Solution Under Exponential Scale Factor In $f(R,T)$ Gravity With Variable G And Λ

Binaya K. Bishi

Department of Applied Mathematics, Lovely Professional University, Phagwara, Punjab, India
Email: binaybc@gmail.com

Abstract : ***This article is devoted to study the solution of field equations of $f(R,T)=R+\lambda T$ gravity under the scale factor of the form $S(t)=S_0 e^{(t-t_0)^{2n}}$ with G and Λ for FRW metric. It is considered that, universe is filled with an exotic fluid of the form $p=A\rho-\frac{B}{\rho^m}$, known as modified Chaplygin gas. Here the derived model is close to ΛCDM model under the limiting case of $t-t_0\to\infty$ and $n\in\mathbb{N}^+$***

1 Introduction

Our universe is considering to be one of the most dynamical system as compare to other physical systems. Observational data from different observations pointed out that, our universe is expanding with acceleration [1]. The study of literatures of modern cosmology revels that, nearly 68.5% of our universe is filled with dark energy. It is usually considered as an exotic entity, which has positive and negative energy density and pressure respectively. It is believed that, the cosmic acceleration is due to the exotic entity usually dubbed as dark energy with equation of state parameter $\omega=-1$. Still today we are not able to provide the scientific evidence of the existence of the dark energy. Therefore, the issue of dark energy is â€˜â€™Mystery of the Millenniumâ€• [2]. The equation of the state parameter is very close to -1 but other dark fluids like k-essence, quintessence, phantom etc. with suitable equation of the state parameter are also allowed. In the era of inflation, acceleration epoch was not driven by standard matter or radiation, which make the researchers to investigate the more general form of the dark fluid along with its solutions and behaviours. Also it is noticed that many of the physical systems with macroscopic nature are approximated through the perfect fluid and at the same time we cannot ignore universe, which has the non-perfect fluid representation.

However, among various form of dark energy cosmological constant is considered to be the one of the simple and prominent candidate. In the Einstein's field equation, we cannot ignore the role of gravitational constant G and cosmological constant Λ. The gravitational constant act as a constant of coupling between matter and geometry in case of Einstein's field equation. In the evolvement cosmic time, it is considered as a function of time rather than treating as a constant. Observational data support the accelerated expansion of the universe or in other way we can say that, some exotic dark fluid varies slowly with space and time, which dominates the present composition of the cosmos. Observations reveals a small and positive value of Λ at the present epoch. The concept of dependence of gravitational constant on cosmic time was first suggested by Dirac [3], which was analysed by several authors [4-7]. It is observed that, different argument based on variable G is available in the literature [8]. Dirac proposed LNH, which leads to G-varying cosmology. In view of Astrophysics variation of G has interesting consequences. The present observational data support the consistent of G-varying cosmology is studied by Canuto and Narlikar [9]. The role of G and

Λ is important in modern cosmology as it may provides hints towards the accelerating universe.

We have discussed above the different candidates of dark energy. Chaplygin gas (CG) is another candidates of dark energy with equation of state like $p = \frac{-B}{\rho}$ $(B > 0)$ [10, 11], where p and ρ stands for pressure and energy density. The equation of state parameter for CG is generalized to $p = \frac{-B}{\rho^n}$, $0 \leq n \leq 1$ [12-14]. This is called GCG (generalized Chaplygin gas). Again it is modified to the form (see [15, 16]):

$$p = A\rho - \frac{B}{\rho^m}, for\ A > 0. \tag{1}$$

This is called MCG (modified Chaplygin gas). The important aspect of the cosmological models is the behaviour of the cosmological solutions. The solutions of the cosmological models have different features like singularities and oscillation depending up on the matter content of the universe. One of the fascinating scenario is cosmological models with bounce solution, which provides a different view point with the Big Bang scenario [17, 18]. Belinsky et al. have investigated the generalized Kasner solution and discussed the oscillatory approach to a singular point, usually referred as BKL instability [19, 20]. Khoury et al. [21] have investigated the Ekpyrotic scenario for the beginning of the hot big bang universe, which favours the string theory and supergravity. Cai et al. [22] have devoted their study on the realization of matter bounce cosmology in view of $f(T)$ gravity. Shabani, and Ziaie [23] have analysed the bouncing behaviour of the universe in in $f(R,T)$ gravity for FLRW model. A bouncing scenario is discussed by Singh et al. [24] by the help of FLW model with a specific form Hubble parameter in $f(R,T)$ gravity.

Motivated by the above research, here I present the bouncing scenario cosmology for a FRW universe in $f(R,T)$ gravity with MCG in presence of variable G and Λ.

2 Gravitational field equations in $f(R,T)$ gravity

Harko et al. [25-28] has developed the $f(R,T)$ gravity, in which gravitational Lagrangian is considered as an arbitrary function of R and T. Here R and T.denotes the Ricci scalar and trace of the energy momentum tensor. Metric formalism is used for the derivation of the gravitational field equation. They have also derived the equation of motion of the test particles through the covariant divergence of T_{ij}. Models based on $f(R,T)$ gravity depends on the source term, representing the variation of T_{ij} w.r.t. g_{ij}. The choice of L_m is important as it generate set of field equations based on L_m. Harko et al. [25, 29], have discussed cases in which the $f(R,T)$ has the forms as $f(R,T) = R + 2g_1(T)$, $f(R,T) = g_1(R) + g_2(T)$ and $f(R,T) = g_1(R) + g_2(R)g_3(T)$. Here I have mainly focused on the case of $f(R,T) = R + 2g_1(T)$. Several research articles have been published since last decade and some of them are discussed above. In order to field the solution of field equations under the case $f(R,T) = R + 2g_1(T)$, let us consider the SF (scale factor) of the form

$$S(t) = S_0 e^{(t-t_0)^{2n}}, \tag{2}$$

where S_0 is a positive constant (dimensional) and $n \in \mathbb{N}^+$. The Hubble parameter is defined and obtained as

$$H = \frac{\dot{S}}{S} = 2n(t - t_0)^{2n-1} \tag{3}$$

Before going to discuss the solution of the field equations, let us analysis the parameters i.e. scale factor, Hubble parameter, n and t_0. In view of the cosmic time t and t_0, there are three possibilities as follows: [i.]

1. For $t = t_0$ or $t \to t_0$, $S(t) = S_1 > 0$(constant) and $H(t) \to \infty$.
2. For $t > t_0$, $S(t) = S_2$(constant) and $H(t) = S_3 > 0$(constant).
3. For $t < t_0$, $S(t) = S_4$(constant) and $H(t) = -S_3$.

Again for $n=\frac{1}{2}$, the scale factor $S(t)=S_0e^{t-t_0}$ and Hubble parameter $H=1$. This case lead to the de Sitter Universe. For $n<\frac{1}{2}$, it is noticed that, $H(t)\to\infty$ when $t\to t_0$. This means that Hubble parameter or some of its derivative diverges at $t=t_0$ leading to the finite time singularity. It is noticed from (ii) and (iii) that, Hubble parameter is negative and positive valued depending on $t>t_0$ and $t<t_0$ respectively. This indicates the two different cosmological history of expansion or contraction. In other words, universe start and end with a singularity. The finite time singularity is classified in to four type, which is presented in the Table 1.

Table 1

Finite time singularities. S_0^* and H_0^* are constants.

Parameters→ / Type of Singularity↓	Scale factor S	Hubble Parameter H	Effective energy density	Effective Pressure	Range of n
Type-I (Big-Rip)	$S(t)\to\infty$	$H(t)\to\infty$ $\dot{H}(t)\to\infty$	diverges	diverges	$n<0$
Type-II (Sudden)	$S(t)\to S_0^*$	$H(t)\to H_0^*$ $\lvert\dot{H}(t)\rvert\to\infty$	…….	diverges	$n\in(0,0.5)$
Type-III	$S(t)\to S_0^*$	$H(t)\to\infty$ $\lvert\dot{H}(t)\rvert\to\infty$	diverges	diverges	$n\in(0.5,1)$
Type-IV (Soft type)	…….	Higher derivatives of H diverges	…….	…….	$n>1$ $n\neq\frac{\alpha}{2}$, $\alpha\in\mathbb{Z}$

Table 2

Behaviour of $G(t)$ under different conditions at $t=t_0$

Conditions	Spherical universe with $S_0\in(0,1)$	Spherical universe with $S_0>1$	Flat universe with $S_0>0$	hyperbolic universe with $S_0>0$
$\lambda>-4\pi, 1+A>BS_0^{\beta(1+m)}$	$G<0$	$G>0$	$G>0$	$G>0$
$\lambda<-4\pi, 1+A<BS_0^{\beta(1+m)}$	$G<0$	$G>0$	$G>0$	$G>0$
$\lambda<-4\pi, 1+A>BS_0^{\beta(1+m)}$	$G>0$	$G<0$	$G<0$	$G<0$
$\lambda>-4\pi, 1+A<BS_0^{\beta(1+m)}$	$G>0$	$G<0$	$G<0$	$G<0$

Let us consider the line-element of the form

$$ds^2=dt^2-S^2(t)\left[\frac{dr^2}{1-kr^2}+r^2(d\theta^2+\sin^2\theta d\phi^2)\right] \tag{4}$$

which is known as Friedmann-Robertson-Walker (FRW) metric. Where $k(=-1,0,1)$ is the curvature parameter.

The energy momentum tensor for a perfect fluid is given by

$$T_i^j=(p+\rho)u_iu^j-p\delta_i^j \tag{5}$$

Here u^i satisfies the condition $g_{ij}u^iu^j=1$ and it is known as flow vector. The ρ and p indicates the energy density and isotropic pressure respectively. The non-vanishing components of T_i^j are $T_i^i=<\rho,-p,-p,-p>$. The generated field equations based on $f(R,T)$ gravity in presence of G and Λ is expressed as

$$\begin{aligned} &f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+(g_{ij}-\nabla_i\nabla_j)f_R(R,T)-\Lambda(t)g_{ij} \\ &=[8\pi-f_T(R,T)]G(t)T_{ij}-f_T(R,T)\Theta_{ij} \end{aligned} \tag{6}$$

where

$$\begin{aligned} &T_{ij}=\frac{-2}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{ij}}L_m\,,\Theta_{ij}=-2G(t)T_{ij}-pg_{ij}, \\ &f_R(R,T)=\frac{\partial f(R,T)}{\partial R}\ and\ f_T(R,T)=\frac{\partial f(R,T)}{\partial T} \end{aligned} \tag{7}$$

Next, we take one of the class of $f(R,T)$ as $f(R,T)=R+2g_1(T)$ [25] along with (5) in

(6) leads to the field equations given by

$$G_{ij} - \Lambda(t)g_{ij} = [8\pi + 2g_1{'}(T)]G(t)T_{ij} + [2pg_1{'}(T) + f(T)]g_{ij} \quad (8)$$

where prime indicates differentiation w.r.t. the argument. For $g_1(T) = \lambda T$ (λ is a constant)one can has the field equations as

$$G_{ij} - \Lambda(t)g_{ij} = [8\pi + 2\lambda]G(t)T_{ij} + (\rho - p)\lambda g_{ij} \quad (9)$$

In view of the above background we have the modified system as

$$3\left(\frac{\dot{S}}{S}\right)^2 + \frac{3k}{S^2} - \Lambda(t) = [(8\pi + 2\lambda)G(t) + \lambda]\rho - p\lambda \quad (10)$$

$$2\frac{\ddot{S}}{S} + \left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} - \Lambda(t) = -[(8\pi + 2\lambda)G(t) + \lambda]p + \lambda\rho \quad (11)$$

3 Solution of the field equations under exponential scale factor

The system of field equations has two equations and five unknowns (S, ρ, p, G, Λ). In order to obtain the exact solution of the highly non-linear system, it is required to make the system closed. Therefore, we have used equation (1), (2) and the relationship among ρ and S in the form

$$\rho = \frac{1}{S^\beta} \quad (12)$$

where $\beta > 0$ is a constant.

Solving equations (10) and (11) we have

$$G(t) = \frac{2}{(8\pi+2\lambda)(\rho+p)}\left[\frac{k}{S^2} + \left(\frac{\dot{S}}{S}\right)^2 - \frac{\ddot{S}}{S}\right] \quad (13)$$

and

$$\Lambda(t) = 3\left[\frac{k}{S^2} + \left(\frac{\dot{S}}{S}\right)^2\right] - \lambda(\rho - p) - \frac{2\rho}{\rho+p}\left[\frac{k}{S^2} + \left(\frac{\dot{S}}{S}\right)^2 - \frac{\ddot{S}}{S}\right] \quad (14)$$

With the help of equations (1), (2) and (12), gravitational constant and cosmological constant are expressed as

$$G(t) = \frac{S_0^\beta e^{(t-t_0)^{2n}\beta}(t-t_0)^{2n-2}(4n^2-2n) - S_0^{\beta-2} k e^{(t-t_0)^{2n}(\beta-2)}}{(4\pi+\lambda)\left(1+A-BS_0^{\beta(1+m)}e^{(t-t_0)^{2n}\beta(1+m)}\right)} \quad (15)$$

$$\Lambda(t) = \frac{\Lambda_0(t-t_0)^{4n-2} + \Lambda_1(t-t_0)^{2n-2} + \Lambda_2}{S_0^2\left(1+A-BS_0^{\beta(1+m)}e^{(t-t_0)^{2n}\beta(1+m)}\right)} \quad (16)$$

where

$$\Lambda_0 = 12n^2S_0^2\left[1 + A - S_0^{\beta(1+m)}Be^{(t-t_0)^{2n}\beta(1+m)}\right]$$
$$\Lambda_1 = S_0^2(8n^2 - 4n)$$
$$\Lambda_2 = (1+3A)ke^{-2(t-t_0)^{2n}} - 3kBS_0^{\beta(1+m)}e^{(t-t_0)^{2n}(-2+\beta(1+m))}$$
$$+\lambda S_0^{2-\beta}e^{-(t-t_0)^{2n}\beta}(A^2-1) - 2\lambda S_0^{2+\beta m}ABe^{(t-t_0)^{2n}\beta m}$$
$$+\lambda S_0^{2\beta m+2+\beta}B^2e^{(t-t_0)^{2n}\beta(2m+1)}$$

The other physical parameters like energy density and pressure takes the form

$$\rho = S_0{}^{-\beta}e^{-(t-t_0)^{2n}\beta} \quad (17)$$

$$p = AS_0{}^{-\beta}e^{-(t-t_0)^{2n}\beta} - BS_0{}^{\beta m}e^{(t-t_0)^{2n}\beta m} \quad (18)$$

Equations (17) and (18) revels that, for flat, spherical and hyperbolic universe the energy density $\rho = S_0^{-\beta}$ and pressure $p = AS_0^{-\beta} - BS_0^{-\beta m}$ at $t = t_0$. For an accelerating universe, pressure most be negative so we have $A < BS_0^{-(\beta m-1)}$ at $t = t_0$. For $t > t_0$ or $t < t_0$, $(t-t_0)^{2n} > 0$. When $t >> t_0$ or $t << t_0$ such that $(t-t_0)^{2n} \to \infty$, under such situation both the physical parameters $\rho \to \infty$ and $p \to -\infty$. Here energy density and pressure are decreasing function of cosmic time t. This can bethought of as a situation after and before the t_0. At $t = t_0$, the gravitational constant $G(t)$ has the following form

$$G(t) = \begin{cases} -\frac{S_0^{\beta-2} k}{(4\pi+\lambda)\left(1+A-BS_0^{\beta(1+m)}\right)}, \text{ for } n > 1 \\ \frac{2S_0^{\beta} - S_0^{\beta-2} k}{(4\pi+\lambda)\left(1+A-BS_0^{\beta(1+m)}\right)}, \text{ for } n = 1 \end{cases} \tag{19}$$

In case of $t > t_0$ or $t < t_0$ we have time dependent gravitational constant as $(t - t_0)^{2n}$ and $(t - t_0)^{2(n-1)}$ for $n \in \mathbb{N}^+$ does not make any difference to the terms present in $G(t)$. At $t = t_0$, the cosmologicaal constant $\Lambda(t)$ takes the form

$$\Lambda(t) = \frac{\Lambda_3}{S_0^2\left(1+A-BS_0^{\beta(1+m)}\right)}, \text{ for } n \geq 1 \tag{20}$$

here $\Lambda_3 = (1 + 3A)k - 3kBS_0^{\beta(1+m)} + \lambda S_0^{2-\beta}(A^2 - 1) - 2\lambda S_0^{2+\beta m} AB + \lambda S_0^{2\beta m+2+\beta} B^2$. Further for $t > t_0$ or $t < t_0$ we have time dependent cosmological constant as $(t - t_0)^{2n}$ is positive in $n \in \mathbb{N}^+$ and it does not make any difference to the terms present in $\Lambda(t)$. The deceleration parameter of the model is defined and obtained as follow

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \left[-1 + \frac{1}{2n}\right] \frac{1}{(t-t_0)^{2n}} - 1 \tag{21}$$

From the expression in (21), it is noticed that $-1 < -1 + \frac{1}{2n} < 0$ for $n \in \mathbb{N}^+$. Thus,

$$q = \begin{cases} -\infty, \text{ at } t = t_0 \\ < 0, \text{ at } t > t_0 \quad or \quad t < t_0. \end{cases} \tag{22}$$

which follow the observational results and we have the accelerating model of the universe. One of the important parameter in cosmology is the Jerk parameter, which is useful to analyze the universe dynamics. Also it helpful to describe the closeness between the ΛCDM and derived models. It is denoted as j and defined as

$$j = \frac{S^2}{\dot{S}^3} \frac{d^3 S}{dt^3}$$

In terms of deceleration parameter, it takes the form

$$j = q + 2q^2 - \frac{\dot{q}}{H}$$

and it is obtained as

$$j = 1 + \left(1 - \frac{3}{2n} + \frac{1}{2n^2}\right) \frac{1}{(t-t_0)^{4n}} + \left(1 - \frac{1}{2n}\right) \frac{3}{(t-t_0)^{2n}} \tag{23}$$

The jerk parameter $j = \infty$ at $t = t_0$ and when the difference between t and t_0 is very large i.e $t - t_0 \to \infty$, $j \to 1$ for $t > t_0$ or $t < t_0$. In such scenario our model is closed to ΛCDM model (See [26] and reference there in).

4 Discussion and Concluding Remarks

FRW field equations are solved for $f(R,T) = R + \lambda T$ gravity under a specific form of scale factor $S(t) = S_0 e^{(t-t_0)^{2n}}$. The presented solution is free from the initial singularity for $n \in \mathbb{N}^+$ but solution has soft type of singularity due to the diverging nature of the higher derivative of H at $t = t_0$. The energy density and pressure have similar qualitative behaviour for flat, spherical and hyperbolic type universe due to the assumed relationship among the scale factor and energy density. For $n > 1$ we have

$$G(t) = \begin{cases} -\frac{S_0^{\beta-2}}{(4\pi+\lambda)\left(1+A-BS_0^{\beta(1+m)}\right)}, & \text{for } k = 1 \\ 0, & \text{for } k = 0 \\ \frac{S_0^{\beta-2}}{(4\pi+\lambda)\left(1+A-BS_0^{\beta(1+m)}\right)}, & \text{for } k = -1 \end{cases}$$

. The above expression revels that for $S_0 > 0$, in spherical universe we have $G(t) > 0$

under $\lambda < -4\pi$, $1 + A > BS_0^{\beta(1+m)}$ or $\lambda > -4\pi$, $1 + A < BS_0^{\beta(1+m)}$ and $G(t) < 0$ under $\lambda > -4\pi,\ 1 + A > BS_0^{\beta(1+m)}$ or $\lambda < -4\pi,\ 1 + A < BS_0^{\beta(1+m)}$ at $t = t_0$. In case of flat universe, no effect of gravitational constant is noticed. In case of hyperbolic universe for $S_0 > 0$, we have $G(t) > 0$ under $\lambda > -4\pi,\ 1 + A > BS_0^{\beta(1+m)}$ or $\lambda < -4\pi,\ 1 + A < BS_0^{\beta(1+m)}$ and $G(t) < 0$ under $\lambda < -4\pi,\ 1 + A > BS_0^{\beta(1+m)}$ or $\lambda > -4\pi,\ 1 + A < BS_0^{\beta(1+m)}$ at $t = t_0$. For $n = 1$ we have

$$G(t) = \begin{cases} \frac{2S_0^{\beta} - S_0^{\beta-2}}{(4\pi+\lambda)\left(1+A-BS_0^{\beta(1+m)}\right)}, & \text{for } k = 1 \\ \frac{2S_0^{\beta}}{(4\pi+\lambda)\left(1+A-BS_0^{\beta(1+m)}\right)}, & \text{for } k = 0 \\ \frac{2S_0^{\beta} + S_0^{\beta-2}}{(4\pi+\lambda)\left(1+A-BS_0^{\beta(1+m)}\right)}, & \text{for } k = -1 \end{cases}$$

. The above expression revels the impact of gravitational constant in flat, spherical and hyperbolic universe and the behaviour is presented in the Table 2. The qualitative behaviour of $\Lambda(t)$ is similar in flat, spherical and hyperbolic universe. For $n \in \mathbb{N}^+$, q is diversing at $t = t_0$ and $q < 0$ at $t > t_0$ or $t < t_0$, which reflects to accelerating expansion history of universe. Further, the constructed model is approaching to ΛCDM model when difference between t and t_0 is very large. More interesting results can be derived for $n \in \mathbb{Z}$ but here emphasis is given to only $n \in \mathbb{N}^+$.

References

[1] Komatsu, E., Dunkley, J., Nolta, M.R., Bennett, C.L., Gold, B., Hinshaw, G., Jarosik, N., Larson, D., Limon, M., Page, L. and Spergel, D.N., 2009. Five-year wilkinson microwave anisotropy probe* observations: cosmological interpretation. The Astrophysical Journal Supplement Series, 180(2), p.330.

[2] Padmanabhan, T., 2006, November. Dark energy: mystery of the millennium. In AIP Conference Proceedings (Vol. 861, No. 1, pp. 179-196). American Institute of Physics.

[3] Dirac, P.A., 1937. The cosmological constants. Nature, 139(3512), pp.323-323.

[4] Hawking, S.W., 1988. Wormholes in spacetime. Physical Review D, 37(4), p.904.

[5] Lavrelashvili, G.V., Rubakov, V.A. and Tinyakov, P.G., 1987. Pisâ€™ ma Zh. Eksp. Teor. Fiz. 46, 134 (1987). ZETP Lett, 46, p.167.

[6] Zel'Dovich, Y.B., 1970. Particle production in cosmology. Soviet Journal of Experimental and Theoretical Physics Letters, 12, p.307.

[7] Dreitlein, J., 1974. Broken symmetry and the cosmological constant. Physical Review Letters, 33(20), p.1243.

[8] Wesson, P.S., 1980. Gravity, Particles, and Astrophysics (D. Reidel Publishing

Co., Dordrecht).

[9] Canuto, V.M. and Narlikar, J.V., 1980. Cosmological tests of the Hoyle-Narlikar conformal gravity. The Astrophysical Journal, 236, pp.6-23.

[10] Kamenshchik, A., Moschella, U. and Pasquier, V., 2001. An alternative to quintessence. Physics Letters B, 511(2-4), pp.265-268.

[11] Gorini, V., Kamenshchik, A., Moschella, U. and Pasquier, V., 2005. The Chaplygin gas as a model for dark energy. In The Tenth Marcel Grossmann Meeting: On Recent Developments in Theoretical and Experimental General Relativity, Gravitation and Relativistic Field Theories (In 3 Volumes) (pp. 840-859).

[12] Gorini, V., Kamenshchik, A. and Moschella, U., 2003. Can the Chaplygin gas be a plausible model for dark energy?. Physical Review D, 67(6), p.063509.

[13] Alam, U., Sahni, V., Deep Saini, T. and Starobinsky, A.A., 2003. Exploring the expanding universe and dark energy using the Statefinder diagnostic. Monthly Notices of the Royal Astronomical Society, 344(4), pp.1057-1074.

[14] Bento, M.C., Bertolami, O. and Sen, A.A., 2002. Generalized Chaplygin gas, accelerated expansion, and dark-energy-matter unification. Physical Review D, 66(4), p.043507.

[15] Debnath, U., 2011. Modified Chaplygin gas with variable G and Λ. Chinese Physics Letters, 28(11), p.119801.

[16] Debnath, U., Banerjee, A. and Chakraborty, S., 2004. Role of modified Chaplygin gas in accelerated universe. Classical and Quantum Gravity, 21(23), p.5609.

[17] Novello, M. and Bergliaffa, S.P., 2008. Bouncing cosmologies. Physics reports, 463(4), pp.127-213.

[18] Brandenberger, R.H., 2009. Alternatives to cosmological inflation. arXiv preprint arXiv:0902.4731.;2010, Cosmology of the very early universe. In AIP Conference Proceedings (Vol. 1268, No. 1, pp. 3-70). American Institute of Physics.; 2011. Introduction to early universe cosmology. arXiv preprint arXiv:1103.2271.; 2012. The matter bounce alternative to inflationary cosmology. arXiv preprint arXiv:1206.4196.

[19] Belinskii, V.A., Khalatnikov, I.M. and Lifshitz, E.M., 1970. Oscillatory approach to a singular point in the relativistic cosmology. Advances in Physics, 19(80), pp.525-573.

[20] Myrzakulov, R. and Sebastiani, L., 2014. Bounce solutions in viscous fluid cosmology. Astrophysics and Space Science, 352(1), pp.281-288.

[21] Khoury, J., Ovrut, B.A., Steinhardt, P.J. and Turok, N., 2001. Ekpyrotic universe: Colliding branes and the origin of the hot big bang. Physical Review D, 64(12), p.123522.

[22] Cai, Y.F., Chen, S.H., Dent, J.B., Dutta, S. and Saridakis, E.N., 2011. Matter bounce cosmology with the f (T) gravity. Classical and Quantum Gravity, 28(21), p.215011.

[23] Shabani, H. and Ziaie, A.H., 2018. Bouncing cosmological solutions from $f(\mathrm{R,T})$ gravity. The European Physical Journal C, 78(5), p.397.

[24] Singh, J.K., Bamba, K., Nagpal, R. and Pacif, S.K.J., 2018. Bouncing cosmology in $f(R,T)$ gravity. Physical Review D, 97(12), p.123536.

[25] Harko, T., Lobo, F.S., Nojiri, S.I. and Odintsov, S.D., 2011. $f(R,T)$ gravity. Physical Review D, 84(2), p.024020.

[26] Kant, N., Wani, M. A., & Kumar, A. (2012). Self-focusing of Hermite–Gaussian laser beams in plasma under plasma density ramp. *Optics Communications*, *285*(21-22), 4483-4487.

[27] Prabhakar, P. K., Prasad, R., Ali, S., & Doble, M. (2013). Synergistic interaction of ferulic acid with commercial hypoglycemic drugs in streptozotocin induced diabetic rats. *Phytomedicine*, *20*(6), 488-494.

[28] Sahoo, P.K., Sahoo, P. and Bishi, B.K., 2017. Anisotropic cosmological models in $f(R,T)$ gravity with variable deceleration parameter. International Journal of Geometric Methods in Modern Physics, 14(06), p.1750097.

[29] Singh, G., Gupta, M. K., Mia, M., & Sharma, V. S. (2018). Modeling and optimization of tool wear in MQL-assisted milling of Inconel 718 superalloy using evolutionary techniques. *The International Journal of Advanced Manufacturing Technology*, *97*(1-4), 481-494.