TRANSFER FUNCTION AND SOLUTION OF DIFFERENT MECHANICAL AND ELECTRICAL SYSTEM IN CONTROL ENGINEERING BY NEW INTEGRAL TRANSFORM

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Abstract - In order to find the time response of a Mechanical and electrical system it is necessary to form and solve the differential equations. By using Sadik Transform we convert time domain(or 't') to Sadik domain (or υ)then we solve equations in terms of υ which is very easy to solve , lastly we apply Inverse Sadik Transform to get solution in terms of t.

Keywords: The Sadik Transform, inverse Sadik Transform, Control Theory, Mechanical and Electrical System, Linear Differential Equations of different Order.

1. INTRODUCTION

Many problems from Mechanical and Electrical systems are Mathematical modules in terms of Linear Differential Equations and there are different techniques to solve differential equations, to solve linear differential equations the most powerful technique is an integral transformation method. Some linear differential equations have been solved by different integral transform like Laplace Transform, Sumudu transform, Natural transform, Elzaki transform, Aboodh transform, Kashuri and Fundo transform, ZZ transform. Laplace transform is the most effective tool to solve some kinds of ordinary and partial differential equations. Actually an electric engineer Oliver Heaviside made Laplace transform popular by developing its operational calculus. After Laplace transforms, in 1993 again an electrical engineer Watugula in proposed a new integral transform named the Sumudu transform and used it for solving problems in control engineering, it is similar to the Laplace transform having the preservation property of unit and change of scale. After that, T. Elzaki introduced a new integral transform named Elzaki transform and applied it for solving partial differential equations, Shaikh Sadikali has been applied Elzaki transform for solving integral equations of convolution type see in. Likewise many integral transforms have been proposed which are similar to the Laplace transform, and each new transform claimed its own superiority over the Laplace transform. Shaikh Sadikali presented solution of some linear partial differential by Sadik Transform. In this paper we considered a new integral transform named the Sadik transform. It is similar to the Laplace transform but the Laplace transform, the Sumudu transform, Elzaki transform and all integral transforms with kernel of an exponential type are particular cases of the Sadik transform. Due to the very general and unified nature of the Sadik transform, we can transport a problem of linear differential equations into the known transformation technique which is available in the literature through the Sadik transform.

2. PRELIMINARY

***Sadik transform*:**

If, 1) (t) piecewise continuous on the interval $0\leq t \leq A$ for any $A\geq 0$.

2) $|(t)| \le K$. When $t \ge M$, for any real constant a. and some positive constant K and M.

Then Sadik transform of $f(t)$ is defined by

$$F(\upsilon^{\alpha}, \beta) = S[f(t)] = \frac{1}{\upsilon^{\beta}}\int_0^{\infty} e^{-\upsilon^{\alpha} t} f(t)dt , \qquad for \ \text{Re}(\upsilon^{\alpha}) > w^{\alpha}$$

Where, υ is complex variable.

α is any non zero real numbers and β is any real number.

Method of applying Sadik transform to different derivatives is given as

"If $\text{S}(\upsilon)$ is Sadik Transform of $f(t)$ then, $S\left[f^{'}(t)\right] = \upsilon^{\alpha} S(\upsilon) - \upsilon^{-\beta} f(0)$

In general, Sadik Transform of nth derivative of $f(t)$ is $"S\left[f^{(n)}(t)\right] = \upsilon^{n\alpha} S(\upsilon) - \sum_{k=o}^{n-1} \upsilon^{k\alpha-\beta} f^{(n-1)-k}(0)"$

3. MAIN RESULT

Generally Mechanical and Electrical system Models has first, second and third order Differential Equations, where input function is $x_i(t)$ and output function is $x_o(t)$ Sadik Transform of input and output functions are represented as $S[x_i(t)] = S_i[\upsilon]$ and $S[x_o(t)] = S_o[\upsilon]$.

If dynamic system produces a first order differential equation with input $x_i(t)$ and output $x_o(t)$ with constant coefficient *a,b,c*...then it will be given as

$$a\frac{dx_o}{dt} + bx_o = Ax_i(t) \qquad \text{..... (1)}$$

Applying Sadik Transform we get,

$$a[\upsilon^{\alpha} S_o(\upsilon) - \upsilon^{-\beta} x_o(0)] + bS_o(\upsilon) = AS_i(\upsilon)$$

$$S_o(\upsilon) = \left[\frac{A}{(a\upsilon^{\alpha} + b)}\right] S_i(\upsilon) + \left[\frac{a\upsilon^{-\beta} x_o(0)}{(a\upsilon^{\alpha} + b)}\right] \qquad \text{..... (2)}$$

If Initial conditions are zero then,

Transfer function is given by,

$$G(\upsilon) = \frac{S_o(\upsilon)}{S_i(\upsilon)} = \frac{A}{(a\upsilon^{\alpha} + b)}$$

Therefore Output function in terms of υ is given by

$$S_o(\upsilon) = \left[\frac{A}{(a\upsilon^{\alpha} + b)}\right] S_i(\upsilon) \qquad \text{..... (3)}$$

By taking Inverse Sadik Transform of above equation we get output of the desired system $x_o(t)$.

If dynamic system produces a second order differential equation with input $x_i(t)$ and output $x_o(t)$ with constant coefficient *a,b,c*,...then it will be given as,

$$a\frac{d^2x_o}{dt^2}+b\frac{dx_o}{dt}+cx_o=Ax_i(t) \quad \text{….. (4)}$$

Applying Sadik Transform we ge,

$$a[\upsilon^{2\alpha}S_o(\upsilon)\ -\ \upsilon^{\alpha-\beta}\ x_o(0)-\upsilon^{-\beta}\ x_o^{'}(0)]+b[\upsilon^{\alpha}S_o(\upsilon)-\upsilon^{-\beta}\ x_o(0)]+\mathrm{c}\,S_o(\upsilon)=AS_i(\upsilon)$$

$$[a\upsilon^{2\alpha}+b\upsilon^{\alpha}+\mathrm{c}]S_o(\upsilon)\ -[a\upsilon^{\alpha}+b]\upsilon^{-\beta}x_o(0)-\ \mathrm{a}\upsilon^{-\beta}\ x_o^{'}(0)=AS_i(\upsilon)$$

$$S_o(\upsilon)=\frac{A}{[a\upsilon^{2\alpha}+b\upsilon^{\alpha}+\mathrm{c}]}S_i(\upsilon)\ -\frac{\upsilon^{-\beta}[(a\upsilon^{\alpha}+b)x_o(0)-\mathrm{a}x_o^{'}(0)]}{[a\upsilon^{2\alpha}+b\upsilon^{\alpha}+\mathrm{c}]} \quad \text{….. (5)}$$

If initial conditions are zero then Transfer function of input and output function is given by,

$$G(\upsilon)=\frac{S_o(\upsilon)}{S_i(\upsilon)}=\frac{A}{[a\upsilon^{2\alpha}+b\upsilon^{\alpha}+\mathrm{c}]} \quad \text{….. (6)}$$

By taking Inverse Sadik Transform of above equation we get output of the desired system $x_o(t)$.

If dynamic system produces a third order differential equation with input $x_i(t)$ and output $x_o(t)$ with constant coefficient *a,b,c,e*...then it will be given as,

$$a\frac{d^3x_o}{dt^3}+b\frac{d^2x_o}{dt^2}+c\frac{dx_o}{dt}+ex_o=Ax_i(t)$$

Applying Sadik Transform we get,

$$a[\upsilon^{3\alpha}S_o(\upsilon)\ -\upsilon^{2\alpha-\beta}\ x_o(0)-\ \upsilon^{\alpha-\beta}\ x_o^{'}(0)-\upsilon^{-\beta}\ x_o^{''}(0)]+b[\upsilon^{2\alpha}S_o(\upsilon)-\upsilon^{\alpha-\beta}\ x_o(0)-\upsilon^{-\beta}\ x_o^{'}(0)]+$$
$$\mathrm{c}[\upsilon^{\alpha}S_o(\upsilon)-\upsilon^{-\beta}\ x_o(0)]+S_o(\upsilon)=AS_i(\upsilon)$$

$$[a\upsilon^{3\alpha}+b\upsilon^{2\alpha}+\mathrm{c}\,\upsilon^{\alpha}+\mathrm{e}]S_o(\upsilon)\ -[\mathrm{a}\,x_o^{''}(0)+(a\upsilon^{\alpha}+\mathrm{b})\,x_o^{'}(0)+(\mathrm{a}\,\upsilon^{2\alpha}+b\upsilon^{\alpha}+c)x_o(0)]\ \upsilon^{-\beta}\ =AS_i(\upsilon)$$

$$S_o(\upsilon)\ =\frac{A}{[a\upsilon^{3\alpha}+b\upsilon^{2\alpha}+\mathrm{c}\,\upsilon^{\alpha}+\mathrm{e}]}S_i(\upsilon)+\frac{[\mathrm{a}\,x_o^{''}(0)+(a\upsilon^{\alpha}+\mathrm{b})\,x_o^{'}(0)+(\mathrm{a}\,\upsilon^{2\alpha}+b\upsilon^{\alpha}+c)x_o(0)]\ \upsilon^{-\beta}}{[a\upsilon^{3\alpha}+b\upsilon^{2\alpha}+\mathrm{c}\,\upsilon^{\alpha}+\mathrm{e}]}$$

…….. (8)

If initial conditions are zero then Transfer function of input and output function is given by,

$$G(\upsilon)=\frac{S_o(\upsilon)}{S_i(\upsilon)}=\frac{A}{[a\upsilon^{3\alpha}+b\upsilon^{2\alpha}+\mathrm{c}\,\upsilon^{\alpha}+\mathrm{e}]} \quad \text{……… (9)}$$

If dynamic system produces a nth order differential equation with input $x_i(t)$ and output $x_o(t)$ with constant coefficients .then it will be given as,

$$a_1\frac{d^nx_o}{dt^n}+a_2\frac{d^{n-1}x_o}{dt^{n-1}}+a_3\frac{d^{n-2}x_o}{dt^{n-2}}+.....+a_{n+1}x_o=Ax_i(t) \quad \text{……… (10)}$$

$$S_o(\upsilon) = \frac{A}{[a_1\upsilon^{n\alpha} + a_2\upsilon^{(n-1)\alpha} + a_3\upsilon^{(n-2)\alpha} + ... + a_{n+1}]} S_i(\upsilon) +$$

$$\frac{[a_1 x_o^{n-1}(0) + (a_1\upsilon^{\alpha} + a_2)\, x_o^{n-2}(0) + (a_1\upsilon^{2\alpha} + a_2\upsilon^{\alpha} + a_3) x_o^{n-3}(0) + ... + (a_1\upsilon^{(n-1)\alpha} + a_2\upsilon^{(n-2)\alpha} + a_3\upsilon^{(n-3)\alpha} + ... + a_{n+1}) x_o(0)]\, \upsilon^{-\beta}}{[a_1\upsilon^{n\alpha} + a_2\upsilon^{(n-1)\alpha} + a_3\upsilon^{(n-2)\alpha} + ... + a_{n+1} a_1\upsilon^{n\alpha} + a_2\upsilon^{(n-1)\alpha} + a_3\upsilon^{(n-2)\alpha} + ... + a_{n+1}]}$$

………… (11)

If initial conditions are zero then Transfer function of input and output function is given by,

$$G(\upsilon) = \frac{S_o(\upsilon)}{S_i(\upsilon)} = \frac{A}{[a_1\upsilon^{n\alpha} + a_2\upsilon^{(n-1)\alpha} + a_3\upsilon^{(n-2)\alpha} + ... + a_{n+1}]} \qquad \text{…..…….. (12)}$$

4. APPLICATIONS OF THE METHOD

Example: Find transfer function by Sadik Transform for spring mass damper system problem

$$\frac{d^2 x_o}{dt^2} + 3\frac{dx_o}{dt} + 2x_o = 5$$

(i) Initial condition $\quad x_o = 4, \dfrac{dx_o}{dt} = 3.$

(ii) If initial conditions are zero.

Solution: (i) Applying result (2)

From given equation if initial condition are non-zero $x_o = 4, \dfrac{dx_o}{dt} = 3$, a=*1, b=3, c=2, A=5.*

$$S_o(\upsilon) = \frac{5}{[\upsilon^{2\alpha} + 3\upsilon^{\alpha} + 2]} S_i(\upsilon) - \frac{\upsilon^{-\beta}[(\upsilon^{\alpha} + 3)(4) + 3]}{[\upsilon^{2\alpha} + 3\upsilon^{\alpha} + 2]} ,$$

$$x_i(t) = 1 \rightarrow S_i(\upsilon) = \frac{1}{\upsilon^{\alpha+\beta}} ,$$

$$S_o(\upsilon) = \frac{5}{[\upsilon^{2\alpha} + 3\upsilon^{\alpha} + 2]} \frac{1}{\upsilon^{\alpha+\beta}} - \frac{[4\upsilon^{\alpha} + 15]}{\upsilon^{\beta}[\upsilon^{2\alpha} + 3\upsilon^{\alpha} + 2]}$$

$$S_o(\upsilon) = \frac{1}{\upsilon^{\beta}}\left[\frac{5}{[\upsilon^{2\alpha} + 3\upsilon^{\alpha} + 2]\upsilon^{\alpha}} - \frac{[4\upsilon^{\alpha} + 15]}{[\upsilon^{2\alpha} + 3\upsilon^{\alpha} + 2]}\right]$$

$$S_o(\upsilon) = \frac{-1}{\upsilon^{\beta}}\left[\frac{-5/2}{\upsilon^{\alpha}} + \frac{16}{\upsilon^{\alpha} + 1} - \frac{19/2}{\upsilon^{\alpha} + 2}\right]$$

By taking Sadik Inverse Transform we get,

$$x_o(t)=\frac{5}{2}+16e^{-t}-\frac{19}{2}e^{-2t}$$

Solution: (ii) if initial conditions are zero then from equation (3),

$$G(\upsilon)=\frac{S_o(\upsilon)}{S_i(\upsilon)}$$

$$S_o(\upsilon)=G(\upsilon).S_i(\upsilon), S_o(\upsilon)=\frac{5}{[\upsilon^{2\alpha}+3\upsilon^{\alpha}+2]}S_i(\upsilon)$$

$$x_i(t)=1\rightarrow S_i(\upsilon)=\frac{1}{\upsilon^{\alpha+\beta}}$$

$$S_o(\upsilon)=\frac{5}{(\upsilon^{\alpha}+2)(\upsilon^{\alpha}+1)}\frac{1}{\upsilon^{\alpha+\beta}}$$

$$S_o(\upsilon)=\frac{1}{\upsilon^{\beta}}\left[\frac{5/2}{\upsilon^{\alpha}}+\frac{5/2}{\upsilon^{\alpha}+2}-\frac{5}{\upsilon^{\alpha}+1}\right]$$

By taking Sadik Inverse Transform we get

$$x_o(t)=\frac{5}{2}+\frac{5}{2}e^{-2t}-5e^{-t}$$

Example: A Torsion spring of Stiffness K, a mass of moment of inertia I and a fluid damper with damping coefficient C are connected together, If angular displacement of free end of the spring is θ_i and the angular displacement of the mass and damper is θ_o. If I=2.5 kg m^2 C=12.5 Nm s/rad, K=250 Nm /rad then Find the solution using transfer function(Sadik).

Solution: By Mathematical modeling Differential Equation of above problem is given by,

$$2.5\frac{d^2\theta_o}{dt^2}+12.5\frac{d\theta_o}{dt}+250\theta_o=250\theta_i(t)$$

$$\frac{d^2\theta_o}{dt^2}+5\frac{d\theta_o}{dt}+100\theta_o=100\theta_i(t)$$

If output is $S_o(\upsilon)=S[\theta_o(t)]$ and input is $S_i(\upsilon)=S[\theta_i(t)]$

Then by Transfer function(by Sadik Transform)

$$G(\upsilon)=\frac{S_o(\upsilon)}{S_i(\upsilon)}$$

$$S_o(\upsilon)=G(\upsilon).S_i(\upsilon)$$

Here a=1, b=5, c=100, A=100 then from equation (6)

$$S_o(\upsilon) = \frac{100}{[\upsilon^{2\alpha} + 5\upsilon^{\alpha} + 100]} S_i(\upsilon)$$

After taking Sadik Transform of input function put it into above equation and by using partial fraction and then Inverse Sadik Transform we can get output function.

Example: Heat flows from a heat source at temperature θ_i (t) through a wall having ideal thermal resistance R_T to a heat sink at θ_o (t) having ideal thermal capacitance C_T. Find the differential equation and Transfer function by using Sadik Transform.

Solution: Heat flow by conduction is given by Fourier's law

$$Q_T = \frac{KA\left(\theta_i(\mathrm{t}) - \theta_o(\mathrm{t})\right)}{t}$$

Where $\theta_i(t) - \theta_o(t)$ = Temperature differential (K)

A = Normal Cross section area (m^2)

l = Thickness (m)

K = Thermal conductivity (W/mk)

Q_T = Heat flow (J/S=W)

If $\frac{l}{KA} = R_T$ then (1) is given as

$$R_T.\mathrm{Q}_T(t) = \theta_i(t) - \theta_o(t) \text{ Since } \mathrm{Q}_T(t) = C_T \frac{d\theta_o}{dt}$$

$$\therefore R_T C_T \frac{d\theta_o}{dt} + \theta_o(t) = \theta_i(t)$$

Where C_T is thermal Capacitance.

If initial conditions are zero then Transfer function by Sadik Transform is given as,

$$G(\upsilon) = \frac{S_o(\upsilon)}{S_i(\upsilon)}$$

$$S_o(\upsilon) = G(\upsilon).S_i(\upsilon)$$

$$S_o(\upsilon) = \left[\frac{A}{(a\upsilon^{\alpha} + b)}\right] S_i(\upsilon)$$

$$S_o(\upsilon) = \left[\frac{1}{(R_T C_T \upsilon^{\alpha} + 1)}\right] S_i(\upsilon)$$

By taking Sadik inverse Transform we get θo (t).

5. CONCLUSIONS

In this paper a new method has been introduced for solving some problems from Mechanical and Electrical systems using transform function by Sadik Transform if initial conditions are zero. Also we can find easy solution if initial conditions are not zero by using this method.

6. REFERENCES

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