

Hence, the problem Minimize  $\tilde{z}(x)$  can be converted into Minimize  $\{[z_L(x), z_R(x)], x \in S\}$ 

Nearest interval approximation used to estimate a fuzzy number by a crisp model. Let  $\check{N}$  and  $\acute{M}$  be two fuzzy number and  $\check{N}_L(\alpha), \check{N}_R(\alpha), \acute{M}_R(\alpha)$  are the  $\alpha$ -cuts of the fuzzy numbers  $\check{N}, \acute{M}$  respectively. Then

$$d(\widetilde{N},\widetilde{M}) = \sqrt{\int_{0}^{1} \left(\widetilde{N}_{L}(\alpha) - \widetilde{M}_{L}(\alpha)\right)^{2} + \int_{0}^{1} \left(\widetilde{N}_{R}(\alpha) - \widetilde{M}_{R}(\alpha)\right)^{2} d\alpha}$$

We need to find a closed interval in  $C_I(\widetilde{N})$  a given fuzzy number  $\check{N}$ , and since every interval is a fuzzy numbers with constant for all  $\alpha \in [0,1]$ .

$$C_I(N) = [C_L, C_R]$$

To minimize 
$$d\left(\widetilde{N}, C_{I}(\widetilde{N})\right) = \sqrt{\int_{0}^{1} \left(\widetilde{N}_{L}(\alpha) - \widetilde{M}_{L}(\alpha)\right)^{2} + \int_{0}^{1} \left(\widetilde{N}_{R}(\alpha) - \widetilde{M}_{R}(\alpha)\right)^{2} d\alpha}$$
  
With respect to  $C_{L}, C_{R}$ 

In minimizing order  $d(\tilde{N}, C_I(\tilde{N}))$  minimizing the function is sufficient

$$D(C_L, C_R) = d^2 \left( \widetilde{N}, C_I(\widetilde{N}) \right)$$

The first order partial derivative is

$$\frac{\partial}{\partial C_L} (D(C_L, C_R)) = -2 \int_0^1 \widetilde{N}_L(\alpha) d\alpha + 2C_L$$
$$\frac{\partial}{\partial C_R} (D(C_L, C_R)) = -2 \int_0^1 \widetilde{N}_R(\alpha) d\alpha + 2C_R$$
To solve
$$\frac{\partial}{\partial C_L} (D(C_L, C_R)) = 0 \to C_L = \int_0^1 \widetilde{N}_L(\alpha) d\alpha$$

$$\frac{\partial}{\partial C_R} (\mathcal{D} (C_L, C_R)) = 0 \to C_R = \int_0^1 \widetilde{N}_R (\alpha) d\alpha$$

Since 
$$\frac{\partial^2}{\partial C_L}$$
 (D ( $C_L, C_R$ ))  $\ge 0$ ,  $\frac{\partial^2}{\partial C_R}$  (D ( $C_L, C_R$ ))  $\ge 0$ 

That is,  $d\left(\widetilde{N}, C_{I}(\widetilde{N})\right)$  is minimum. Hence the fuzzy interval is,  $C_{I}(\widetilde{N})$ =  $\left\{\int_{0}^{1}\widetilde{N}_{L}(\alpha)d\alpha, \int_{0}^{1}\widetilde{N}_{R}(\alpha)d\alpha\right\}$  which is the fuzzy number of the nearest interval approximation method.

Let us take a arbitrary triangular fuzzy number  $B = (b_1, b_2, b_3)$  with the  $\alpha$ -cuts  $[B_L(\alpha), B_R(\alpha)]$  and the membership functions are defined as,

$$\mu_{\tilde{B}}(x) = \begin{cases} \frac{x - b_1}{b_2 - b_1} & : b_1 \le x < b_2 \\ \frac{b_3 - x}{b_3 - b_2} & : b_2 \le x \le b_3 \\ 0 & : otherwise \end{cases}$$

To find a lower  $C_L$  and upper  $C_R$  limit of the interval using nearest interval approximation method as follows,

$$C_{L} = \int_{0}^{1} B_{L}(\alpha) d\alpha = \int_{0}^{1} [b_{1} + (b_{2} - b_{1})\alpha] d\alpha = \frac{b_{1} + b_{2}}{2}$$

$$C_{R} = \int_{0}^{1} B_{R}(\alpha) d\alpha = \int_{0}^{1} [b_{3} - (b_{3} - b_{2})\alpha] d\alpha = \frac{b_{2} + b_{3}}{2}$$
[h+h-h+h]

Finally, the interval number of the triangular fuzzy number is  $\left[\frac{b_1+b_2}{2}, \frac{b_2+b_3}{2}\right]$ 

## **Model formulation**

In this model an inventory of shortages is considered. This EOQ model attempts to calculate the optimal order quantity of the product item by reducing the total average cost.

# **NOTATIONS:**

 $C_{B1} \rightarrow$  Price for carrying per unit time per quantity

 $C_{B2} \rightarrow Cost$  of shortages per unit time by quantity

 $C_{B3} \rightarrow Cost of setup per time$ 

 $D_B \rightarrow$  Total number of units generated by period of time.

 $Q_{B1} \rightarrow$ The volume that will go into the inventory

 $Q_{B2} \rightarrow$  The need remains unfulfilled

 $Q_B \rightarrow$  In each production run the lot size.

 $C_{B1L} \rightarrow$  left limit of the holding cost

 $C_{B1R} \rightarrow right limit of the holding cost$ 

 $C_{B2L} \rightarrow left limit of the shortage cost$ 

 $C_{B2R} \rightarrow right limit of the shortage cost$ 

 $C_{B3L} \rightarrow$  left limit of the ordering cost

 $C_{B3R} \rightarrow right limit of the ordering cost$ 

 $h_L(\mathbf{Q}) \rightarrow$  left limit of the interval objective function

 $h_{\mathcal{C}}(\mathbf{Q}) \rightarrow$  centre or half width limit of the interval objective function

 $h_L(\mathbf{Q}) \rightarrow$  right limit of the interval objective function

# **ASSUMPTIONS:**

- $\checkmark$  Zero is the Lead time
- ✓ Possible shortages
- ✓ Demand is well known and stable
- ✓ Commodity development or delivery is instantaneous

Let Q be the quantity of stock for the item at the time (t=0)in the period(0,t(t<sub>1</sub>+t<sub>2</sub>) the inventory level decreases slowly to meet the requirements. Continuing this process the inventory level reaches zero at the time t<sub>1</sub> and then shortages will occur at the interval (t<sub>1</sub>,t).the cycle itself repeats. To minimize the average total cost per unit time for which order level Q>0 is given by Min  $C_B(Q)=\frac{1}{2}C_{B1}\left(\frac{Q_{B1}^2}{Q_B}\right)+\frac{1}{2}C_{B2}\left(\frac{Q_{B2}^2}{Q_B}\right)+C_{B3}\left(\frac{D_B}{Q_B}\right)$ 

Up to this point, we are considering that the demand, ordering costs, holding costs, etc as real number is of fixed. But all these components are not always fixed in real-life

business situations; rather they are different in different situations. To address this uncertainty we approach with fuzzy variables where demand and other cost components are treated as triangular fuzzy numbers.

Now let us take a fuzzy demand  $\check{D}_B = (D_B - \alpha, D_B, D_B + \beta)$ fuzzy holding cost $\check{C}_{B1} = (C_{B1} - \alpha, C_{B1}, C_{B1} + \beta)$ 

fuzzy shortage cost  $\check{C}_{B2}=(C_{B2}-\alpha, C_{B2}, C_{B2}+\beta)$ 

fuzzy ordering cost
$$\check{C}_{B3}$$
=( $C_{B3}$ - $\alpha$ ,  $C_{B3}$ ,  $C_{B3}$ + $\beta$ ).

Real valued variables  $D_B, C_{B1}, C_{B2}, C_{B3}$  are replaced by the triangular fuzzy variables  $\check{D}_B, \check{C}_{B1}, \check{C}_{B2}, \check{C}_{B3}$  we get the equation

$$\check{\mathbf{C}}_{\mathrm{B}}(\mathbf{Q}) = \frac{1}{2} \check{\mathbf{C}}_{B1} \left( \frac{Q_{B1}^2}{Q_B} \right) + \frac{1}{2} \check{\mathbf{C}}_{B2} \left( \frac{Q_{B2}^2}{Q_B} \right) + \check{\mathbf{C}}_{B3} \left( \frac{D_B}{Q_B} \right)$$

Using the nearest interval approximation method the fuzzy numbers are converted into interval numbers as,  $\check{D}_B = (D_B - \alpha, D_B, D_B + \beta) = [D_{BL}, D_{BR}]$ 

$$\begin{split} \check{C}_{B1} &= (C_{B1} - \alpha, C_{B1}, C_{B1} + \beta) = [C_{B1L}, C_{B1R}] \\ \check{C}_{B2} &= (C_{B2} - \alpha, C_{B2}, C_{B2} + \beta) = [C_{B2L}, C_{B2R}] \\ \check{C}_{B3} &= (C_{B3} - \alpha, C_{B3}, C_{B3} + \beta) = [C_{B3L}, C_{B3R}] \\ The expression \check{C}_{B}(Q) \text{ becomes} \\ \check{C}_{B}(Q) &= [h_{L}, h_{R}], \text{ where } h_{L} &= \frac{1}{2}C_{B1L} \left(\frac{Q_{B1}^{2}}{Q_{B}}\right) + \frac{1}{2}C_{B2L} \left(\frac{Q_{B2}^{2}}{Q_{B}}\right) + C_{B3L} \left(\frac{D_{BL}}{Q_{B}}\right) \\ &h_{R} &= \frac{1}{2}C_{B1R} \left(\frac{Q_{B1}^{2}}{Q_{B}}\right) + \frac{1}{2}C_{B2R} \left(\frac{Q_{B2}^{2}}{Q_{B}}\right) + C_{B3R} \left(\frac{D_{BR}}{Q_{B}}\right) \end{split}$$

In these equations we used composition rules of intervals

Hence, minimize  $\{h_L(Q), h_R(Q)\},\$ 

In case of minimization problem the multi objective optimization problem can be formulated from (3) as,

Minimize {  $h_L(Q), h_R(Q)$  },

Subject to Q $\ge 0$ , where  $h_C = \frac{h_L + h_R}{2}$ 

Representation of interval valued problem is

Minimize  $\{h_L(Q), h_C(Q), h_R(Q)\},\$ 

Subject to  $Q \ge 0$ 

In the this above expression gives a best approximate value.

# FUZZY PROGRAMMING TECHNIQUE FOR SOLUTION

We used the following fuzzy programming methodology to solve multi-objective minimization problems. we first consider the lower boundaries  $L_L$ ,  $L_C$ ,  $L_R$  and the upper boundaries  $U_L$ ,  $U_C$ ,  $U_R$ , For each of the objective functions  $h_L(Q)$ ,  $h_C(Q)$ ,  $h_R(Q)$ , where  $L_L$ ,  $L_C$ ,  $L_R$  is the aspired level achievement and  $U_L$ ,  $U_C$ ,  $U_R$ , is the maximum appropriate level achievement for Objectives  $h_L(Q)$ ,  $h_C(Q)$ ,  $h_R(Q)$  respectively and  $d_b=U_b-L_b$  is the degradation allowance for Objective  $h_b(Q)$ , b=L,C,R.

Once the aspiration rates and degradation allowance have been defined for each of the objective functions, we developed a fuzzy model and then transformed the fuzzy model into a crisp model.

# Stride 1:

Resolve multi-objective cost function as a single objective cost function using one target at a time and ignoring all others.

# Stride 2:

We obtain a result from stride 1, then we have to find the corresponding values for each objective at each solution derived.

## Stride 3:

From the previous stride 2, We find the best  $L_b$  and worst  $U_b$  value for every objective the corresponding to the set of solutions. The initial fuzzy model of (EOQ) can be described as follows in terms of the aspiration rates for each objective. Then to find the value Q which satisfying the inequality  $h_b \leq L_b, b=L, C, R$  subject to the non negative restriction

# Stride 4:

For each membership function we define a fuzzy linear membership function  $\mu_{h_h}$ ; b=L,C,R.

$$\mu_{h_b} = \begin{cases} 1 & : h_b \le L_b \\ \frac{h_b - L_b}{d_b} & : L_b \le h_b \le U_b \\ 0 & : h_b \ge U_b \end{cases}$$

## Stride 5:

For each objective functions we determine the linear membership function after that the above problem can be formulated as a crisp model

Max  $\alpha$ 

 $\alpha \leq \mu_{h_b}$ ; b= L,C,R

# **HEXAGONAL FUZZY NUMBERS:**

A fuzzy number on  $\tilde{B}_h$  is a hexagonal fuzzy number denoted by  $\tilde{B}_h = (b_1, b_2, b, b, b, b_6)$  where  $(b_1 \le b_2 \le b_3 \le b_4 \le b_5 \le b_6)$  are real numbers satisfying  $b_2 - b_1 \le b_3 - b_2$  and  $b_5 - b_4 \le b_6 - b_5$  and its membership function  $\mu_{\tilde{B}}(x)$  is given as:

$$\mu_{\bar{B}}(x) = \begin{cases} 0 & ; x < b_1 \\ \frac{1}{2} \left[ \frac{x - b_1}{b_2 - b_1} \right] & ; b_1 \le x \le b_2 \\ \frac{1}{2} + \frac{1}{2} \left[ \frac{x - b_2}{b_3 - b_2} \right] & ; b_2 \le x \le b_3 \\ 1 & ; b_3 \le x \le b_4 \\ 1 - \frac{1}{2} \left[ \frac{x - b_4}{b_5 - b_4} \right] & ; b_4 \le x \le b_5 \\ \frac{1}{2} \left[ \frac{b_6 - x}{b_6 - b_5} \right] & ; b_5 \le x \le b_6 \\ 0 & ; x > b_6 \end{cases}$$

# **REMARK:**

A hexagonal fuzzy numbers can be defined as  $\tilde{A}_h = (D_1(u), S_1(v), S_2(v), D_2(u))$  for  $u \in [0,0.5]$  and v = [0.5,1], where

- $D_1(u)$  is a bounded left continuous non-decreasing function over [0,0.5].
- $S_1(v)$  is a bounded left continuous non-decreasing function over [0.5,1].
- $S_2(v)$  is a bounded continuous non-increasing function over [1,0.5].
- $D_2(u)$  is a bounded left continuous non-increasing function over [0.5,0].

# FUZZY MODEL FOR THE HEXAGONAL FUZZY NUMBER

The lower upper limit of the interval is then approximated by the nearest interval approximation method for each case of hexagonal fuzzy number

$$C_{BL} = \int_{0}^{1} B_{L}(\alpha) \, d\alpha \qquad C_{BR} = \int_{0}^{1} B_{R}(\alpha) \, d\alpha$$
  

$$C_{BL} = \int_{0}^{1} B_{L}(\alpha) \, d\alpha = \int_{0}^{1} b_{3} + (b_{2} - b_{3}) \alpha \, d\alpha = \frac{b_{3} + b_{2}}{2}$$
  

$$C_{BR} = \int_{0}^{1} B_{R}(\alpha) \, d\alpha = \int_{0}^{1} b_{5} + (b_{4} - b_{5}) \alpha \, d\alpha = \frac{b_{4} + b_{5}}{2}$$
  

$$C_{BL} = \frac{b_{3} + b_{2}}{2} \qquad C_{BR} = \frac{b_{4} + b_{5}}{2}$$
  

$$C_{BL} = \int_{0}^{1} B_{L}(\alpha) \, d\alpha = \int_{0}^{1} b_{1} + (b_{2} - b_{1}) \alpha \, d\alpha = \frac{b_{1} + b_{2}}{2}$$
  

$$C_{BR} = \int_{0}^{1} B_{R}(\alpha) \, d\alpha = \int_{0}^{1} b_{6} + (b_{5} - b_{6}) \alpha \, d\alpha = \frac{b_{5} + b_{6}}{2}$$
  

$$C_{BL} = \frac{b_{1} + b_{2}}{2} \qquad C_{BR} = \frac{b_{5} + b_{6}}{2}$$
  
Here, Consider two cases,

Case I,	$C_{BL} = \frac{b_3 + b_2}{2}$	:	$C_{BR} = \frac{b_4 + b_5}{2}$
Case II,	$C_{BL} = \frac{b_1 + b_2}{2}$	:	$C_{BR} = \frac{b_5 + b_6}{2}$

# NUMERICAL EXAMPLES:

In this numerical example the parameters demand, holding cost, ordering cost, shortage cost are considered as hexagonal fuzzy numbers.

 $C_{B1} = (3.5, 4, 4.5, 5.5, 6, 6.5)$   $C_{B2} = (23.5, 24, 24.5, 25.5, 26, 26.5)$   $C_{B3} = (98.5, 99, 99.5, 101.5, 102, 102.5)$  $D_{B} = (2500, 3500, 4500, 5500, 6500, 7500)$ 

# CASE I

Hence, the each objective functions  $h_L$ ,  $h_C$ ,  $h_R$  of individual minimum and maximum are in the given table

Objective function	bjective function $h_L$		$h_R$
$h_L$	1694.486674	1708.587666	1698.511604

$h_{C}$	2050.7609		2049.671033	2046.338194
$h_R$	2412.37041		2395.659518	2399.338194
$L_{BL}$ =min ( $h_L$ ) = 1694.486674		UB	$h_L = \max(h_L) = 1708.5876$	566
$L_{BR} = \min(h_R) = 2395$	.659518	U <sub>BR</sub> = n	$\max(h_R) = 2412.37041$	

 $L_{BC} = \min(h_c) = 2046.338194$   $U_{BC} = \max(h_c) = 1000$ 

 $U_{BC} = \max(h_C) = 2050.7609$ 

From the above expression we construct the problem as;

Max  $\alpha$ 

$$h_{L} = \frac{1}{2} \times 4.25 \times \left(\frac{Q_{B1}^{2}}{Q_{B}}\right) + \frac{1}{2} \times 24.25 \times \left(\frac{Q_{B2}^{2}}{Q_{B}}\right) + 99.25 \left(\frac{4000}{Q_{B}}\right) + (14.100992) \alpha = 1708.587666$$

$$h_{R} = \frac{1}{2} \times 5.75 \times \left(\frac{Q_{B1}^{2}}{Q_{B}}\right) + \frac{1}{2} \times 25.75 \times \left(\frac{Q_{B2}^{2}}{Q_{B}}\right) + 101.75 \times \left(\frac{6000}{Q_{B}}\right) + (4.422706) \alpha = 2412.37041$$

$$h_{C} = \frac{1}{2} \times 5 \times \left(\frac{Q_{B1}^{2}}{Q_{B}}\right) + \frac{1}{2} \times 25 \times \left(\frac{Q_{B2}^{2}}{Q_{B}}\right) + 100.5 \times \left(\frac{5000}{Q_{B}}\right) + (16.710892) \alpha = 2050.7609$$

to solve this equations by using the non linear programming techniques and found the optimum value of  $\alpha$ ,  $\ddot{Q}_B$ ,  $\ddot{Q}_{B1}$ 

$Q_B$	$Q_{B1}$		
489.7904704	408.2022742		

Objective functions  $h_L$ ,  $h_C$ ,  $h_R$  which is the minimum total cost

$h_L$	$h_{C}$	$h_R$	
1698.274191	2046.345783	2399.521599	

## CASE II

Hence, the each objective functions  $h_L$ ,  $h_C$ ,  $h_R$  of individual minimum and maximum are in the given table

Objective function	$h_L$	$h_{C}$	$h_R$
$h_L$	1385.24079	1441.550413	1404.698174
h <sub>C</sub>	2071.806542	2058.293602	2046.338194
$h_R$	2774.738007	2688.181935	2702.231315

$L_{BL}=\min(h_L)=1385.24079$	$U_{BL}=max(h_L)=1441.550413$
$L_{BR} = \min(h_R) = 2688.181935$	$U_{BR} = \max(h_R) = 2774.738007$
$L_{BC} = \min(h_c) = 2046.338194$	$U_{BC} = \max(h_c) = 2071.806542$

From the above expression we construct the problem as;

Max  $\alpha$ 

$$h_{L} = \frac{1}{2} \times 3.75 \times \left(\frac{Q_{B1}^{2}}{Q_{B}}\right) + \frac{1}{2} \times 23.75 \times \left(\frac{Q_{B2}^{2}}{Q_{B}}\right) + 98.755 \times \left(\frac{3000}{Q_{B}}\right) + (56.309623) \alpha = 1441.550413$$
$$h_{R} = \frac{1}{2} \times 6.25 \times \left(\frac{Q_{B1}^{2}}{Q_{B}}\right) + \frac{1}{2} \times 26.25 \times \left(\frac{Q_{B2}^{2}}{Q_{B}}\right) + 102.25 \times \left(\frac{7000}{Q_{B}}\right) + (25.468348) \alpha = 2774.738007$$
$$h_{C} = \frac{1}{2} \times 5 \times \left(\frac{Q_{B1}^{2}}{Q_{B}}\right) + \frac{1}{2} \times 25 \times \left(\frac{Q_{B2}^{2}}{Q_{B}}\right) + 100.5 \times \left(\frac{5000}{Q_{B}}\right) + (86.556072) \alpha = 2071.806542$$

to solve this equation using non linear programming techniques and found the optimum value of  $\alpha$ 

value of  $\alpha$ ,  $\ddot{Q}_B$ ,  $\ddot{Q}_{B1}$ 

$Q_B$	$Q_{B1}$		
483.7868929	402.924764		

Objective functions  $h_L$ ,  $h_C$ ,  $h_R$  which is the minimum total cost

$h_L$	$h_{C}$	$h_R$
1402.063732	2046.57149	2705.54843

# **SENSITIVITY ANALYSIS:**

Cases	c <sub>B1</sub>	c <sub>B2</sub>	c <sub>B3</sub>	Q	$\ddot{\mathbf{Q}_1}$	C(Ö)
Crisp	5	25	100	489.898	408.248	2041.241452
Fuzzy	[4,6]	[23,28]	[94,106]	[491.050]	[411.142]	[1699.432,2402.011]
Fuzzy	[4.25,5.75]	[24.25,25.75]	[99.25,101.75]	[489.790]	[408.202]	[1698.274,2399.523]

Case I						
Case II	[3.75,6.25]	[23.75,26.25]	[98.25,102.25]	[483.787]	[402.925]	[1402.064,2705.548]

## CONCLUSION

In this project, we have applied the method of nearest interval approximation for solving a fuzzy inventory model with shortage using hexagonal fuzzy number. Here demand, holding cost, ordering cost, shortage cost are hexagonal fuzzy number. Consider two cases for this hexagonal fuzzy number using nearest interval approximation method. The fuzzy costs are converted into intervals. In order to minimizing the Interval objective function are converted into multi objective function which is solved by the fuzzy optimization techniques and found the optimum value.

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