# An Approach for Solving FuzzyTransportation Problemusing Ranking function

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***Abstract:***

***In this paper we shall study fuzzy transportation problem, and we introduce an approach for solving a wide range of such problem by using a method which apply it for ranking of the fuzzy numbers. Some of the quantities in a fuzzy transportation problem may be fuzzy or crisp quantities In many fuzzy decision problems, the quantities are represented in terms of fuzzy numbers Fuzzy numbers may , triangular or trapezoidal or any LR fuzzy number. Thus,some fuzzy numbers are not directly comparable. First, we transform the fuzzy quantities as the cost, coefficients, supply and demands, in to crisp quantities by using our method and then by using the classical algorithms we solve and obtain the solution of the problem. The new method is a systematic procedure, easy to apply and can be utilized for all types of transportation problem whether maximize or minimize objective function.At the end, this method is illustrated with a numerical example.***

***Keywords: Optimization, Transportation problem, ranking of fuzzy numbers***

## 1. Introduction:

The theory of fuzzy set introduced by Zadeh in 1965 has achieved successful applications in various fields. The concept of fuzzy mathematical programming was introduced by Tanaka et al in 1947 the frame work of fuzzy decision of Bellman and Zadeh. Transportation models have wide applications in logistics and supply chain for reducing the cost. When the cost coefficients and the supply and demand quantities are known exactly, many algorithms have been developed for solving the transportation problem. But, in the real world, there are many cases that the cost coefficients, and the supply and demand quantities
are fuzzy quantities

A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. Most of the existing techniques provide only crisp solutions for the fuzzy transportation problem. Shiang-Tai Liu and Chiang Kao Chanas et al Chanas and Kuchta, proposed a method for solving fuzzy transportation problem. In many fuzzy decision problems, the data are represented in terms of fuzzy numbers. In a fuzzy transportation problem, all parameters are fuzzy numbers. Fuzzy numbers may be triangular or trapezoidal.

## 2. Preliminaries:

**2.1 Definition:** A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0,1]. (i,e) A = {(x, $\mu_A(x)$ ; x Є X}, Here $\mu_A$: X →[0,1] is a mapping called the degree of membership function of the fuzzy set A and

$\mu_A(x)$ is called the membership value of x ε X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

**2.2 Definition :** A fuzzy set A of the universe of discourse X is called a *normal* fuzzy set implying that there exist at least one x ε X such that $\mu_A(x) = 1$.

**2.3 Definition:** The fuzzy set A is *convex* if and only if, for any x1, x2 Є X, the membership function of A satisfiesthe inequality $\mu_A\{\lambda x_1+(1-\lambda)x_2\} \geq \min\{\mu_A(x1), \mu_A(x2)\}$. $0 \leq \lambda \leq 1$.

**2.4 Definition (*Triangular fuzzy number*) :** For a triangular fuzzy number A(x), it can be represented by A(a,b,c ; 1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} (x-a)/(b-a), & a \leq x \leq b \\ 1, & x=b \\ \mu(x) = (c-x)/(c-b), & c \leq x \leq d \\ 0, & \text{otherwise} \end{cases}$$

**2.5 Definition: (*Trapezoidal fuzzy number*):** For a trapezoidal number A(x), it can be represented by A(a,b,c,d;1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} (x-a)/(b-a), & a \leq x \leq b \\ 1, & b \leq x \leq c \\ (d-x)/(d-c), & c \leq x \leq d \\ 0, & \text{otherwise} \end{cases}$$

**2.6 Definition: (*α-cut of a trapezoidal fuzzy number*):** The α-cut of a fuzzy number A(x) is defined as

$A(\alpha) = \{ x : \mu(x) \geq \alpha, \alpha \in [0,1]\}$

Addition of two fuzzy numbers can be performed as $(a_1,b_1,c_1) + (a_2,b_2,c_2) = (a_1+a_2, b_1+b_2, c_1+c_2)$

Addition of two trapezoidal fuzzy numbers can be performed as

$(a_1,b_1,c_1,d_1)+(a_2,b_2,c_2,d_2) = (a_1+a_2,b_1+b_2,c_1+c_2,d_1+d2)$.

**Robust Ranking Technique:**

Roubast ranking technique which satisfy commen ratio , linearity, and additively properties and provides results which are consist human intuition. If ã is a fuzzy number then the Roubast Ranking is defined by

$R(\tilde{a}) = \int_0^1 0.5\ (a\alpha La\alpha U10) d\alpha$, where $a_\alpha{}^L a_\alpha{}^U$ is the $\alpha$ level cut of the fuzzy number ã In this paper we use this method for ranking the objective values. The Roubast ranking index R(ã) gives the representative value of fuzzy number ã.

## 3.Algorithm to solve fuzzy transportation problem using IVAM

Step 1:

Convert the fuzzy values in the transportation problem into crisp values using robust ranking technique.

Step 2:

Check whether the transportation problem is balanced. If not, introduce dummy row (or column) to balance the problem.

Step 3:

Find the row opportunity cost and column opportunity cost. To find therow opportunity cost, for each row the smallest cost of that row is subtracted from each element of the same row. To find the column opportunity cost for each column of the original transportation cost matrix the smallest cost of that column is subtracted from each element of the samecolumn.

Step 4:

Obtain the $R + C$matrix, by adding the low row opportunity cost with the column opportunity cost.

Step 5:

In the $R + C$matrix, determine the penalty cost for each row and column by finding the difference between the lowest cell cost in the row (or column) and the next lowest cell cost in the row (orcolumn).

Step 6:

Now from this select the maximum penalty cost and allocate the minimum of supply (or demand) to the minimum element of the row (or column). Delete the row (or column) in which the corresponding supply (or demand) is exhausted.

Step 7:

Repeat steps 5 and 6 until satisfaction of all the supply and demand met (i.e. both demand and supply will be exhausted).

Step 8:

Compute the total transportation cost for the feasible allocations using the original balance transportation cost matrix.

Numerical Example

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Example 3.1.1:

Consider the fuzzy transportation problem with destinations $D_1$, $D_2$, $D_{3,4}$ and sources $S_1$, $S_2$, $S_3$, $S_4$. And the cost of shipping one unit of the product from the $i^{th}$ source to the $j^{th}$ destination is given in the following table in the form of triangular fuzzy numbers. Now let us find the minimum transportation cost for this transportation problem.

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | (2,4,6) | (2,4,8) | (2,6,8) | (2,4,6) | (4,6,8) |
| $S_2$ | (2,4,8) | (2,6,8) | (2,6,4) | (4,6,8) | (2,4,6) |
| $S_3$ | (2,4,8) | (4,6,8) | (4,6,8) | (2,4,6) | (8,12,16) |
| $S_4$ | (4,6,10) | (2,4,6) | (4,8,12) | (4,6,10) | (6,8,10) |
| Demand | (10,12,14) | (4,6,8) | (2,6,8) | (4,6,10) | |

**Solution:**
**Step 1**:

Applying robust ranking technique to convert the fuzzy numbers into crisp numbers,
(i)(2,4,6)

$$( f^L, f^U) = [2 + (4 - 2), 6 - (6 - 4)]$$

$$\alpha\alpha$$

$$= (2 + 2\alpha, 6 - 2\alpha)$$

$$(a) = \int_0^1 0.5(0.8)d\alpha = 4$$

Proceeding similarly, the Robust's ranking indices for the fuzzy costs ãij are calculated as:
R(a12) = 4.5, R(a13) = 5.5, R(a14) = 4, R(a21) = 4.5,
R(a22) = 5.5, R(a23) = 4.5, R(a24) = 6, R(a31) = 4.5,
R(a32) = 6, R(a33) =6, R(a34) = 4, R(a41) = 6.5,
R(a42) = 4, R(a43) = 8, R(a44) = 6.5. we replace these
values for their corresponding ãij in the above table we get as follows

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 4 | 4.5 | 5.5 | 4 | 6 |
| $S_2$ | 4.5 | 5. | 4. | 6 | 4 |

| | | | | | |
|---|---|---|---|---|---|
| | | 5 | 5 | | |
| $S_3$ | 4.5 | 6 | 6 | 4 | 12 |
| $S_4$ | 6.5 | 4 | 8 | 6.5 | 8 |
| Demand | 12 | 6 | 5.5 | 6.5 | |

**Step 2:**

Sum of the supply = 6 + 4 + 12 + 8 = 30

Sum of the demand = 12 + 6 + 5.5 + 6.5 = 30

Thus the above transportation problem is balanced (or consistent) as $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

**Step 3:**

The following matrix is the row opportunity cost matrix and the column matrix found by subtracting the smallest cost of that row (or column) from each element of the same row (or column).

$$\text{Row opportunity cost matrix} = \begin{bmatrix} 0 & 0.5 & 1.5 & 0 \\ 0 & 1 & 0 & 1.5 \\ 0.5 & 2 & 2 & 0 \\ 2.5 & 0 & 4 & 2.5 \end{bmatrix}$$

$$\text{Column opportunity cost matrix} = \begin{bmatrix} 0 & 0.5 & 1 & 0 \\ 0.5 & 1.5 & 0 & 2 \\ 0.5 & 2 & 1.5 & 0 \\ 2 & 0 & 3.5 & 2.5 \end{bmatrix}$$

**Step 4:**

The $R + C$matrix is obtained by adding the row opportunity matrix with the column opportunity matrix.

R+C matrix = Row opportunity cost matrix + column opportunity cost matrix

$$= \begin{bmatrix} 0 & 1 & 2.5 & 0 \\ 0.5 & 2.5 & 0 & 3.5 \\ 1 & 4 & 3.5 & 0 \\ 4.5 & 0 & 7.5 & 5 \end{bmatrix}$$

**Step 5:**

In the table, the penalty cost for each row and column is obtained by finding the difference between the lowest cell cost in the row (or column) and the next lowest cell cost in the row (or column).

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| | $D_1$ | $D_2$ | $D_1$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | [4.5] 0 | 1 | [1. 5] 2.5 | 0 | 6 |
| $S_2$ | 0.5 | 2.5 | [4] 0 | 3.5 | 4 |
| $S_3$ | [5.5] 1 | 4 | 3.5 | [6.5] 0 | 12 |
| $S_4$ | [2] 4.5 | [6] 0 | 7.5 | 5 | 8 |
| Demand | 12 | 6 | 5.5 | 6.5 | |

Here the maximum penalty cost is 4.5 and the corresponding minimum cost is 0 present in the cell (4,2).

$$x_{42} = min(6,8) = 6$$

So the cell (4,2) gets allotted with the demand 6 and the supply reduces to $8 - 6 = 2$. And the column $D_2$ getsdeleted.

**Step 7:**

Repeating the above step, we find that the maximum penalty cost is 2.5 and the corresponding minimum cost is 0 in the cell (2,3).

$$x_{23} = min(4,5.5) = 4$$

So the supply 4 gets allotted in the cell (2,3) and the demand becomes $5.5 - 4 = 1.5$ and row $S_2$ gets deleted

Again finding the maximum penalty cost, we get 1 and the minimum cost as 2.5 in the cell (1,3).

$$x_{13} = min(1.5,6) = 1.5$$

Thus the column $D_3$ gets deleted and the supply becomes $6 - 1.5 = 4.5$

Repeating we get the maximum penalty cost as 1 and the minimum cost as o corresponding to the cell (1,1) .

$$x_{11} = min(4.5,12) = 4.5$$

So the supply in row $S_1$ gets exhausted and the supply reduces to $12 - 4.5 = 7.5$

Again finding the penalty cost, we get 5 and the minimum cost as 0 corresponding to the cell (3,4).

$$x_{34} = \min(12,6.5) = 6.5$$

So the demand in the row $D_4$ gets depleted and supply becomes $12 - 6.5 = 5.5$

The next allocation is made in the cell (3,1) to which the supply 5.5 gets allotted and the demand reduces to 7.5 − 5.5 = 2

$$x_{31} = min(7.5,5.5) = 5.5$$

The next allocation is made in the final cell (4,1) to which the supply/demand 2 gets allocated.

$$x_{41} = \min(2,2) = 2$$

Therefore we have the following allocations

$x_{11} = 4.5$, $x_{13} = 1.5$, $x_{23} = 4$, $x_{31} = 5.5$, $x_{34} = 6.5$, $x_{41} = 2$, $x_{42} = 6$

**Step 8:**

The total transportation cost is found using the original balanced transportation cost matrix.

$$\text{Transportation cost} = (4.5 \times 4) + (1.5 \times 5.5) + (4 \times 4.5) + (5.5 \times 4.5) + (6.5 \times 4) + (2 \times 6.5) + (6 \times 4)$$

$= 132$

**Conclusions:**

In this paper new approach is proposed for finding the IFBFS and the fuzzy optimal solution of fuzzy transportation problemsin which the transportation cost availability and demand of the product are represented as generalized trapezoidal fuzzynumbers. The advantages of the proposed methods are discussed and a numerical example is solved to illustrate theproposed methods. The proposed method is very easy to understand and to apply for solving the fuzzy transportation.

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