SOME PROPERTIES AND OPERATIONS OF SPHERICAL FERMATEAN FUZZY MATRICES

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Abstract - In this paper, we define an operation on SphericalFermateanfuzzy matrices and discuss their properties in relation with standard operations available in the literature. Also, we proved some new results on SphericalFermateanfuzzy matrices.

Keywords: Fuzzy matrices, Spherical Fuzzy Matrices, Spherical Fermatean Fuzzy Matrices, Arithmetic operations, properties.

1. INTRODUCTION

The fuzzy set (FS) was developed by Zadeh[28] to model these imprecise evaluations in the decision-making process. As an extension of FS, the intuitionistic fuzzy set (IFS) is characterized by the membership degree and the non-membership degree satisfying the condition that their sum is less than or equal to 1 was developed by Atanassov [4].Fuzzy matrices were introduced for the first time by Thomason [23], who discussed theconvergence of powers of fuzzy matrix. Fuzzy matrices engage in recreation to a vital role in scientific development. The elementary and necessary fuzzy matrix theory is given. Instead, the authors have solely tried to convey those essentially required to develop the fuzzy model. The authors don't gift detailed mathematical theories to shape with fuzzy matrices; instead they need given exclusively the required properties by method of examples. Some authors have presented a number of results on the convergence of the power sequence of fuzzy matrices. Ragab et al[16,17] presented some properties on determinant and adjoint of square fuzzy matrix. Kim and Roush [10,11] studied the canonical form of an idempotent matrix. Hashimoto studied the canonical form of a transitive matrix.Adak et.al [1,2,3] develops some properties of generalized intuitionistic fuzzy, fuzzy block matrices and distributive lattices.Meenakshi and Kaliraja [10] have extended Sanchez's approach for medical diagnosis using representation of a interval valued fuzzy matrix. For decision making in fuzzy environment one may refer Bellman and Zadeh [5].

Madhumangal pal and SanjibMontal [14,15] introduced the bipolar matrix and their algebraic operations. Yager [25,26,27] recently proposed the concept of Pythagorean fuzzy set (PFS) which is characterized by the membership and the non-membership degree satisfying the condition that their sum of square is not larger than 1.Hesitant fuzzy sets can be used as a functional tool allowing many potential membership degrees of an element to a set. These fuzzy sets let several membership degrees of an element to be possible between zero developed by Torra [22].Neutrosophic logic and neutrosophic sets were developed by Smarandache[19,20] as an extension of intuitionistic fuzzy sets. A neutrosophic set is defined as the set where each element of the universe has a degree of truthiness, indeterminacy and falsity. They can be determined independently and their sum may be between 0 and 3. Picture fuzzy sets were developed by Cuong [8], Picture fuzzy sets basedmodels may be adequate in situations when we face human opinions involving moreanswers of types: yes, abstain, no, and refusal. Voting can be a good example of sucha situation as the human voters may be divided into four groups of those who: vote for, abstain, and vote against, refusal of the voting. These sets let decision makers use alarger area for assigning membership, non-membership, and hesitancy degrees.

Spherical fuzzy sets (SFS) were introduced by Kahraman and Gündogdu [13] as an extension of Pythagorean, neutrosophic and picture fuzzy sets. The idea behindSFS is to let decision makers to generalize other extensions of fuzzy sets by defining amembership function on a spherical surface and independently assign the parametersof that membership function with a larger domain.

Fermatean fuzzy sets were introduced by TapanSenapati and Ronald.R.Yager[24] , this is an extension of Pythagorean fuzzy sets. Fermatean fuzzy matrices were introduced by I.Silambarasan[21].

In this paper, we introduce the concept of Spherical fermateanfuzzy matrices. Also, give some operators bringing to Spherical fermateanfuzzy Matrices. Also some properties are discussed.

2. PRELIMINARIES

This section presents the basic definitions.

Definition 2.1

Consider a matrix $A = \left[(a_{ij})\right]_{m\times n}$ where $a_{ij} \in [0,1] 1 \leq i \leq m$ and $1 \leq i \leq n$. Then A is a fuzzy matrix.

Definition 2.2

An intuitionistic fuzzy matrix (IFM) is a matrix of pairs $A = \left[(a_{ij}, a_{ij}')\right]_{m\times n}$ of nonnegative real numbers $a_{ij}, a_{ij}' \in [0,1]$, satisfying $0 \leq a_{ij} + a_{ij}' \leq 1$ for all i, j.

Definition 2.3

Let a set X be a universe of discourse, A Pythagorean fuzzy set (PFS) P is an object having the form, $\left(\langle x, P\left(\mu_p(x), \vartheta_p(x)\right) / x \epsilon X\rangle\right)$ Where the function $\mu_p: X \to [0,1]$ and $\vartheta_p: X \to [0,1]$ such that $0 \leq \left(\mu_p(x)\right)^2 + \left(\vartheta_p(x)\right)^2 \leq 1. \mu_p(x)$ and $\vartheta_p(x)$ denote the degree of membership and degree of non-membership of $x \in X$to P.

Definition 2.4

A Pythagorean fuzzy matrix (PFM) is a matrix of pairs $A = \left[(a_{ij}, a_{ij}')\right]_{m\times n}$ of nonnegative real numbers, $a_{ij}, a_{ij}' \in [0,1]$, satisfying $a_{ij}^2 + {a_{ij}'}^2 \leq 1$ for all i, j.

Definition 2.5

A Spherical Fuzzy Matrix (SFM) of order $(m \times n)$ is denoted by $\tilde{A}_{s_{(m\times n)}}$ and that of order$(m \times m)$, that is square SFM is denoted by$\tilde{A}_{s_{(m\times m)}}$. We conclude that the SFM is of the form$\tilde{A}_s = \left[(a_{ij}, a_{ij}^{'}, a_{ij}^{*})\right]_{(m\times n)}$, where $a_{ij}, a_{ij}^{'}, a_{ij}^{*}$ are the Membership, Non-membership and Hesitancy of$\tilde{A}_s$. Also satisfies the condition$0 \leq {a_{ij}}^2 + {a_{ij}^{'}}^2 + {a_{ij}^{*}}^2 \leq 1$.

Definition 2.6

The SFM is of the form$\tilde{A}_s = \left[(a_{ij}, a_{ij}^{'}, a_{ij}^{*})\right]_{(m\times n)}$, where $a_{ij}, a_{ij}^{'}, a_{ij}^{*}$ are the Membership, Non-membership and Hesitancy of$\tilde{A}_s$.

Now $\Re_{\tilde{A}_s} = \sqrt{1 - {a_{ij}}^2 - {a_{ij}^{'}}^2 - {a_{ij}^{*}}^2}$ is called as a refusal degree of SFM.

Definition 2.7

A Fermatean Fuzzy Matrix (FFM) is a pair $\tilde{A}_s = \left[\left(\mu_{a_{ij}}, \nu_{a_{ij}}\right)\right]_{(m\times n)}$ of non negative real numbers $\mu_{a_{ij}}, \nu_{a_{ij}} \in [0,1]$ satisfying the condition $0 \leq \left(\mu_{a_{ij}}\right)^3 + \left(\nu_{a_{ij}}\right)^3 \leq 1$, for all i,j, where $\mu_{a_{ij}} \in [0,1]$is called the degree of membership and$\nu_{a_{ij}} \in [0,1]$ is called the degree of non-membership.

For understanding the FFM better, we give an instance to illustrate the understand- ability of the FFM. We can definitely get $0.8 + 0.7 > 1$ and therefore, it does not follow the condition of intuitionistic fuzzy matrices. Also we can get $(0.8)^2 + (0.7)^2 = 0.64 + 0.49 = 1.13 > 1$, which does not obey the constraint condition of Pythagorean fuzzy matrix. However, we can get $(0.8)^3 + (0.7)^3 = 0.512 + 0.343 = 0.855 \leq 1$, which is good enough to apply the FFM to control it.

Definition 2.8

A Spherical Fermatean Fuzzy Matrix (SFFM) of order $(m \times n)$ is denoted by $\tilde{A}_{s_{(m\times n)}}$ and that of order$(m \times m)$, that is square SFFM is denoted by$\check{A}_{F(m\times m)}$. We conclude that the SFM is of the form$\check{A}_F = \left[\left(x_{ij}^m, x_{ij}^n, x_{ij}^h\right)\right]_{(m\times n)}$, where $x_{ij}^m, x_{ij}^n, x_{ij}^h$ are the Membership, Non-membership and Hesitancy of$\check{A}_F$. Also satisfies the condition$0 \leq \left(x_{ij}^m\right)^3 + \left(x_{ij}^n\right)^3 + \left(x_{ij}^h\right)^3 \leq 1$.

Definition 2.8

The SFM is of the form$\check{A}_F = \left[\left(x_{ij}^m, x_{ij}^n, x_{ij}^h\right)\right]_{(m\times n)}$, where $x_{ij}^m, x_{ij}^n, x_{ij}^h$ are the Membership, Non-membership and Hesitancy of$\check{A}_F$.

Now $\Re_{\check{A}_F} = \sqrt{1 - \left(x_{ij}^m\right)^3 - \left(x_{ij}^n\right)^3 - \left(x_{ij}^h\right)^3}$ is called as a refusal degree of SFFM.

3. OPERATIONS ON SPHERICAL FERMATEAN FUZZY MATRICES

In this section, we have defined some operations on spherial fermatean fuzzi matrices

Let us consider the Spherical Fermatean Fuzzy matrices$\check{A}_F$ and$\check{B}_F$ such that

$$\check{A}_F \vee_F \check{B}_F = \left\{\begin{matrix} max\left(x_{ij}^m, y_{ij}^m\right), min\left(x_{ij}^n, y_{ij}^n\right), \\ min\left\{\begin{matrix}\left[1 - \langle\left(max\{x_{ij}^m, y_{ij}^m\}\right)^2 + \left(min\{x_{ij}^n, y_{ij}^n\}\right)^3\rangle\right]^{\frac{1}{3}}, \\ max\left(x_{ij}^h, y_{ij}^h\right)\end{matrix}\right\}\end{matrix}\right\}$$

$$\check{A}_F \wedge_F \check{B}_F = \left\{\begin{matrix} min\left(x_{ij}^m, y_{ij}^m\right), max\left(x_{ij}^n, y_{ij}^n\right), \\ max\left\{\begin{matrix}\left[1 - \langle\left(min\{x_{ij}^m, y_{ij}^m\}\right)^2 + \left(max\{x_{ij}^n, y_{ij}^n\}\right)^3\rangle\right]^{\frac{1}{3}}, \\ min\left(x_{ij}^h, y_{ij}^h\right)\end{matrix}\right\}\end{matrix}\right\}$$

$$\check{A}_F \oplus \check{B}_F = \left\{\begin{matrix} \left(\left(x_{ij}^m\right)^3 + \left(y_{ij}^m\right)^3 - \left(x_{ij}^m\right)^3\left(y_{ij}^m\right)^3\right)^{\frac{1}{3}}, x_{ij}^n y_{ij}^n, \\ \left[\left(1 - \left(y_{ij}^m\right)^3\right)\left(x_{ij}^h\right)^3 + \left(1 - \left(x_{ij}^m\right)^3\right)\left(y_{ij}^h\right)^3 - \left(x_{ij}^h\right)^3\left(y_{ij}^h\right)^3\right]^{1/3}\end{matrix}\right\}$$

$$\check{A}_F \otimes \check{B}_F = \left\{\begin{matrix} x_{ij}^m y_{ij}^m, \left(\left(x_{ij}^n\right)^3 + \left(y_{ij}^n\right)^3 - \left(x_{ij}^n\right)^3\left(y_{ij}^n\right)^3\right)^{\frac{1}{3}}, \\ \left[\left(1 - \left(y_{ij}^n\right)^3\right)\left(x_{ij}^h\right)^3 + \left(1 - \left(x_{ij}^n\right)^3\right)\left(y_{ij}^h\right)^3 - \left(x_{ij}^h\right)^3\left(y_{ij}^h\right)^3\right]^{1/3}\end{matrix}\right\}$$

Multiplication by a scalar, $\lambda \geq 0$

[

$$\lambda.\breve{A}_F = \left\{\left[1-\left(1-\left(x_{ij}^m\right)^3\right)^\lambda\right]^{1/3}, y_{ij}^{n\,\lambda}, \left[\left(1-\left(x_{ij}^m\right)^3\right)^\lambda-\left(1-\left(x_{ij}^m\right)^3-\left(x_{ij}^h\right)^3\right)^\lambda\right]^{1/3}\right\}$$

λ . Power of $\breve{A}_F$, $\lambda \geq 0$

$$\breve{A}_F{}^\lambda = \left\{\left(x_{ij}^m\right)^\lambda, \left[1-\left(1-\left(y_{ij}^m\right)^3\right)^\lambda\right]^{1/3}, \left[\left(1-\left(y_{ij}^m\right)^3\right)^\lambda-\left(1-\left(y_{ij}^m\right)^3-\left(x_{ij}^h\right)^3\right)^\lambda\right]^{1/3}\right\}$$

4. SOME PROPERTIES OF SPHERICAL FERMATEAN FUZZY MATRICES

In the section, we discussed some property of spherical fermatean fuzzy matrices

Let us consider the Spherical FermateanFuzzy matrices $\breve{A}_F$ and $\breve{B}_F$ such that $\breve{A}_F = \left[\left(x_{ij}^m, x_{ij}^n, x_{ij}^h\right)\right]_{(m\times n)}$ and $\breve{B}_F = \left[\left(y_{ij}^m, y_{ij}^n, y_{ij}^h\right)\right]_{(m\times n)}$ then,

(i). $\breve{A}_F \vee_F \breve{B}_F = \breve{B}_F \vee_F \breve{A}_F$

(ii). $\breve{A}_F \wedge_F \breve{B}_F = \breve{B}_F \wedge_F \breve{A}_F$

(iii). $\breve{A}_F \oplus \breve{B}_F = \breve{B}_F \oplus \breve{A}_F$

(iv). $\breve{A}_F \otimes \breve{B}_F = \breve{B}_F \otimes \breve{A}_F$

(v). $\lambda\left(\breve{A}_F \oplus \breve{B}_F\right) = \lambda\breve{A}_F \oplus \lambda\breve{B}_F$

(vi). $\lambda_1\breve{A}_F \oplus \lambda_2\breve{A}_F = (\lambda_1 \oplus \lambda_2)\breve{A}_F$

(vii). $\left(\breve{A}_F \otimes \breve{B}_F\right)^\lambda = \breve{A}_F{}^\lambda \otimes \breve{B}_F{}^\lambda$

(viii). $\breve{A}_F{}^{\lambda_1} \otimes \breve{A}_F{}^{\lambda_2} = \breve{A}_F{}^{\lambda_1+\lambda_2}$

Proof:

(i). To prove $\breve{\mathbf{A}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{B}}_{\mathbf{F}} = \breve{\mathbf{B}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{A}}_{\mathbf{F}}$

$$\breve{A}_F \vee_F \breve{B}_F = \left\{\begin{matrix} max\left(x_{ij}^m, y_{ij}^m\right), min\left(x_{ij}^n, y_{ij}^n\right), \\ min\left\{\begin{matrix}\left[1-\langle\left(max\{x_{ij}^m, y_{ij}^m\}\right)^2+\left(min\{x_{ij}^n, y_{ij}^n\}\right)^3\rangle\right]^{\frac{1}{3}}, \\ max\left(x_{ij}^h, y_{ij}^h\right)\end{matrix}\right\}\end{matrix}\right\}$$

$$= \left\{\begin{matrix} max\left(y_{ij}^m, x_{ij}^m\right), min\left(y_{ij}^n, x_{ij}^n\right), \\ min\left\{\begin{matrix}\left[1-\langle\left(max\{y_{ij}^m, x_{ij}^m\}\right)^2+\left(min\{y_{ij}^n, x_{ij}^n\}\right)^3\rangle\right]^{\frac{1}{3}}, \\ max\left(y_{ij}^h, x_{ij}^h\right)\end{matrix}\right\}\end{matrix}\right\}$$

$$= \breve{B}_F \vee_F \breve{A}_F$$

Thus $\breve{A}_F \vee_F \breve{B}_F = \breve{B}_F \vee_F \breve{A}_F$ holds.

(ii). To Prove $\breve{\mathbf{A}}_{\mathbf{F}} \wedge_{\mathbf{F}} \breve{\mathbf{B}}_{\mathbf{F}} = \breve{\mathbf{B}}_{\mathbf{F}} \wedge_{\mathbf{F}} \breve{\mathbf{A}}_{\mathbf{F}}$

$$\breve{A}_F \wedge_F \breve{B}_F = \left\{ \begin{matrix} min\left(x_{ij}^m, y_{ij}^m\right), max\left(x_{ij}^n, y_{ij}^n\right), \\ max\left\{ \begin{matrix} \left[1 - \langle\left(min\{x_{ij}^m, y_{ij}^m\}\right)^3 + \left(max\{x_{ij}^n, y_{ij}^n\}\right)^3\rangle\right]^{\frac{1}{3}}, \\ min\left(x_{ij}^h, y_{ij}^h\right) \end{matrix} \right\} \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} min\left(y_{ij}^m, x_{ij}^m\right), max\left(y_{ij}^n, x_{ij}^n\right), \\ max\left\{ \begin{matrix} \left[1 - \langle\left(min\{y_{ij}^m, x_{ij}^m\}\right)^3 + \left(max\{y_{ij}^n, x_{ij}^n\}\right)^3\rangle\right]^{\frac{1}{3}}, \\ min\left(y_{ij}^h, x_{ij}^h\right) \end{matrix} \right\} \end{matrix} \right\}$$

$$= \tilde{B}_s \cdot \tilde{A}_s$$

Thus $\breve{A}_F \wedge_F \breve{B}_F = \breve{B}_F \wedge_F \breve{A}_F$ holds

(iii). To Prove $\breve{\mathbf{A}}_{\mathbf{F}} \oplus \breve{\mathbf{B}}_{\mathbf{F}} = \breve{\mathbf{B}}_{\mathbf{F}} \oplus \breve{\mathbf{A}}_{\mathbf{F}}$

$$\breve{A}_F \oplus \breve{B}_F = \left\{ \begin{matrix} \left(\left(x_{ij}^m\right)^3 + \left(y_{ij}^m\right)^3 - \left(x_{ij}^m\right)^3\left(y_{ij}^m\right)^3\right)^{\frac{1}{3}}, x_{ij}^n y_{ij}^n, \\ \left[\left(1 - \left(y_{ij}^m\right)^3\right)\left(x_{ij}^h\right)^3 + \left(1 - \left(x_{ij}^m\right)^3\right)\left(y_{ij}^h\right)^3 - \left(x_{ij}^h\right)^3\left(y_{ij}^h\right)^3\right]^{1/3} \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} \left(\left(y_{ij}^m\right)^3 + \left(x_{ij}^m\right)^3 - \left(y_{ij}^m\right)^3\left(x_{ij}^m\right)^3\right)^{\frac{1}{3}}, y_{ij}^n x_{ij}^n, \\ \left[\left(1 - \left(x_{ij}^m\right)^3\right)\left(y_{ij}^h\right)^3 + \left(1 - \left(y_{ij}^m\right)^3\right)\left(x_{ij}^h\right)^3 - \left(y_{ij}^m\right)^3\left(x_{ij}^h\right)^3\right]^{1/3} \end{matrix} \right\}$$

$$= \breve{B}_F \oplus \breve{A}_F$$

Thus $\breve{A}_F \oplus \breve{B}_F = \breve{B}_F \oplus \breve{A}_F$ holds.

(iv). To Prove $\breve{\mathbf{A}}_{\mathbf{F}} \otimes \breve{\mathbf{B}}_{\mathbf{F}} = \breve{\mathbf{B}}_{\mathbf{F}} \otimes \breve{\mathbf{A}}_{\mathbf{F}}$

$$\breve{\mathrm{A}}_{\mathrm{F}} \otimes \breve{\mathrm{B}}_{\mathrm{F}} = \left\{ \begin{matrix} x_{ij}^m y_{ij}^m, \left(\left(x_{ij}^n\right)^3 + \left(y_{ij}^n\right)^3 - \left(x_{ij}^n\right)^3\left(y_{ij}^n\right)^3\right)^{\frac{1}{3}}, \\ \left[\left(1 - \left(y_{ij}^n\right)^3\right)\left(x_{ij}^h\right)^3 + \left(1 - \left(x_{ij}^n\right)^3\right)\left(y_{ij}^h\right)^3 - \left(x_{ij}^h\right)^3\left(y_{ij}^h\right)^3\right]^{1/3} \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} y_{ij}^m x_{ij}^m, \left(\left(y_{ij}^n\right)^3 + \left(x_{ij}^n\right)^3 - \left(y_{ij}^n\right)^3\left(x_{ij}^n\right)^3\right)^{\frac{1}{3}}, \\ \left[\left(1 - \left(x_{ij}^n\right)^3\right)\left(y_{ij}^h\right)^3 + \left(1 - \left(y_{ij}^n\right)^3\right)\left(x_{ij}^h\right)^3 - \left(y_{ij}^h\right)^3\left(x_{ij}^h\right)^3\right]^{1/3} \end{matrix} \right\}$$

$$= \breve{\mathbf{B}}_{\mathbf{F}} \otimes \breve{\mathbf{A}}_{\mathbf{F}}$$

Thus $\breve{\mathrm{A}}_{\mathrm{F}} \otimes \breve{\mathrm{B}}_{\mathrm{F}} = \breve{\mathrm{B}}_{\mathrm{F}} \otimes \breve{\mathrm{A}}_{\mathrm{F}}$ holds.

(v). To Prove $\boldsymbol{\lambda}\left(\breve{\mathbf{A}}_{\mathbf{F}} \oplus \breve{\mathbf{B}}_{\mathbf{F}}\right) = \boldsymbol{\lambda}\breve{\mathbf{A}}_{\mathbf{F}} \oplus \boldsymbol{\lambda}\breve{\mathbf{B}}_{\mathbf{F}}$

$$\lambda\left(\breve{\mathrm{A}}_{\mathrm{F}} \oplus \breve{\mathrm{B}}_{\mathrm{F}}\right) = \lambda \left\{ \begin{matrix} \left(\left(x_{ij}^m\right)^3 + \left(y_{ij}^m\right)^3 - \left(x_{ij}^m\right)^3\left(y_{ij}^m\right)^3\right)^{\frac{1}{3}}, x_{ij}^n y_{ij}^n, \\ \left[\left(1 - \left(y_{ij}^m\right)^3\right)\left(x_{ij}^h\right)^3 + \left(1 - \left(x_{ij}^m\right)^3\right)\left(y_{ij}^h\right)^3 - \left(x_{ij}^h\right)^3\left(y_{ij}^h\right)^3\right]^{1/3} \end{matrix} \right\}$$

$$= \left\{ \begin{array}{c} \left[1-\left(1-\left(\left(x_{ij}^{m}\right)^3+\left(y_{ij}^{m}\right)^3-\left(x_{ij}^{m}\right)^3\left(y_{ij}^{m}\right)^3\right)\right)^{\lambda}\right]^{\frac{1}{3}}, x_{ij}^{n\,\lambda} y_{ij}^{n\,\lambda}, \\ \left[\begin{array}{c} \left\{1-\left(\left(x_{ij}^{m}\right)^3+\left(y_{ij}^{m}\right)^3-\left(x_{ij}^{m}\right)^3\left(y_{ij}^{m}\right)^3\right)\right\}^{\lambda} \\ -\left\{\left(\begin{array}{c} 1-\left(\left(x_{ij}^{m}\right)^3+\left(y_{ij}^{m}\right)^3-\left(x_{ij}^{m}\right)^3\left(y_{ij}^{m}\right)^3\right)-\left(1-\left(y_{ij}^{m}\right)^3\right)\left(x_{ij}^{h}\right)^3 \\ -\left(1-\left(x_{ij}^{m}\right)^3\right)\left(y_{ij}^{h}\right)^3+\left(x_{ij}^{h}\right)^3\left(y_{ij}^{h}\right)^3 \end{array} \right)\right\}^{\lambda} \end{array} \right]^{1/3} \end{array} \right\} \dots \quad (1)$$

$$\lambda\breve{A}_F \oplus \lambda\breve{B}_F = \left\{\left[1-\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda}\right]^{\frac{1}{3}}, x_{ij}^{n\,\lambda}, \left[\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda}-\left(1-\left(x_{ij}^{m}\right)^3-\left(x_{ij}^{h}\right)^3\right)^{\lambda}\right]^{\frac{1}{3}}\right\}$$

$$\oplus \left\{\left[1-\left(1-\left(y_{ij}^{m}\right)^3\right)^{\lambda}\right]^{\frac{1}{3}}, y_{ij}^{n\,\lambda}, \left[\left(1-\left(y_{ij}^{m}\right)^3\right)^{\lambda}-\left(1-\left(y_{ij}^{m}\right)^3-\left(y_{ij}^{h}\right)^3\right)^{\lambda}\right]^{\frac{1}{3}}\right\}$$

$$= \left\{ \begin{array}{c} \left[1-\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda}+1-\left(1-\left(y_{ij}^{m}\right)^3\right)^{\lambda}-\left(1-\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda}\right)\left(1-\left(1-\left(y_{ij}^{m}\right)^3\right)^{\lambda}\right)\right]^{\frac{1}{2}}, x_{ij}^{n\,\lambda} y_{ij}^{n\,\lambda}, \\ \left\{ \begin{array}{c} 1-\left(1-\left(1-\left(y_{ij}^{m}\right)^3\right)^{\lambda}\right)\left[\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda}-\left(1-\left(x_{ij}^{m}\right)^3-\left(x_{ij}^{h}\right)^3\right)^{\lambda}\right]+ \\ \left\{ \begin{array}{c} 1-\left(1-\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda}\right)\left[\left(1-\left(y_{ij}^{m}\right)^3\right)^{\lambda}-\left(1-\left(y_{ij}^{m}\right)^3-\left(y_{ij}^{h}\right)^3\right)^{\lambda}\right] \\ -\left[\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda}-\left(1-\left(x_{ij}^{m}\right)^3-\left(x_{ij}^{h}\right)^3\right)^{\lambda}\right] \\ \left[\left(1-\left(y_{ij}^{m}\right)^3\right)^{\lambda}-\left(1-\left(y_{ij}^{m}\right)^3-\left(y_{ij}^{h}\right)^3\right)^{\lambda}\right] \end{array} \right\}^{1/2} \end{array} \right\} \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left[1-\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda}\left(1-\left(y_{ij}^{m}\right)^3\right)^{\lambda}\right]^{\frac{1}{2}}, x_{ij}^{n\,\lambda} y_{ij}^{n\,\lambda}, \\ \left[\begin{array}{c} \left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda}\left(1-\left(y_{ij}^{m}\right)^3\right)^{\lambda}-\left[\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda}-\left(x_{ij}^{h}\right)^3\right]^{\lambda} \\ \left[\left(1-\left(y_{ij}^{m}\right)^3\right)^{\lambda}-\left(y_{ij}^{h}\right)^3\right]^{\lambda} \end{array} \right]^{1/2} \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left[1-\left(1-\left(\left(x_{ij}^{m}\right)^3+\left(y_{ij}^{m}\right)^3-\left(x_{ij}^{m}\right)^3\left(y_{ij}^{m}\right)^3\right)\right)^{\lambda}\right]^{\frac{1}{2}}, x_{ij}^{n\,\lambda} y_{ij}^{n\,\lambda}, \\ \left[\begin{array}{c} \left\{1-\left(\left(x_{ij}^{m}\right)^3+\left(y_{ij}^{m}\right)^3-\left(x_{ij}^{m}\right)^3\left(y_{ij}^{m}\right)^3\right)\right\}^{\lambda} \\ -\left\{\left(\begin{array}{c} 1-\left(\left(x_{ij}^{m}\right)^3+\left(y_{ij}^{m}\right)^3-\left(x_{ij}^{m}\right)^3\left(y_{ij}^{m}\right)^3\right)-\left(1-\left(y_{ij}^{m}\right)^{3^2}\right)\left(x_{ij}^{h}\right)^3 \\ -\left(1-\left(x_{ij}^{m}\right)^3\right)\left(y_{ij}^{h}\right)^3+\left(x_{ij}^{h}\right)^3\left(y_{ij}^{h}\right)^3 \end{array} \right)\right\}^{\lambda} \end{array} \right]^{1/2} \end{array} \right\} \dots \quad (2)$$

By (A) and (B), $\lambda\left(\breve{A}_F \oplus \breve{B}_F\right) = \lambda\breve{A}_F \oplus \lambda\breve{B}_F$ holds.

(vi). To Prove $\boldsymbol{\lambda_1\breve{A}_F \oplus \lambda_2\breve{A}_F = (\lambda_1 \oplus \lambda_2)\breve{A}_F}$

$$\lambda_1 \breve{A}_F \oplus \lambda_2 \breve{A}_F = \left\{ \left[1-\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_1}\right]^{\frac{1}{3}}, {x_{ij}^{n}}^{\lambda_1}, \left[\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_1} - \left(1-\left(x_{ij}^{m}\right)^3-\left(x_{ij}^{h}\right)^3\right)^{\lambda_1}\right]^{\frac{1}{2}} \right\}$$

$$\oplus \left\{ \left[1-\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_2}\right]^{\frac{1}{2}}, {x_{ij}^{n}}^{\lambda_1}, \left[\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_2} - \left(1-\left(x_{ij}^{m}\right)^3-\left(x_{ij}^{h}\right)^3\right)^{\lambda_2}\right]^{\frac{1}{2}} \right\}$$

$$= \left\{ \begin{array}{c} \left(1-\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_1} + 1-\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_2} - \left(1-\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_1}\right)\left(1-\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_2}\right)\right)^{\frac{1}{3}}, \\ {x_{ij}^{n}}^{\lambda_1+\lambda_2}, \left\{ \begin{array}{c} \left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_2}\left[\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_1} - \left(1-\left(x_{ij}^{m}\right)^3-\left(x_{ij}^{h}\right)^3\right)^{\lambda_1}\right] \\ +\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_1}\left[\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_2} - \left(1-\left(x_{ij}^{m}\right)^3-\left(x_{ij}^{h}\right)^3\right)^{\lambda_2}\right] \\ -\left[\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_1} - \left(1-\left(x_{ij}^{m}\right)^3-\left(x_{ij}^{h}\right)^3\right)^{\lambda_1}\right]\left[\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_2} - \left(1-\left(x_{ij}^{m}\right)^3-\left(x_{ij}^{h}\right)^3\right)^{\lambda_2}\right] \end{array} \right\}^{1/3} \end{array} \right\}$$

$$= \left\{ \left[1-\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_1+\lambda_2}\right]^{\frac{1}{3}}, {x_{ij}^{n}}^{\lambda_1+\lambda_2}, \left[\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_1+\lambda_2} - \left(1-\left(x_{ij}^{m}\right)^3-\left(x_{ij}^{h}\right)^3\right)^{\lambda_1+\lambda_2}\right]^{\frac{1}{3}} \right\} \quad \ldots (3)$$

$(\lambda_1 + \lambda_2)\tilde{A}_S$

$$= \left\{ \left[1-\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_1+\lambda_2}\right]^{\frac{1}{3}}, {x_{ij}^{n}}^{\lambda_1+\lambda_2}, \left[\left(1-\left(x_{ij}^{m}\right)^3\right)^{\lambda_1+\lambda_2} - \left(1-\left(x_{ij}^{m}\right)^3-\left(x_{ij}^{h}\right)^3\right)^{\lambda_1+\lambda_2}\right]^{\frac{1}{3}} \right\} \ldots (4)$$

By (C) and (D), $\lambda_1 \breve{A}_F \oplus \lambda_2 \breve{A}_F = (\lambda_1 \oplus \lambda_2)\breve{A}_F$ holds

(vii). To Prove $\left(\breve{\mathbf{A}}_{\mathbf{F}} \otimes \breve{\mathbf{B}}_{\mathbf{F}}\right)^{\boldsymbol{\lambda}} = {\breve{\mathbf{A}}_{\mathbf{F}}}^{\boldsymbol{\lambda}} \otimes {\breve{\mathbf{B}}_{\mathbf{F}}}^{\boldsymbol{\lambda}}$

$$\left(\breve{A}_F \otimes \breve{B}_F\right)^{\lambda} = \left\{ \begin{array}{c} x_{ij}^{m} x_{ij}^{n}, \left(\left(x_{ij}^{n}\right)^3 + \left(y_{ij}^{n}\right)^3 - \left(x_{ij}^{n}\right)^3\left(y_{ij}^{n}\right)^3\right)^{\frac{1}{3}}, \\ \left[\left(1-\left(y_{ij}^{n}\right)^3\right) {a_{ij}^{*}}^{2} + \left(1-\left(x_{ij}^{n}\right)^3\right)\left(y_{ij}^{h}\right)^3 - \left(x_{ij}^{h}\right)^3\left(y_{ij}^{h}\right)^3\right]^{1/3} \end{array} \right\}^{\lambda}$$

$$= \left\{ \begin{array}{c} {x_{ij}^{m}}^{\lambda} {x_{ij}^{n}}^{\lambda}\left(1-\left[1-\left(x_{ij}^{n}\right)^3 + \left(y_{ij}^{n}\right)^3 - \left(x_{ij}^{n}\right)^3\left(y_{ij}^{n}\right)^3\right]^{\lambda}\right)^{\frac{1}{3}}, \\ \left\{ \begin{array}{c} \left[1-\left(\left(x_{ij}^{n}\right)^3 + \left(y_{ij}^{n}\right)^3 - \left(x_{ij}^{n}\right)^3\left(y_{ij}^{n}\right)^3\right)\right]^{\lambda} \\ -\left[\left(\begin{array}{c} 1-\left(\left(x_{ij}^{n}\right)^3 + \left(y_{ij}^{n}\right)^3 - \left(x_{ij}^{n}\right)^3\left(y_{ij}^{n}\right)^3\right) - \left(1-\left(y_{ij}^{n}\right)^3\right)\left(x_{ij}^{h}\right)^3 \\ -\left(1-\left(x_{ij}^{n}\right)^3\right)\left(y_{ij}^{h}\right)^3 + \left(x_{ij}^{h}\right)^3\left(y_{ij}^{h}\right)^3 \end{array} \right)\right]^{\lambda} \end{array} \right\}^{1/3} \end{array} \right\} \quad \ldots \quad (5)$$

$${\breve{A}_F}^{\lambda} \otimes {\breve{B}_F}^{\lambda} = \left\{ {x_{ij}^{m}}^{\lambda}, \left[1-\left(1-\left(x_{ij}^{n}\right)^3\right)^{\lambda}\right]^{\frac{1}{3}}, \left[\left(1-\left(x_{ij}^{n}\right)^3\right)^{\lambda} - \left(1-\left(x_{ij}^{n}\right)^3-\left(x_{ij}^{h}\right)^3\right)^{\lambda}\right]^{\frac{1}{3}} \right\}$$

$$\otimes\left\{y_{ij}^{m\,\lambda},\left[1-\left(1-\left(y_{ij}^{n}\right)^{3}\right)^{\lambda}\right]^{\frac{1}{3}},\left[\left(1-\left(y_{ij}^{n}\right)^{3^{2}}\right)^{\lambda}-\left(1-\left(y_{ij}^{n}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\right)^{\lambda}\right]^{\frac{1}{3}}\right\}$$

$$=\left\{\begin{array}{c} x_{ij}^{m\,\lambda}y_{ij}^{n\,\lambda}\left(1-\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda}+1-\left(1-\left(y_{ij}^{n}\right)^{3}\right)^{\lambda}-\left[1-\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda}\right]\left[1-\left(1-\left(y_{ij}^{n}\right)^{3}\right)^{\lambda}\right]\right)^{\frac{1}{3}}, \\ \left\{\begin{array}{c} 1-\left(1-\left(1-\left(y_{ij}^{n}\right)^{3}\right)^{\lambda}\right)\left[\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda}-\left(1-\left(x_{ij}^{n}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\right)^{\lambda}\right] \\ +\left\{\begin{array}{c} 1-\left(1-\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda}\right)\left[\left(1-\left(y_{ij}^{n}\right)^{3}\right)^{\lambda}-\left(1-\left(y_{ij}^{n}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\right)^{\lambda}\right] \\ -\left[\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda}-\left(1-\left(x_{ij}^{n}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\right)^{\lambda}\right]\left[\left(1-\left(y_{ij}^{n}\right)^{3}\right)^{\lambda}-\left(1-\left(y_{ij}^{n}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\right)^{\lambda}\right] \end{array}\right\} \end{array}\right\}^{1/3} \end{array}\right\}$$

$$=\left\{\begin{array}{c} x_{ij}^{m\,\lambda}y_{ij}^{n\,\lambda},\left[1-\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda}\left(1-\left(y_{ij}^{n}\right)^{3}\right)^{\lambda}\right]^{\frac{1}{3}}, \\ \left[\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda}\left(1-\left(y_{ij}^{n}\right)^{3}\right)^{\lambda}-\left[\left(1-\left(x_{ij}^{n}\right)^{3}\right)-\left(x_{ij}^{h}\right)^{3}\right]^{\lambda}\left[\left(1-\left(y_{ij}^{n}\right)^{3}\right)-\left(y_{ij}^{h}\right)^{3}\right]^{\lambda}\right]^{1/3} \end{array}\right\}$$

$$=\left\{\begin{array}{c} x_{ij}^{m\,\lambda}y_{ij}^{n\,\lambda}\left(1-\left[1-\left(x_{ij}^{n}\right)^{3}+\left(y_{ij}^{n}\right)^{3}-\left(x_{ij}^{n}\right)^{3}\left(y_{ij}^{n}\right)^{3}\right]^{\lambda}\right)^{\frac{1}{3}}, \\ \left\{\begin{array}{c} \left[1-\left(\left(x_{ij}^{n}\right)^{3}+\left(y_{ij}^{n}\right)^{3}-\left(x_{ij}^{n}\right)^{3}\left(y_{ij}^{n}\right)^{3}\right)\right]^{\lambda} \\ -\left[\left(\begin{array}{c} 1-\left(\left(x_{ij}^{n}\right)^{3}+\left(y_{ij}^{n}\right)^{3}-\left(x_{ij}^{n}\right)^{3}\left(y_{ij}^{n}\right)^{3}\right)-\left(1-\left(y_{ij}^{n}\right)^{3}\right)\left(x_{ij}^{h}\right)^{3} \\ -\left(1-\left(x_{ij}^{n}\right)^{3}\right)b_{ij}^{*\,2}+\left(x_{ij}^{h}\right)^{3}\left(y_{ij}^{h}\right)^{3} \end{array}\right)\right]^{\lambda} \end{array}\right\}^{1/3} \end{array}\right\} \ldots \quad (6)$$

By (E) and (F), $\left(\breve{A}_F\otimes\breve{B}_F\right)^{\lambda}=\breve{A}_F{}^{\lambda}\otimes\breve{B}_F{}^{\lambda}$ holds.

(viii). To Prove $\mathbf{\breve{A}_F{}^{\lambda_1}\otimes\breve{A}_F{}^{\lambda_2}=\breve{A}_F{}^{\lambda_1+\lambda_2}}$

$$\breve{A}_F{}^{\lambda_1}\otimes\breve{A}_F{}^{\lambda_2}=\left\{\begin{array}{c} x_{ij}^{m\,\lambda_1},\left[1-\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_1}\right]^{\frac{1}{3}}, \\ \left[\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_1}-\left(1-\left(x_{ij}^{n}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\right)^{\lambda_1}\right]^{\frac{1}{3}} \end{array}\right\}$$

$$\otimes\left\{x_{ij}^{m\,\lambda_2},\left[1-\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_2}\right]^{\frac{1}{3}},\left[\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_2}-\left(1-\left(x_{ij}^{n}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\right)^{\lambda_2}\right]^{\frac{1}{3}}\right\}$$

$$=\left\{\begin{array}{c} x_{ij}^{m\,\lambda_1+\lambda_2},\left[1-\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_1}+1-\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_2}-\left[1-\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_1}\right]\left[1-\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_2}\right]\right]^{\frac{1}{3}} \\ \left\{\begin{array}{c} \left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_2}\left[\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_1}-\left(1-\left(x_{ij}^{n}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\right)^{\lambda_1}\right] \\ +\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_1}\left[\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_2}-\left(1-\left(x_{ij}^{n}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\right)^{\lambda_2}\right] \\ -\left[\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_1}-\left(1-\left(x_{ij}^{n}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\right)^{\lambda_1}\right]\left[\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_2}-\left(1-\left(x_{ij}^{n}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\right)^{\lambda_2}\right] \end{array}\right\}^{1/3} \end{array}\right\}$$

$$= \left\{ \begin{array}{c} {x_{ij}^{m}}^{\lambda_1+\lambda_2}, \left[1-\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_1+\lambda_2}\right]^{\frac{1}{3}}, \\ \left[\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_1+\lambda_2} - \left(1-\left(x_{ij}^{n}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\right)^{\lambda_1+\lambda_2}\right]^{1/3} \end{array} \right\} \quad \ldots.. \quad (7)$$

$$\breve{A}_F^{\ \lambda_1+\lambda_2} = \left\{ \begin{array}{c} {x_{ij}^{m}}^{\lambda_1+\lambda_2}, \left[1-\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_1+\lambda_2}\right]^{\frac{1}{3}}, \\ \left[\left(1-\left(x_{ij}^{n}\right)^{3}\right)^{\lambda_1+\lambda_2} - \left(1-\left(x_{ij}^{n}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\right)^{\lambda_1+\lambda_2}\right]^{1/3} \end{array} \right\} \quad \ldots.. \quad (8)$$

By (G) and (H) $\breve{A}_F^{\ \lambda_1} \otimes \breve{A}_F^{\ \lambda_2} = \breve{A}_F^{\ \lambda_1+\lambda_2}$ holds.

Proposition 4.1

Let us consider the Spherical Fuzzy matrices $\breve{A}_F$, $\breve{B}_F$ and $\breve{C}_F$ of order $m \times n$ are defined as $\breve{A}_F = \left[\left(x_{ij}^{m}, x_{ij}^{n}, x_{ij}^{h}\right)\right]_{(m\times n)}$, $\breve{B}_F = \left[\left(y_{ij}^{m}, y_{ij}^{n}, y_{ij}^{h}\right)\right]_{(m\times n)}$ and $\breve{C}_F = \left[\left(z_{ij}^{m}, z_{ij}^{n}, z_{ij}^{h}\right)\right]_{(m\times n)}$ then satisfies the conditions,

(i) $\breve{\mathbf{A}}_{\mathbf{F}} \vee_{\mathbf{F}} \left(\breve{\mathbf{B}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}}\right) = \left(\breve{\mathbf{A}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{B}}_{\mathbf{F}}\right) \vee_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}}$

(ii) $\breve{\mathbf{A}}_{\mathbf{F}} \wedge_{F} \left(\breve{\mathbf{B}}_{\mathbf{F}} \wedge_{F} \breve{\mathbf{C}}_{\mathbf{F}}\right) = \left(\breve{\mathbf{A}}_{\mathbf{F}} \wedge_{F} \breve{\mathbf{B}}_{\mathbf{F}}\right) \wedge_{F} \breve{\mathbf{C}}_{\mathbf{F}}$

(iii) $\breve{\mathbf{A}}_{\mathbf{F}} \wedge_{F} \left(\breve{\mathbf{B}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}}\right) = \left(\breve{\mathbf{A}}_{\mathbf{F}} \wedge_{F} \breve{\mathbf{B}}_{\mathbf{F}}\right) \vee_{\mathbf{F}} \left(\breve{\mathbf{A}}_{\mathbf{F}} \wedge_{F} \breve{\mathbf{C}}_{\mathbf{F}}\right)$

(iv) $\breve{\mathbf{A}}_{\mathbf{F}} \vee_{\mathbf{F}} \left(\breve{\mathbf{B}}_{\mathbf{F}} \wedge_{F} \breve{\mathbf{C}}_{\mathbf{F}}\right) = \left(\breve{\mathbf{A}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{B}}_{\mathbf{F}}\right) \wedge_{F} \left(\breve{\mathbf{A}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}}\right)$

Proof:

(i) To Prove $\breve{\mathbf{A}}_{\mathbf{F}} \vee_{\mathbf{F}} \left(\breve{\mathbf{B}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}}\right) = \left(\breve{\mathbf{A}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{B}}_{\mathbf{F}}\right) \vee_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}}$

$$\breve{A}_F \vee_F \left(\breve{B}_F \vee_F \breve{C}_F\right) = \breve{A}_F \vee_F \left\{ \begin{array}{c} max\left(y_{ij}^{m}, z_{ij}^{m}\right), min\left(y_{ij}^{n}, z_{ij}^{n}\right), \\ min\left\{ \begin{array}{c} \left[1-\langle\left(max\{y_{ij}^{m}, z_{ij}^{m}\}\right)^{3} + \left(min\{y_{ij}^{n}, z_{ij}^{n}\}\right)^{3}\rangle\right]^{\frac{1}{3}}, \\ max\left(y_{ij}^{h}, z_{ij}^{h}\right) \end{array} \right\} \end{array} \right\}$$

$$= \left\{ \begin{array}{c} max\left(x_{ij}^{m}, d_{ij}^{m}\right), min\left(x_{ij}^{n}, e_{ij}^{n}\right), \\ min\left\{ \begin{array}{c} \left[1-\langle\left(max\{x_{ij}^{m}, f_{ij}^{m}\}\right)^{3} + \left(min\{x_{ij}^{n}, g_{ij}^{n}\}\right)^{3}\rangle\right]^{\frac{1}{3}}, \\ max\left(x_{ij}^{h}, d_{ij}^{h}\right) \end{array} \right\} \end{array} \right\}$$

$$= \left[\left(d_{ij}^{m}, d_{ij}^{n}, d_{ij}^{h}\right)\right] \quad \ldots\ldots\ldots \quad (9)$$

$$\left(\breve{A}_F \vee_F \breve{B}_F\right) \vee_F \breve{C}_F = \left\{ \begin{array}{c} max\left(x_{ij}^{m}, y_{ij}^{m}\right), min\left(x_{ij}^{n}, y_{ij}^{n}\right), \\ min\left\{ \begin{array}{c} \left[1-\langle\left(max\{x_{ij}^{m}, y_{ij}^{m}\}\right)^{3} + \left(min\{x_{ij}^{n}, y_{ij}^{n}\}\right)^{3}\rangle\right]^{\frac{1}{3}}, \\ max\left(x_{ij}^{h}, y_{ij}^{h}\right) \end{array} \right\} \end{array} \right\} \vee_F \breve{C}_F$$

$$= \left\{ \begin{matrix} max\left(z_{ij}^{m}, d_{ij}^{m}\right), min\left(x_{ij}^{n}, d_{ij}'\right), \\ min\left\{ \begin{matrix} \left[1 - \langle\left(max\left\{x_{ij}^{m}, f_{ij}^{m}\right\}\right)^{3} + \left(min\left\{x_{ij}^{n}, g_{ij}^{n}\right\}\right)^{3}\rangle\right]^{\frac{1}{3}}, \\ max\left(x_{ij}^{h}, d_{ij}^{h}\right) \end{matrix} \right\} \end{matrix} \right\}$$

$$= \left[\left(d_{ij}^{m}, d_{ij}^{n}, d_{ij}^{h}\right)\right] \qquad \text{.........} \qquad (10)$$

By (I) and (J), $\breve{A}_F \vee_F \left(\breve{B}_F \vee_F \breve{C}_F\right) = \left(\breve{A}_F \vee_F \breve{B}_F\right) \vee_F \breve{C}_F$ holds.

(ii) To Prove $\breve{\mathbf{A}}_{\mathbf{F}} \wedge_{\mathbf{F}} \left(\breve{\mathbf{B}}_{\mathbf{F}} \wedge_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}}\right) = \left(\breve{\mathbf{A}}_{\mathbf{F}} \wedge_{\mathbf{F}} \breve{\mathbf{B}}_{\mathbf{F}}\right) \wedge_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}}$

$$\breve{\mathbf{A}}_{\mathbf{F}} \wedge_F \left(\breve{\mathbf{B}}_{\mathbf{F}} \wedge_F \breve{\mathbf{C}}_{\mathbf{F}}\right) = \breve{\mathbf{A}}_{\mathbf{F}} \wedge_F \left\{ \begin{matrix} min\left(y_{ij}^{m}, z_{ij}^{m}\right), max\left(y_{ij}^{n}, z_{ij}^{n}\right), \\ max\left\{ \begin{matrix} \left[1 - \langle\left(min\left\{y_{ij}^{m}, z_{ij}^{m}\right\}\right)^{3} + \left(max\left\{y_{ij}^{n}, z_{ij}^{n}\right\}\right)^{3}\rangle\right]^{\frac{1}{3}}, \\ min\left(x_{ij}^{h}, z_{ij}^{h}\right) \end{matrix} \right\} \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} min\left(x_{ij}^{m}, p_{ij}^{m}\right), max\left(x_{ij}^{n}, p_{ij}^{n}\right), \\ max\left\{ \begin{matrix} \left[1 - \langle\left(min\left\{x_{ij}^{m}, q_{ij}^{m}\right\}\right)^{3} + \left(max\left\{x_{ij}^{n}, r_{ij}^{n}\right\}\right)^{3}\rangle\right]^{\frac{1}{3}}, \\ min\left(x_{ij}^{h}, p_{ij}^{h}\right) \end{matrix} \right\} \end{matrix} \right\}$$

$$= \left[\left(p_{ij}^{m}, p_{ij}^{n}, p_{ij}^{h}\right)\right] \qquad \text{.............} \qquad (11)$$

$$\left(\breve{\mathbf{A}}_{\mathbf{F}} \wedge_F \breve{\mathbf{B}}_{\mathbf{F}}\right) \wedge_F \breve{\mathbf{C}}_{\mathbf{F}} = \left\{ \begin{matrix} min\left(x_{ij}^{m}, y_{ij}^{m}\right), max\left(x_{ij}^{n}, y_{ij}^{n}\right), \\ max\left\{ \begin{matrix} \left[1 - \langle\left(min\left\{x_{ij}^{m}, y_{ij}^{m}\right\}\right)^{3} + \left(max\left\{x_{ij}^{n}, y_{ij}^{n}\right\}\right)^{3}\rangle\right]^{\frac{1}{3}}, \\ min\left(x_{ij}^{h}, y_{ij}^{h}\right) \end{matrix} \right\} \end{matrix} \right\} \wedge_F \breve{\mathbf{C}}_{\mathbf{F}}$$

$$= \left\{ \begin{matrix} min\left(p_{ij}^{m}, z_{ij}^{m}\right), max\left(p_{ij}^{n}, z_{ij}^{n}\right), \\ max\left\{ \begin{matrix} \left[1 - \langle\left(min\left\{q_{ij}, z_{ij}^{m}\right\}\right)^{2} + \left(max\left\{r_{ij}', z_{ij}^{n}\right\}\right)^{2}\rangle\right]^{\frac{1}{2}}, \\ min\left(p_{ij}^{h}, z_{ij}^{h}\right) \end{matrix} \right\} \end{matrix} \right\}$$

$$= \left[\left(p_{ij}^{m}, p_{ij}^{n}, p_{ij}^{h}\right)\right] \qquad \text{.............} \qquad (12)$$

By (K) and (L), $\breve{\mathbf{A}}_{\mathbf{F}} \wedge_F \left(\breve{\mathbf{B}}_{\mathbf{F}} \cdot \breve{\mathbf{C}}_{\mathbf{F}}\right) = \left(\breve{\mathbf{A}}_{\mathbf{F}} \wedge_F \breve{\mathbf{B}}_{\mathbf{F}}\right) \wedge_F \breve{\mathbf{C}}_{\mathbf{F}}$ holds.

(iii). To Prove $\breve{\mathbf{A}}_{\mathbf{F}} \wedge_{\mathbf{F}} \left(\breve{\mathbf{B}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}}\right) = \left(\breve{\mathbf{A}}_{\mathbf{F}} \wedge_{\mathbf{F}} \breve{\mathbf{B}}_{\mathbf{F}}\right) \vee_{\mathbf{F}} \left(\breve{\mathbf{A}}_{\mathbf{F}} \wedge_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}}\right)$

$$\breve{\mathbf{A}}_{\mathbf{F}} \wedge_F \left(\breve{\mathbf{B}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}}\right) = \breve{\mathbf{A}}_{\mathbf{F}} \wedge_F \left\{ \begin{matrix} max\left(y_{ij}^{m}, z_{ij}^{m}\right), min\left(y_{ij}^{n}, z_{ij}^{n}\right), \\ min\left\{ \begin{matrix} \left[1 - \langle\left(max\left\{y_{ij}^{m}, z_{ij}^{m}\right\}\right)^{3} + \left(min\left\{y_{ij}^{n}, z_{ij}^{n}\right\}\right)^{3}\rangle\right]^{\frac{1}{3}}, \\ max\left(y_{ij}^{h}, z_{ij}^{h}\right) \end{matrix} \right\} \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} min\left(x_{ij}^{m}, u_{ij}^{m}\right), max\left(x_{ij}^{n}, u_{ij}^{n}\right), \\ max\left\{ \begin{matrix} \left[1 - \langle\left(min\left\{x_{ij}^{m}, v_{ij}^{m}\right\}\right)^{2} + \left(max\left\{x_{ij}^{n}, w_{ij}^{n}\right\}\right)^{2}\rangle\right]^{\frac{1}{2}}, \\ min\left(x_{ij}^{h}, u_{ij}^{h}\right) \end{matrix} \right\} \end{matrix} \right\}$$

$$= \left[\left(u_{ij}^{m}, u_{ij}^{n}, u_{ij}^{h}\right)\right] \qquad \text{.............} \qquad (13)$$

$$(\breve{\mathbf{A}}_{\mathbf{F}}\wedge_F\breve{\mathbf{B}}_{\mathbf{F}})\vee_{\mathbf{F}}(\breve{\mathbf{A}}_{\mathbf{F}}\wedge_F\breve{\mathbf{C}}_{\mathbf{F}}) = \left\{\begin{matrix} min(x_{ij}^m, y_{ij}^m), max(x_{ij}^n, y_{ij}^n), \\ max\left\{\begin{matrix}\left[1-\langle\left(min\{x_{ij}^m, y_{ij}^m\}\right)^3+\left(max\{x_{ij}^n, y_{ij}^n\}\right)^3\rangle\right]^{\frac{1}{3}}, \\ min(x_{ij}^h, y_{ij}^h)\end{matrix}\right\}\end{matrix}\right\}$$

$$\vee_{\mathbf{F}}\left\{\begin{matrix} min(x_{ij}^m, z_{ij}^m), max(x_{ij}^n, z_{ij}^n), \\ max\left\{\begin{matrix}\left[1-\langle\left(min\{x_{ij}^m, z_{ij}^m\}\right)^3+\left(max\{x_{ij}^n, z_{ij}^n\}\right)^3\rangle\right]^{\frac{1}{3}}, \\ min(x_{ij}^h, z_{ij}^h)\end{matrix}\right\}\end{matrix}\right\}$$

$$= \left\{\begin{matrix} max(l_{ij}^m, u_{ij}^m), min(l_{ij}^n, u_{ij}^n), \\ min\left\{\begin{matrix}\left[1-\langle\left(max\{l_{ij}^m, u_{ij}^m\}\right)^2+\left(min\{l_{ij}^n, u_{ij}^n\}\right)^2\rangle\right]^{\frac{1}{2}}, \\ max(l_{ij}^h, u_{ij}^h)\end{matrix}\right\}\end{matrix}\right\}$$

$= \left[\left(u_{ij}^m, u_{ij}^n, u_{ij}^h\right)\right]$ ………….. (14)

By (M) and (N), $\breve{\mathbf{A}}_{\mathbf{F}}\wedge_F\left(\breve{\mathbf{B}}_{\mathbf{F}}\vee_{\mathbf{F}}\breve{\mathbf{C}}_{\mathbf{F}}\right) = \left(\breve{\mathbf{A}}_{\mathbf{F}}\wedge_F\breve{\mathbf{B}}_{\mathbf{F}}\right)\vee_{\mathbf{F}}\left(\breve{\mathbf{A}}_{\mathbf{F}}\wedge_F\breve{\mathbf{C}}_{\mathbf{F}}\right)$ holds.

(iv). To Prove $\breve{\mathbf{A}}_{\mathbf{F}}\vee_{\mathbf{F}}\left(\breve{\mathbf{B}}_{\mathbf{F}}\wedge_{\mathbf{F}}\breve{\mathbf{C}}_{\mathbf{F}}\right) = \left(\breve{\mathbf{A}}_{\mathbf{F}}\vee_{\mathbf{F}}\breve{\mathbf{B}}_{\mathbf{F}}\right)\wedge_{\mathbf{F}}\left(\breve{\mathbf{A}}_{\mathbf{F}}\vee_{\mathbf{F}}\breve{\mathbf{C}}_{\mathbf{F}}\right)$

$$\breve{\mathbf{A}}_{\mathbf{F}}\vee_{\mathbf{F}}(\breve{\mathbf{B}}_{\mathbf{F}}\wedge_F\breve{\mathbf{C}}_{\mathbf{F}}) = \breve{\mathbf{A}}_{\mathbf{F}}\vee_{\mathbf{F}}\left\{\begin{matrix} min(y_{ij}^m, z_{ij}^m), max(y_{ij}^n, z_{ij}^n), \\ max\left\{\begin{matrix}\left[1-\langle\left(min\{y_{ij}^m, z_{ij}^m\}\right)^3+\left(max\{y_{ij}^n, z_{ij}^n\}\right)^3\rangle\right]^{\frac{1}{3}}, \\ min(x_{ij}^h, z_{ij}^h)\end{matrix}\right\}\end{matrix}\right\}$$

$$= \left\{\begin{matrix} max(x_{ij}^m, r_{ij}^m), min(x_{ij}^n, r_{ij}^n), \\ min\left\{\begin{matrix}\left[1-\langle\left(max\{x_{ij}^m, s_{ij}^m\}\right)^2+\left(min\{x_{ij}^n, t_{ij}^n\}\right)^2\rangle\right]^{\frac{1}{2}}, \\ max(x_{ij}^h, r_{ij}^h)\end{matrix}\right\}\end{matrix}\right\}$$

$= \left[\left(r_{ij}^m, r_{ij}^n, r_{ij}^h\right)\right]$ ………….. (15)

$$(\breve{\mathbf{A}}_{\mathbf{F}}\vee_{\mathbf{F}}\breve{\mathbf{B}}_{\mathbf{F}})\wedge_F(\breve{\mathbf{A}}_{\mathbf{F}}\vee_{\mathbf{F}}\breve{\mathbf{C}}_{\mathbf{F}}) = \left\{\begin{matrix} max(x_{ij}^m, y_{ij}^m), min(x_{ij}^n, y_{ij}^n), \\ min\left\{\begin{matrix}\left[1-\langle\left(max\{x_{ij}^m, y_{ij}^m\}\right)^3+\left(min\{x_{ij}^n, y_{ij}^n\}\right)^3\rangle\right]^{\frac{1}{3}}, \\ max(x_{ij}^h, y_{ij}^h)\end{matrix}\right\}\end{matrix}\right\}$$

$$\wedge_F\left\{\begin{matrix} max(x_{ij}^m, z_{ij}^m), min(x_{ij}^n, z_{ij}^n), \\ min\left\{\begin{matrix}\left[1-\langle\left(max\{x_{ij}^m, z_{ij}^m\}\right)^3+\left(min\{x_{ij}^n, z_{ij}^n\}\right)^3\rangle\right]^{\frac{1}{3}}, \\ max(x_{ij}^h, z_{ij}^h)\end{matrix}\right\}\end{matrix}\right\}$$

$$= \left\{\begin{matrix} min(x_{ij}^m, r_{ij}^m), max(x_{ij}^n, r_{ij}^n), \\ min\left\{\begin{matrix}\left[1-\langle\left(min\{x_{ij}^m, s_{ij}^m\}\right)^2+\left(max\{x_{ij}^n, t_{ij}^n\}\right)^2\rangle\right]^{\frac{1}{2}}, \\ min(x_{ij}^h, h_{ij}^h)\end{matrix}\right\}\end{matrix}\right\}$$

$= \left[\left(r_{ij}^m, r_{ij}^n, r_{ij}^h\right)\right]$ ………….. (16)

By (O) and (P), $\breve{\mathbf{A}}_{\mathbf{F}} \vee_{\mathbf{F}} (\breve{\mathbf{B}}_{\mathbf{F}} \wedge_F \check{\mathbf{C}}_{\mathbf{F}}) = (\breve{\mathbf{A}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{B}}_{\mathbf{F}}) \wedge_F (\breve{\mathbf{A}}_{\mathbf{F}} \vee_{\mathbf{F}} \check{\mathbf{C}}_{\mathbf{F}})$ holds.

Proposition 4.2

Let us consider the Spherical Fuzzy matrices $\breve{\mathrm{A}}_{\mathrm{F}}$, $\breve{\mathrm{B}}_{\mathrm{F}}$ and $\check{\mathrm{C}}_{\mathrm{F}}$ of order $m \times n$ are defined as $\breve{\mathrm{A}}_{\mathrm{F}} = [(x_{ij}^m, x_{ij}^n, x_{ij}^h)]_{(m\times n)}$, $\breve{\mathrm{B}}_{\mathrm{F}} = [(y_{ij}^m, y_{ij}^n, y_{ij}^h)]_{(m\times n)}$ and $\check{\mathrm{C}}_{\mathrm{F}} = [(z_{ij}^m, z_{ij}^n, z_{ij}^h)]_{(m\times n)}$ then satisfies the conditions,

(i) $(\breve{\mathrm{A}}_{\mathrm{F}} \vee_{\boldsymbol{F}} \breve{\mathrm{B}}_{\mathrm{F}}) \oplus_F \check{\mathrm{C}}_{\mathrm{F}} = (\breve{\mathrm{A}}_{\mathrm{F}} \oplus_F \check{\mathrm{C}}_{\mathrm{F}}) \vee_{\boldsymbol{F}} \left((\breve{\mathbf{B}}_{\mathrm{F}} \oplus_F \check{\mathrm{C}}_{\mathrm{F}})\right)$

(ii) $(\breve{\mathrm{A}}_{\mathrm{F}} \wedge_{\boldsymbol{F}} \breve{\mathrm{B}}_{\mathrm{F}}) \oplus_F \check{\mathrm{C}}_{\mathrm{F}} = (\breve{\mathrm{A}}_{\mathrm{F}} \oplus_F \check{\mathrm{C}}_{\mathrm{F}}) \wedge_{\boldsymbol{F}} \left((\breve{\mathbf{B}}_{\mathrm{F}} \oplus_F \check{\mathrm{C}}_{\mathrm{F}})\right)$

(iii) $(\breve{\mathrm{A}}_{\mathrm{F}} \vee_{\boldsymbol{F}} \breve{\mathrm{B}}_{\mathrm{F}}) \otimes_F \check{\mathrm{C}}_{\mathrm{F}} = (\breve{\mathrm{A}}_{\mathrm{F}} \otimes_F \check{\mathrm{C}}_{\mathrm{F}}) \vee_{\boldsymbol{F}} \left((\breve{\mathbf{B}}_{\mathrm{F}} \otimes_F \check{\mathrm{C}}_{\mathrm{F}})\right)$

(iv) $(\breve{\mathrm{A}}_{\mathrm{F}} \wedge_{\boldsymbol{F}} \breve{\mathrm{B}}_{\mathrm{F}}) \otimes_F \check{\mathrm{C}}_{\mathrm{F}} = (\breve{\mathrm{A}}_{\mathrm{F}} \otimes_F \check{\mathrm{C}}_{\mathrm{F}}) \wedge_{\boldsymbol{F}} \left((\breve{\mathbf{B}}_{\mathrm{F}} \otimes_F \check{\mathrm{C}}_{\mathrm{F}})\right)$

Proof:

(i) To Prove $(\breve{\mathbf{A}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{B}}_{\mathbf{F}}) \oplus_{\mathbf{F}} \check{\mathbf{C}}_{\mathbf{F}} = (\breve{\mathbf{A}}_{\mathbf{F}} \oplus_{\mathbf{F}} \check{\mathbf{C}}_{\mathbf{F}}) \vee_{\mathbf{F}} \left((\breve{\mathbf{B}}_{\mathbf{F}} \oplus_{\mathbf{F}} \check{\mathbf{C}}_{\mathbf{F}})\right)$

$$(\breve{\mathrm{A}}_{\mathrm{F}} \vee_{\boldsymbol{F}} \breve{\mathrm{B}}_{\mathrm{F}}) \oplus_F \check{\mathrm{C}}_{\mathrm{F}} = \left\{\begin{matrix} max(x_{ij}^m, y_{ij}^m), min(x_{ij}^n, y_{ij}^n), \\ min\left\{\begin{matrix} \left[1 - \langle (max\{x_{ij}^m, y_{ij}^m\})^3 + (min\{x_{ij}^n, y_{ij}^n\})^3 \rangle\right]^{\frac{1}{3}}, \\ max(x_{ij}^h, y_{ij}^h) \end{matrix}\right\} \end{matrix}\right\} \oplus_F \check{\mathrm{C}}_{\mathrm{F}}$$

$$= \left\{\begin{matrix} \left((x_{ij}^m)^3 + (z_{ij}^m)^3 - (x_{ij}^m)^3 (z_{ij}^m)^3\right)^{\frac{1}{3}}, x_{ij}^n z_{ij}^n, \\ \left[\left(1 - (z_{ij}^m)^3\right)(x_{ij}^h)^3 + \left(1 - (x_{ij}^m)^3\right)(z_{ij}^h)^3 - (x_{ij}^h)^3 (z_{ij}^h)^3\right]^{1/3} \end{matrix}\right\} \ldots\ldots. \quad (17)$$

$$(\breve{\mathrm{A}}_{\mathrm{F}} \oplus_F \check{\mathrm{C}}_{\mathrm{F}}) \vee_{\boldsymbol{F}} \left((\breve{\mathbf{B}}_{\mathrm{F}} \oplus_F \check{\mathrm{C}}_{\mathrm{F}})\right) = \left[\begin{matrix} \left\{\begin{matrix} \left((x_{ij}^m)^3 + (z_{ij}^m)^3 - (x_{ij}^m)^3 (z_{ij}^m)^3\right)^{\frac{1}{3}}, x_{ij}^n z_{ij}^n, \\ \left[\left(1 - (z_{ij}^m)^3\right)(x_{ij}^h)^3 + \left(1 - (x_{ij}^m)^3\right)(z_{ij}^h)^3 - (x_{ij}^h)^3 (z_{ij}^h)^3\right]^{\frac{1}{3}} \end{matrix}\right\} \vee_{\boldsymbol{F}} \\ \left\{\begin{matrix} \left((y_{ij}^m)^3 + (z_{ij}^m)^3 - (y_{ij}^m)^3 (z_{ij}^m)^3\right)^{\frac{1}{3}}, y_{ij}^n z_{ij}^n, \\ \left[\left(1 - (z_{ij}^m)^3\right)(y_{ij}^h)^3 + \left(1 - (y_{ij}^m)^3\right)(z_{ij}^h)^3 - (y_{ij}^h)^3 (z_{ij}^h)^3\right]^{\frac{1}{3}} \end{matrix}\right\} \end{matrix}\right]$$

$$= \left\{ \begin{array}{c} max\left(\left(\left(x_{ij}^{m}\right)^{3}+\left(z_{ij}^{m}\right)^{3}-\left(x_{ij}^{m}\right)^{3}\left(z_{ij}^{m}\right)^{3}\right)^{\frac{1}{3}},\left(\left(y_{ij}^{m}\right)^{3}+\left(z_{ij}^{m}\right)^{3}-\left(y_{ij}^{m}\right)^{3}\left(z_{ij}^{m}\right)^{3}\right)^{\frac{1}{3}}\right), \\ min\left(x_{ij}^{n}z_{ij}^{n}, y_{ij}^{n}z_{ij}^{n}\right), \\ min\left\{\left[1-\left\langle \begin{array}{c} \left(max\left\{\begin{array}{c}\left(1-\left(z_{ij}^{m}\right)^{3}\right)\left(x_{ij}^{h}\right)^{3}+\left(1-\left(x_{ij}^{m}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}, \\ \left(1-\left(z_{ij}^{m}\right)^{3}\right)\left(y_{ij}^{h}\right)^{3}+\left(1-\left(y_{ij}^{m}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\end{array}\right\}\right)^{3}+ \\ \left(min\left\{\begin{array}{c}\left(1-\left(z_{ij}^{m}\right)^{3}\right)\left(x_{ij}^{h}\right)^{3}+\left(1-\left(x_{ij}^{m}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}, \\ \left(1-\left(z_{ij}^{m}\right)^{3}\right)\left(y_{ij}^{h}\right)^{3}+\left(1-\left(y_{ij}^{m}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\end{array}\right\}\right)^{3} \end{array}\right\rangle\right]^{\frac{1}{3}}, \\ max\left(x_{ij}^{h}, y_{ij}^{h}\right) \right\} \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left(\left(x_{ij}^{m}\right)^{3}+\left(z_{ij}^{m}\right)^{3}-\left(x_{ij}^{m}\right)^{3}\left(z_{ij}^{m}\right)^{3}\right)^{\frac{1}{3}}, x_{ij}^{n}z_{ij}^{n}, \\ \left[\left(1-\left(z_{ij}^{m}\right)^{3}\right)\left(x_{ij}^{h}\right)^{3}+\left(1-\left(x_{ij}^{m}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right]^{1/3} \end{array} \right\} \ldots\ldots. \quad (18)$$

By (P) and (R) ,$(\breve{A}_F \vee_{F} \breve{B}_F) \oplus_F \check{C}_F = (\breve{A}_F \oplus_F \check{C}_F) \vee_{F} \left((\breve{\mathbf{B}}_F \oplus_F \check{C}_F)\right)$holds.

(ii) To Prove $(\breve{\mathbf{A}}_{\mathbf{F}} \wedge_{\mathbf{F}} \breve{\mathbf{B}}_{\mathbf{F}}) \oplus_{\mathbf{F}} \check{\mathbf{C}}_{\mathbf{F}} = (\breve{\mathbf{A}}_{\mathbf{F}} \oplus_{\mathbf{F}} \check{\mathbf{C}}_{\mathbf{F}}) \wedge_{\mathbf{F}} \left((\breve{\mathbf{B}}_{\mathbf{F}} \oplus_{\mathbf{F}} \check{\mathbf{C}}_{\mathbf{F}})\right)$

$$(\breve{A}_F \wedge_{F} \breve{B}_F) \oplus_F \check{C}_F = \left\{ \begin{array}{c} min\left(x_{ij}^{m}, y_{ij}^{m}\right), max\left(x_{ij}^{n}, y_{ij}^{n}\right), \\ max\left\{\begin{array}{c}\left[1-\left\langle\left(min\{x_{ij}^{m}, y_{ij}^{m}\}\right)^{3}+\left(max\{x_{ij}^{n}, y_{ij}^{n}\}\right)^{3}\right\rangle\right]^{\frac{1}{3}}, \\ min\left(x_{ij}^{h}, y_{ij}^{h}\right)\end{array}\right\} \end{array} \right\} \oplus_F \check{C}_F$$

$$= \left\{ \begin{array}{c} \left(\left(y_{ij}^{m}\right)^{3}+\left(z_{ij}^{m}\right)^{3}-\left(y_{ij}^{m}\right)^{3}\left(z_{ij}^{m}\right)^{3}\right)^{\frac{1}{3}}, y_{ij}^{n}z_{ij}^{n}, \\ \left[\left(1-\left(z_{ij}^{m}\right)^{3}\right)\left(y_{ij}^{h}\right)^{3}+\left(1-\left(y_{ij}^{m}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right]^{1/3} \end{array} \right\} \ldots\ldots. \quad (19)$$

$$(\breve{A}_F \oplus_F \check{C}_F) \wedge_{F} \left((\breve{\mathbf{B}}_F \oplus_F \check{C}_F)\right) = \left[\begin{array}{c} \left\{ \begin{array}{c} \left(\left(x_{ij}^{m}\right)^{3}+\left(z_{ij}^{m}\right)^{3}-\left(x_{ij}^{m}\right)^{3}\left(z_{ij}^{m}\right)^{3}\right)^{\frac{1}{3}}, x_{ij}^{n}z_{ij}^{n}, \\ \left[\left(1-\left(z_{ij}^{m}\right)^{3}\right)\left(x_{ij}^{h}\right)^{3}+\left(1-\left(x_{ij}^{m}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right]^{\frac{1}{3}} \end{array} \right\} \\ \wedge_{F} \left\{ \begin{array}{c} \left(\left(y_{ij}^{m}\right)^{3}+\left(z_{ij}^{m}\right)^{3}-\left(y_{ij}^{m}\right)^{3}\left(z_{ij}^{m}\right)^{3}\right)^{\frac{1}{3}}, y_{ij}^{n}z_{ij}^{n}, \\ \left[\left(1-\left(z_{ij}^{m}\right)^{3}\right)\left(y_{ij}^{h}\right)^{3}+\left(1-\left(y_{ij}^{m}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right]^{1/3} \end{array} \right\} \end{array} \right]$$

$$= \left\{ \begin{array}{c} min\left(\left(\left(x_{ij}^{m}\right)^{3}+\left(z_{ij}^{m}\right)^{3}-\left(x_{ij}^{m}\right)^{3}\left(z_{ij}^{m}\right)^{3}\right)^{\frac{1}{3}},\left(\left(y_{ij}^{m}\right)^{3}+\left(z_{ij}^{m}\right)^{3}-\left(y_{ij}^{m}\right)^{3}\left(z_{ij}^{m}\right)^{3}\right)^{\frac{1}{3}}\right), max\left(x_{ij}^{n}z_{ij}^{n}, y_{ij}^{n}z_{ij}^{n}\right), \\ max\left\{ \begin{array}{c} \left[1-\langle \begin{array}{c} \left(min\left\{ \begin{array}{c} \left[\left(1-\left(z_{ij}^{m}\right)^{3}\right)\left(x_{ij}^{h}\right)^{3}+\left(1-\left(x_{ij}^{m}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right], \\ \left(1-\left(z_{ij}^{m}\right)^{3}\right)\left(y_{ij}^{h}\right)^{3}+\left(1-\left(y_{ij}^{m}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3} \end{array}\right\}\right)^{3} \\ +\left(max\left\{ \begin{array}{c} \left[\left(1-\left(z_{ij}^{m}\right)^{3}\right)\left(x_{ij}^{h}\right)^{3}+\left(1-\left(x_{ij}^{m}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right], \\ \left(1-\left(z_{ij}^{m}\right)^{3}\right)\left(y_{ij}^{h}\right)^{3}+\left(1-\left(y_{ij}^{m}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3} \end{array}\right\}\right)^{3} \end{array} \rangle\right]^{\frac{1}{3}}, \\ min\left(x_{ij}^{h}, y_{ij}^{h}\right) \end{array}\right\} \end{array}\right\}$$

$$= \left\{ \begin{array}{c} \left(\left(y_{ij}^{m}\right)^{3}+\left(z_{ij}^{m}\right)^{3}-\left(y_{ij}^{m}\right)^{3}\left(z_{ij}^{m}\right)^{3}\right)^{\frac{1}{3}}, y_{ij}^{n}z_{ij}^{n}, \\ \left[\left(1-\left(z_{ij}^{m}\right)^{3}\right)\left(y_{ij}^{h}\right)^{3}+\left(1-\left(y_{ij}^{m}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right]^{1/3} \end{array}\right\} \ldots\ldots \quad (20)$$

By (P) and (R), $\left(\breve{A}_F \wedge_F \breve{B}_F\right) \oplus_F \breve{C}_F = \left(\breve{A}_F \oplus_F \breve{C}_F\right) \wedge_F \left(\left(\breve{B}_F \oplus_F \breve{C}_F\right)\right)$ holds.

(iii) To Prove $\left(\breve{\mathbf{A}}_{\mathbf{F}} \vee_{\mathbf{F}} \breve{\mathbf{B}}_{\mathbf{F}}\right) \otimes_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}} = \left(\breve{\mathbf{A}}_{\mathbf{F}} \otimes_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}}\right) \vee_{\mathbf{F}} \left(\left(\breve{\mathbf{B}}_{\mathbf{F}} \otimes_{\mathbf{F}} \breve{\mathbf{C}}_{\mathbf{F}}\right)\right)$

$$\left(\breve{A}_F \vee_F \breve{B}_F\right) \otimes_F \breve{C}_F = \left\{ \begin{array}{c} max\left(x_{ij}^{m}, y_{ij}^{m}\right), min\left(x_{ij}^{n}, y_{ij}^{n}\right), \\ min\left\{ \begin{array}{c} \left[1-\langle\left(max\left\{x_{ij}^{m}, y_{ij}^{m}\right\}\right)^{3}+\left(min\left\{x_{ij}^{n}, y_{ij}^{n}\right\}\right)^{3}\rangle\right]^{\frac{1}{3}}, \\ max\left(x_{ij}^{h}, y_{ij}^{h}\right) \end{array}\right\} \end{array}\right\} \otimes_F \breve{C}_F$$

$$= \left\{ \begin{array}{c} x_{ij}^{m}z_{ij}^{m}, \left(\left(x_{ij}^{n}\right)^{3}+\left(z_{ij}^{n}\right)^{3}-\left(x_{ij}^{n}\right)^{3}\left(z_{ij}^{n}\right)^{3}\right)^{\frac{1}{3}}, \\ \left[\left(1-\left(z_{ij}^{n}\right)^{3}\right)\left(x_{ij}^{h}\right)^{3}+\left(1-\left(x_{ij}^{n}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right]^{1/3} \end{array}\right\} \ldots\ldots(21)$$

$$\left(\breve{A}_F \otimes_F \breve{C}_F\right) \vee_F \left(\left(\breve{\mathbf{B}}_F \otimes_F \breve{C}_F\right)\right) = \left\{ \begin{array}{c} x_{ij}^{m}z_{ij}^{m}, \left(\left(x_{ij}^{n}\right)^{3}+\left(z_{ij}^{n}\right)^{3}-\left(x_{ij}^{n}\right)^{3}\left(z_{ij}^{n}\right)^{3}\right)^{\frac{1}{3}}, \\ \left[\left(1-\left(z_{ij}^{n}\right)^{3}\right)\left(x_{ij}^{h}\right)^{3}+\left(1-\left(x_{ij}^{n}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right]^{\frac{1}{3}} \end{array}\right\}$$

$$\vee_F \left\{ \begin{array}{c} y_{ij}^{m}z_{ij}^{m}, \left(\left(y_{ij}^{n}\right)^{3}+\left(z_{ij}^{n}\right)^{3}-\left(y_{ij}^{n}\right)^{3}\left(z_{ij}^{n}\right)^{3}\right)^{\frac{1}{3}}, \\ \left[\left(1-\left(z_{ij}^{n}\right)^{3}\right)\left(y_{ij}^{h}\right)^{3}+\left(1-\left(y_{ij}^{n}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right]^{\frac{1}{3}} \end{array}\right\}$$

$$= \left\{ \begin{array}{c} max\left(x_{ij}^{m}z_{ij}^{m}, y_{ij}^{m}z_{ij}^{m}\right), min\left(x_{ij}^{n}z_{ij}^{n}, y_{ij}^{n}z_{ij}^{n}\right), \\ min\left\{ \begin{array}{c} \left[1-\langle \begin{array}{c} \left(max\left\{\left(\left(x_{ij}^{n}\right)^{3}+\left(z_{ij}^{n}\right)^{3}-\left(x_{ij}^{n}\right)^{3}\left(z_{ij}^{n}\right)^{3}\right), \left(\left(y_{ij}^{n}\right)^{3}+\left(z_{ij}^{n}\right)^{3}-\left(y_{ij}^{n}\right)^{3}\left(z_{ij}^{n}\right)^{3}\right)\right\}\right)^{3} \\ +\left(min\left\{ \begin{array}{c} \left[\left(1-\left(z_{ij}^{n}\right)^{3}\right)\left(x_{ij}^{h}\right)^{3}+\left(1-\left(x_{ij}^{n}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right], \\ \left[\left(1-\left(z_{ij}^{n}\right)^{3}\right)\left(y_{ij}^{h}\right)^{3}+\left(1-\left(y_{ij}^{n}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right] \end{array}\right\}\right)^{3} \end{array} \rangle\right]^{\frac{1}{3}}, \\ max\left(x_{ij}^{h}, y_{ij}^{h}\right) \end{array}\right\} \end{array}\right\}$$

$$=\begin{Bmatrix} x_{ij}^{m} z_{ij}^{m}, \left(\left(x_{ij}^{n}\right)^{3}+\left(z_{ij}^{n}\right)^{3}-\left(x_{ij}^{n}\right)^{3}\left(z_{ij}^{n}\right)^{3}\right)^{\frac{1}{3}}, \\ \left[\left(1-\left(z_{ij}^{n}\right)^{3}\right)\left(x_{ij}^{h}\right)^{3}+\left(1-\left(x_{ij}^{n}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right]^{1/3} \end{Bmatrix} \ldots\ldots \quad (22)$$

By (S) and (T), $\left(\breve{A}_F \vee_F \breve{B}_F\right) \otimes_F \breve{\mathbf{C}}_\mathbf{F} = \left(\breve{\mathbf{A}}_\mathbf{F} \otimes_\mathbf{F} \breve{\mathbf{C}}_\mathbf{F}\right) \vee_\mathbf{F} \left(\left(\breve{\mathbf{B}}_\mathbf{F} \otimes_\mathbf{F} \breve{\mathbf{C}}_\mathbf{F}\right)\right)$

(iv) To Prove $\left(\breve{\mathbf{A}}_\mathbf{F} \wedge_\mathbf{F} \breve{\mathbf{B}}_\mathbf{F}\right) \otimes_\mathbf{F} \breve{\mathbf{C}}_\mathbf{F} = \left(\breve{\mathbf{A}}_\mathbf{F} \otimes_\mathbf{F} \breve{\mathbf{C}}_\mathbf{F}\right) \wedge_\mathbf{F} \left(\left(\breve{\mathbf{B}}_\mathbf{F} \otimes_\mathbf{F} \breve{\mathbf{C}}_\mathbf{F}\right)\right)$

$$\left(\breve{A}_F \wedge_F \breve{B}_F\right) \otimes_F \breve{C}_F = \begin{Bmatrix} min\left(x_{ij}^{m}, y_{ij}^{m}\right), max\left(x_{ij}^{n}, y_{ij}^{n}\right), \\ max\begin{Bmatrix} \left[1-\langle\left(min\{x_{ij}^{m}, y_{ij}^{m}\}\right)^{3}+\left(max\{x_{ij}^{n}, y_{ij}^{n}\}\right)^{3}\rangle\right]^{\frac{1}{3}}, \\ min\left(x_{ij}^{h}, y_{ij}^{h}\right) \end{Bmatrix} \end{Bmatrix} \otimes_F \breve{C}_F$$

$$=\begin{Bmatrix} y_{ij}^{m} z_{ij}^{m}, \left(\left(x_{ij}^{n}\right)^{3}+\left(z_{ij}^{n}\right)^{3}-\left(y_{ij}^{n}\right)^{3}\left(z_{ij}^{n}\right)^{3}\right)^{\frac{1}{3}}, \\ \left[\left(1-\left(z_{ij}^{n}\right)^{3}\right)\left(y_{ij}^{h}\right)^{3}+\left(1-\left(y_{ij}^{n}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right]^{1/3} \end{Bmatrix} \ldots\ldots \quad (23)$$

$$\left(\breve{A}_F \otimes_F \breve{C}_F\right) \wedge_F \left(\left(\breve{\mathbf{B}}_F \otimes_F \breve{C}_F\right)\right) = \begin{Bmatrix} x_{ij}^{m} z_{ij}^{m}, \left(\left(x_{ij}^{n}\right)^{3}+\left(z_{ij}^{n}\right)^{3}-\left(x_{ij}^{n}\right)^{3}\left(z_{ij}^{n}\right)^{3}\right)^{\frac{1}{3}}, \\ \left[\left(1-\left(z_{ij}^{n}\right)^{3}\right)\left(x_{ij}^{h}\right)^{3}+\left(1-\left(x_{ij}^{n}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right]^{\frac{1}{3}} \end{Bmatrix}$$

$$\wedge_F \begin{Bmatrix} y_{ij}^{m} z_{ij}^{m}, \left(\left(y_{ij}^{n}\right)^{3}+\left(z_{ij}^{n}\right)^{3}-\left(y_{ij}^{n}\right)^{3}\left(z_{ij}^{n}\right)^{3}\right)^{\frac{1}{3}}, \\ \left[\left(1-\left(z_{ij}^{n}\right)^{3}\right)\left(y_{ij}^{h}\right)^{3}+\left(1-\left(y_{ij}^{n}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right]^{\frac{1}{3}} \end{Bmatrix}$$

$$=\begin{Bmatrix} min\left(x_{ij}^{m} z_{ij}^{m}, y_{ij}^{m} z_{ij}^{m}\right), max\left(x_{ij}^{n} z_{ij}^{n}, y_{ij}^{n} z_{ij}^{n}\right), \\ max\begin{Bmatrix} \left[1-\langle\left(min\{x_{ij}^{m} z_{ij}^{m}, y_{ij}^{m} z_{ij}^{m}\}\right)^{3}+\left(max\{x_{ij}^{n} z_{ij}^{n}, y_{ij}^{n} z_{ij}^{n}\}\right)^{3}\rangle\right]^{\frac{1}{3}}, \\ min\begin{pmatrix} \left(1-\left(z_{ij}^{n}\right)^{3}\right)\left(x_{ij}^{h}\right)^{3}+\left(1-\left(x_{ij}^{n}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(x_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}, \\ \left(1-\left(z_{ij}^{n}\right)^{3}\right)\left(y_{ij}^{h}\right)^{3}+\left(1-\left(y_{ij}^{n}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3} \end{pmatrix} \end{Bmatrix} \end{Bmatrix}$$

$$=\begin{Bmatrix} y_{ij}^{m} z_{ij}^{m}, \left(\left(x_{ij}^{n}\right)^{3}+\left(z_{ij}^{n}\right)^{3}-\left(y_{ij}^{n}\right)^{3}\left(z_{ij}^{n}\right)^{3}\right)^{\frac{1}{3}}, \\ \left[\left(1-\left(z_{ij}^{n}\right)^{3}\right)\left(y_{ij}^{h}\right)^{3}+\left(1-\left(y_{ij}^{n}\right)^{3}\right)\left(z_{ij}^{h}\right)^{3}-\left(y_{ij}^{h}\right)^{3}\left(z_{ij}^{h}\right)^{3}\right]^{1/3} \end{Bmatrix} \ldots\ldots \quad (24)$$

By (U) and (V), $\left(\breve{A}_F \wedge_F \breve{B}_F\right) \otimes_F \breve{C}_F = \left(\breve{A}_F \otimes_F \breve{C}_F\right) \wedge_F \left(\left(\breve{\mathbf{B}}_F \otimes_F \breve{C}_F\right)\right)$holds.

CONCLUSION

In this paper, we have introduced the notions of Spherical Fermatean Fuzzy Matrix, addition and multiplication operators. We have also discussed some of their properties.

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