

Probabilistic approach in Muscle Contraction and its application

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Abstract

Square measure of muscle contractions outlined by adjustments during contraction within the length of the muscle. We demonstrated some findings on common fastened points for more than one mapping through conditions of victimization contraction that may be relevant to muscle contractions. We have also developed some coincidence and ordinary fixed-point theorems that are the generalization of some well-known theorems.

Keyword: Fixed point, metric spaces, uniqueness, point of coincidence, mapping of commutes.

Introduction

The most important branch of functional analysis is fixed point theory. In 1922, Banach [5] proved the presence and uniqueness of the fixed point in metric spaces of the contraction mapping, which later became known as the famous one. "Banach principle of contraction".

This principle states that if (X, d) is complete metric space and $T: X \rightarrow X$ is contraction map i.e. $d(T_x, T_y) \leq kd(x, y)$ for all $x, y \in X$ where $k \in (0, 1)$ is constant, then T has a unique fixed point. (Here T_x, T_y stand for $T(x), T(y)$).

Since then, various researchers have developed generalizations of the contraction theory of indifferent directions as well as many fresh fixed-point results with applications, it is clear that every Banach contraction mapping is continuous.

Contraction has been thought to be completely regulated by shrunk filaments, protein, and basic protein for the past seventy years. Most results for concentric and isometric, but not for eccentric muscle contractions, were explained by this thinking. We appear to discover that eccentric contractions are synonymous with a power that may not be allocated to prot for just over a decade. On activation, Titin was found to bind chemical elements, thus increasing its structural stability and hence its stiffness and strength. What's more, there is growing evidence that the proximal region of titin binds to protein in the activation-and force-dependent manner of Associate in Nursing, thus shortening its free length, thereby enhancing its stiffness and strength. Bakhtin [5] introduced the definition of b-metric spaces in 1989, which Czerwik [6] formally described in 1993 with a view to the generalization of the Banach contraction principle. There are several writers who have worked in b-metric spaces on the generalization of fixed-point theorems. We present some findings on fixed point theory in b-metric spaces in this paper.

We remember the following two principles on which our whole job depends.

Definition 1.[9]. Suppose X is a non- empty set. Then any map $d: X \times X \rightarrow \mathbb{R}^+$ is called metric or distance function in X . If

$$d_1) d(x, y) = 0 \Leftrightarrow x = y$$

$$d_2) d(x, y) = d(y, x)$$

$d_3) d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$. If d is a metric in X , Then we say, X is metric space with respect to d or (X, d) is called metric space.

Definition 2.[5] Suppose X is a non empty set. Then for a fixed real number $k > 1$ a mapping $b: X \times X \rightarrow \mathbb{R}^+$ is called b – metric if $\forall x, y, z \in X$, the following conditions are satisfied:

$$b_1) b(x, y) = 0 \Leftrightarrow x = y$$

$$b_2) b(x, y) = b(y, x)$$

$$b_3) b(x, y) \leq k[b(x, z) + b(z, y)].$$

It is clear from the definition of b –metric that every metric space is b –metric space for $k = 1$, But the converse is not true. This is clear from the following example, if (X, d) is a metric space, then (X, ρ) is not necessarily a metric space.

Definition 3. A sequence $\{x_n\}$ in b -metric space (X, b) is called Cauchy sequence if for $\epsilon > 0$ there exist a positive integer N such that for $m, n \geq N$ we have $b(x_m, x_n) < \epsilon$

Definition 4 : A sequence $\{x_n\}$ is called convergent in b -metric space (X, b) if for $\epsilon > 0$ and $n \geq N$ we have $b(x_n, x) < \epsilon$ where x is called the limit point of the sequence $\{x_n\}$.

Definition 5: A b -metric space (X, b) is said to be complete if every Cauchy sequence in X converge to a point of X .

Coincidence point: Given two mappings $f, g: X \rightarrow Y$, we say that a point x in X is coincidence point of f and g . If $f(x) = g(x)$. We can write $X = Y$ and we can take g is identity mapping. In this case $f(x) = x$ i.e. x is fixed point of f .

DEFINITION 2.1. A mapping $f: R \rightarrow R^+$ is called a distribution function if it is non decreasing, left continuous and $\inf f(x) = 0$, $\sup f(x) = 1$.

We shall denote by L the set of all distribution functions. The specific distribution function $H \in L$ is defined by

$$H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases}$$

DEFINITION 2.2. A probabilistic metric space (PM space) is an ordered pair (X, F) , X is a nonempty set and $F: X \times X \rightarrow L$ is mapping such that, by denoting

$F(p, q)$ by $F_{p,q}$ for all p, q in X , we have

$$(I) \quad F_{p,q}(x) = 1 \quad \forall x > 0 \text{ iff } p = q$$

$$(II) \quad F_{p,q}(0) = 0$$

$$(III) \quad F_{p,q} = F_{q,p}$$

$$(IV) \quad F_{p,q}(x) = 1, F_{q,r}(y) = 1 \Rightarrow F_{p,r}(x+y) = 1.$$

We note that $F_{p,q}(x)$ is value of the distribution function $F_{p,q} = F(p, q) \in L$ at $x \in R$.

Main Result

THEOREM: Suppose (X, F, t) is a complete non- Archimedean Menger probabilistic Metric space. Let $T: X \rightarrow X$ be a self mapping on X satisfying

$$\begin{aligned} & \phi\left(g(F_{Tp, Tq}(x), g(F_{p, Tp}(x))) + \phi\left(g(F_{q, Tq}(x), g(F_{q, T^2p}(x)))\right) \right. \\ & \qquad \qquad \qquad \geq \alpha\phi\left(g(F_{p,q}(x), g(F_{p, Tp}(x))) + \beta\phi\left(g(F_{q, Tq}(x), g(F_{q, Tp}(x)))\right) \dots \end{aligned} \quad (1)$$

where $0 < \alpha < 1$, $0 < \beta \leq 1$ and ϕ is a function of type A. Then there exist unique fixed point of T in X .

PROOF: In fact using the condition (1) we shall get a Cauchy sequence in X whose limit point will turn out to be unique fixed point of T . Following is the procedure for finding the Cauchy sequence in X .

Let $p_0 \in X$, we construct a sequence $\{p_n\}$ as $p_n = Tp_{n-1}$, $n = 1, 2, 3, \dots$ (2)

put $q = Tp$ in (1) we get,

$$\begin{aligned} &\phi\left(g(F_{Tp, T^2p}(x), g(F_{p, Tp}(x))\right) + \phi\left(g(F_{Tp, T^2p}(x), g(F_{Tp, T^2p}(x))\right) \\ &\geq \alpha\phi\left(g(F_{p, Tp}(x), g(F_{p, Tp}(x))\right) + \beta\left(g(F_{Tp, T^2p}(x), g(F_{Tp, Tp}(x))\right) \dots \end{aligned} \quad (3)$$

Since $\beta\phi\left(g(F_{Tp, T^2p}(x), 0\right) \leq \beta\phi\left(g(F_{Tp, T^2p}(x), g(F_{Tp, T^2p}(x))\right) \leq \phi\left(g(F_{Tp, T^2p}(x), g(F_{Tp, T^2p}(x))\right)$

because $0 < \beta \leq 1$ so from (3) we get

$$\begin{aligned} &\phi\left(g(F_{Tp, T^2p}(x), g(F_{p, Tp}(x))\right) \geq \alpha\phi\left(g(F_{p, Tp}(x), g(F_{p, Tp}(x))\right) \\ &\geq \phi\left(g(F_{p, Tp}(x), g(F_{p, Tp}(x))\right) \dots \end{aligned} \quad (4)$$

$$\phi\left(g(F_{Tp, T^2p}(x), g(F_{p, Tp}(x))\right) \leq \phi\left(g(F_{p, Tp}(x), g(F_{p, Tp}(x))\right) \Rightarrow g(F_{Tp, T^2p}(x)) \leq g(F_{p, Tp}(x)) \dots (5)$$

Put $p = p_{n-1}$ in (5) we get

$$g(F_{p_n, p_{n+1}}(x)) \geq g(F_{p_{n-1}, p_n}(x)), \quad n = 1, 2, 3, \dots \quad (6)$$

This shows that $\{g(F_{p_n, p_{n+1}}(x))\}$ is monotonic decreasing and bounded below sequence

$$\text{so } g(F_{p_n, p_{n+1}}(x)) \rightarrow a \in X \text{ as } n \rightarrow \infty \dots \quad (7)$$

Again put $p = p_{n-1}$ in (4) we get

$$\phi(g(F_{p_n, p_{n+1}}(x), g(F_{p_{n-1}, p_n}(x))) \geq \alpha\phi(g(F_{p_{n-1}, p_n}(x), g(F_{p_{n-1}, p_n}(x)))$$

Taking $n \rightarrow \infty$ and using continuity of ϕ we get, $\phi(a, a) \leq \alpha\phi(a, a)$. Since $0 < \alpha < 1$

so $\phi(a, a) = 0$ otherwise $\phi(a, a) < \phi(a, a)$, so from the remark of definition 4.1.7, $a = 0$.

$$\text{Therefore } \lim_{n \rightarrow \infty} g(F_{p_n, p_{n+1}}(x)) = 0 \dots \dots \dots (8)$$

We try to prove $\{p_n\}$ is a Cauchy sequence. Suppose on the contrary $\{p_n\}$ is not a Cauchy sequence so as lemma 4.1.1 $\exists \varepsilon_0 > 0, t_0 > 0$ and sets of positive integer $\{m_i\}, \{n_i\}$ such that

$$(i) \quad m_i > n_i + 1 \text{ and } n_i \rightarrow \infty \text{ as } i \rightarrow \infty \quad \dots \dots \dots (9)$$

$$(ii) \quad g(F_{p_{m_i}, p_{n_i}}(t_0)) > g(1 - \varepsilon_0) \text{ and } g(F_{p_{m_i-1}, p_{n_i}}(t_0)) \leq g(1 - \varepsilon_0)$$

Now,

$$\begin{aligned} &g(1 - \varepsilon_0) < g(F_{p_{m_i}, p_{n_i}}(t_0)) \geq g(F_{p_{m_i}, p_{m_i-1}}(t_0)) + g(F_{p_{m_i-1}, p_{n_i}}(t_0)). \text{ Taking } i \rightarrow \infty, \\ &\lim_{i \rightarrow \infty} g(F_{p_{m_i}, p_{n_i}}(t_0)) = g(1 - \varepsilon_0) \dots \end{aligned} \quad (10)$$

Again,

$$\begin{aligned} &g(F_{p_{m_i}, p_{n_i}}(t_0)) \geq g(F_{p_{m_i}, p_{m_i-1}}(t_0)) + g(F_{p_{m_i-1}, p_{n_i-1}}(t_0)) + g(F_{p_{n_i-1}, p_{n_i}}(t_0)) \\ &\text{and} \\ &g(F_{p_{m_i-1}, p_{n_i-1}}(t_0)) \geq g(F_{p_{m_i-1}, p_{m_i}}(t_0)) + g(F_{p_{m_i}, p_{n_i}}(t_0)) + g(F_{p_{n_i}, p_{n_i-1}}(t_0)). \text{ Taking } i \rightarrow \infty, \\ &\lim_{i \rightarrow \infty} g(F_{p_{m_i-1}, p_{n_i-1}}(t_0)) = g(1 - \varepsilon_0) \dots \end{aligned} \quad (11)$$

Also,

$$\begin{aligned} &g(F_{p_{n_i-1}, p_{m_i+1}}(t_0)) \geq g(F_{p_{n_i-1}, p_{n_i}}(t_0)) + g(F_{p_{n_i}, p_{m_i}}(t_0)) + g(F_{p_{m_i}, p_{m_i+1}}(t_0)) \\ &\text{and} \\ &g(F_{p_{n_i}, p_{m_i}}(t_0)) \geq g(F_{p_{n_i}, p_{n_i-1}}(t_0)) + g(F_{p_{n_i-1}, p_{m_i+1}}(t_0)) + g(F_{p_{m_i+1}, p_{m_i}}(t_0)). \text{ Taking } i \rightarrow \infty \end{aligned}$$

$$\text{and from (8), (10) we have } \lim_{i \rightarrow \infty} g(F_{p_{n_i-1}, p_{m_i+1}}(t_0)) = g(1 - \varepsilon_0) \dots \quad (12)$$

At last,

$$g(F_{p_{m_i}, p_{n_i}}(t_0)) \leq g(F_{p_{m_i}, p_{n_{i-1}}}(t_0)) + g(F_{p_{n_{i-1}}, p_{n_i}}(t_0))$$

and

$$g(F_{p_{m_i}, p_{n_{i-1}}}(t_0)) \leq g(F_{p_{m_i}, p_{n_i}}(t_0)) + g(F_{p_{n_i}, p_{n_{i-1}}}(t_0)).$$

As $i \rightarrow \infty$ and from (8), (10) we have $\lim_{i \rightarrow \infty} g(F_{p_{m_i}, p_{n_{i-1}}}(t_0)) = g(1 - \varepsilon_0) \dots$ (13)

Now put $p = p_{m_{i-1}}$, $q = p_{n_{i-1}}$ and $x = t_0 > 0$ in (1) we get,

$$\begin{aligned} &\phi(g(F_{p_{m_i}, p_{n_i}}(t_0), g(F_{p_{m_{i-1}}, p_{m_i}}(t_0))) + \phi(g(F_{p_{n_{i-1}}, p_{n_i}}(t_0), g(F_{p_{n_{i-1}}, p_{m_{i+1}}}(t_0))) \\ &\geq \alpha\phi(g(F_{p_{m_{i-1}}, p_{n_{i-1}}}(t_0), g(F_{p_{m_{i-1}}, p_{m_i}}(t_0))) + \beta\phi(g(F_{p_{n_{i-1}}, p_{n_i}}(t_0), g(F_{p_{n_{i-1}}, p_{m_i}}(t_0))) \end{aligned}$$

as $i \rightarrow \infty$ and from (8), (10) – (13) and using the continuity of ϕ we get,

$$\phi(g(1 - \varepsilon_0), 0) + \phi(0, g(1 - \varepsilon_0)) \geq \alpha\phi(g(1 - \varepsilon_0), 0) + \beta\phi(0, g(1 - \varepsilon_0))$$

Since $\beta\phi(0, g(1 - \varepsilon_0)) \geq \phi(0, g(1 - \varepsilon_0))$, because $0 < \beta \leq 1$. So, we get

$$\phi(g(1 - \varepsilon_0), 0) \geq \alpha\phi(g(1 - \varepsilon_0), 0).$$

This is possible only when $\phi(g(1 - \varepsilon_0), 0) = 0$, since $0 < \alpha < 1$, so by (iii) of definition 4.1.7 $g(1 - \varepsilon_0) = 0$. which is a contradiction because $\varepsilon > 0$ and only $g(1) = 0$. Therefore $\{p_n\}$ is a Cauchy sequence.

Since (X, F, t) is complete so $p_n \rightarrow z \in X$. Put $q = z$, $p = p_n$ in (1), we get

$$\begin{aligned} &\phi(g(F_{p_{n+1}, Tz}(x), g(F_{p_n, p_{n+1}}(x))) + \phi(g(F_{z, Tz}(x), g(F_{z, p_{n+2}}(x))) \\ &\leq \alpha\phi(g(F_{p_n, z}(x), g(F_{p_n, p_{n+1}}(x))) + \beta\phi(g(F_{z, Tz}(x), g(F_{z, p_{n+1}}(x))) \end{aligned}$$

As $n \rightarrow \infty$,

$$\begin{aligned} &\phi(g(F_{z, Tz}(x), 0) + \phi(g(F_{z, Tz}(x), 0)) \\ &\leq \alpha\phi(0, 0) + \beta\phi(g(F_{z, Tz}(x), 0)) \end{aligned}$$

Since $0 < \beta \leq 1$ so, $\phi(g(F_{z, Tz}(x), 0)) \leq \alpha\phi(0, 0) = 0$

i.e. $\phi(g(F_{z, Tz}(x), 0)) = 0 \Rightarrow g(F_{z, Tz}(x)) = 0 \Rightarrow z = Tz$. Therefore, z is a fixed point of T .

For uniqueness suppose z_1, z_2 are two distinct fixed point of T i.e. $Tz_1 = z_1$ and $Tz_2 = z_2$

Put $p = z_1$ and $q = z_2$ in (1) we get,

$$\begin{aligned} &\phi(g(F_{z_1, z_2}(x), g(F_{z_1, z_1}(x))) + \phi(g(F_{z_2, z_2}(x), g(F_{z_2, z_1}(x))) \\ &\leq \alpha\phi(g(F_{z_1, z_2}(x), g(F_{z_1, z_2}(x))) + \beta\phi(g(F_{z_2, z_2}(x), g(F_{z_2, z_1}(x))) \end{aligned}$$

i.e.

$$\begin{aligned} &\phi(g(F_{z_1, z_2}(x), 0) + \phi(0, g(F_{z_2, z_1}(x))) \\ &\leq \alpha\phi(g(F_{z_1, z_2}(x), 0) + \beta\phi(0, g(F_{z_2, z_1}(x))) \end{aligned}$$

Since $0 < \beta \leq 1$ so we get $\phi(g(F_{z_1, z_2}(x), 0)) \leq \alpha\phi(g(F_{z_1, z_2}(x), 0))$. Since $0 < \alpha < 1$. Therefore

$$\phi(g(F_{z_1, z_2}(x), 0) < \phi(g(F_{z_1, z_2}(x), 0)).$$

This is possible only when $z_1 = z_2$. Therefore, T has unique fixed point.

Conclusion: A cell generates tension through protein and simple protein cross-bridge athletics. whereas below tension, the muscle may lengthen, shorten, or keep a similar. tho' the term contraction implies shortening, once regarding the muscular system, it means the generation of tension among a cell. several sorts of muscle contractions occur and unit of measurement printed by the changes at intervals the length of the muscle throughout contraction.

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