AN OPTIMAL SOLUTIONS OF MULTI OBJECTIVE TRANSPORTATION PROBLEMS UNDER FERMATEAN FUZZY ENVIRONMENT

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Abstract: Various investigation have introduced different methods like Northwest corner method, Least cost method, Vogel's approximation to crack the transportation problem. The main aim of research article to solve the TP is a function under fermatean fuzzy environment. In this article, we introduce Geometric mean technique to resolve fermatean fuzzy transportion problem (FFTP) and an algorithm of the suggested method in presented. A numerical example is illustrated with the new technique and compare with existing method.

Keywords: Transportation problem, fuzzy set, pythogorean fuzzy set, fermatean fuzzy set, optimal solutions, geometric mean, score function, operations, VAM,comparison table.
AMS Subject classification (2010): 03E72, 90C08, 90B06.

1.Introduction: In the real world, it is not necessary that all the parameters i.e., cost, demand and supply related to the transportation problem (TP) are known precisely one of the recent ways to take the impression in the fermatean fuzzy sets(FFS), an extension of pythogarean fuzzy sets (PFS). The basic transportation problem was introduced in by Hitchcock [3] for the intend of product distribution from several sources to numerous localities. Charnesetial [1] suggested the stepping stone method which applicable the simplex method to solve the transportation problem. Later on the primal simplex transportation scheme was introduced by Dantzig [2] in 1963. In the recent world, all the constraint of the transportation problems may not be known completely due to intractable characteristics. In order to overcome this situation, Uncertainity numbers are initiated by Zadeh [16] in 1965 and later developed by Zimmermann [18] in 1978. Yager [[13],[14]] in 2013, 2014 established an additional category of non-standard uncertainity collection of orthopair fuzzy set is called pythogorean fuzzy set (PFS), which is a special case used to overcome the situation that if the sum of the membership function and non-membership function is greater than one. In PFS, the square sum of the membership and the non-membership degree is equal to 0 less than one. A new method for solving transportation problems using trapezoidal fuzzy numbers was suggested by Kadhirvel and Balamurgan [5] in 2012. Narayanamoorthy et.al [9] in 2013 have introduced a new procedure for solving fuzzy transportation problems. The extension of topic to n multiple criteria decision making with pythogorean fuzzy collections was introduced by Zang et.al [17] in 2014. A pythogorean fuzzy techniques to solve the transportation problem was introduced by Kumar et.al [6] in 2019. Jeyalakshmi et.al [4] in 2021, introduced monalisha technique to unravel pythogorean fuzzy transportation problem. But, if orthopair fuzzy set as <0.9, 0.6>, where 0.9 is the support of the membership of certain criteria of a parameter and 0.6 is the support against membership then it does not obey the condition of IFS as well as PFS. However, the cubic sum of the support of membership and support against membership degrees is equal to or less than one. And in the situation Senapati and Yager [[10], [11]] very recently introduced Fermatean fuzzy set (FFS). They also showed that FFS have more uncertain than IFS's and PFS's and capable of handling higher level of uncertainties.

Based on the earlier discussion transportation problem and recently available several research articles on TP, there are no existing methodologies which are available on TP under fermatean fuzzy environment. Hence, there is an essential urgency to introduce a new solution methodology for solving TP in the light of fermatean fuzzy environment. A modified algorithm using geometric mean to unravel the fermatean fuzzy transportation problem is suggested in this work. For this fact, in the article, we have treated cost parameters, demand and supply parameters of a TP as FFS and FFS's are most fruitful fuzzy set which are more completed to manage higher level of uncertainties. This is new frame work of fuzzy transportation problems.

The main contribution of this research work is as follows.

(i) In fermatean fuzzy TP, all the parameters Viz, transportation cost, supply and demand are considered as fermatean fuzzy numbers (FFNS).

(ii) Three newly score functions are proposed for the ranking of FFS's.

(iii) Ranking/order relation of two FFS's are proposed.

Table 1: List of abbreviations

Abbreviations	Full name
FN	Fuzzy number
TP	Transportation problem
FP	Fuzzy parameter
FFS	Fermatean fuzzy set
FFN	Fermatean fuzzy number
FS	Fuzzy set
DM	Decision maker
FFTP	Fermatean fuzzy transportation problem
FTP	Fuzzy transportation problem
IFS	Intuitionistic fuzzy set
PFS	Pythogorean fuzzy set
PFTP	Pythogorean fuzzy transportation problem
LPP	Linear programming problems

Table-2 Significance difference of the various authors towards TP under fuzzy environment

Chanas etial	1984	Fuzzy approach to the transportation problem

Li and Lal	2000	Fuzzy approach to the multi -objective transportation problem
Dinagar and Palanivel	2009	Transportation problem in fuzzy Environment
Kaur and Kumar	2011	Fuzzy transportation problems using ranking function
Singh and Yadev	2016	Solving intuitionistic fuzzy solid transportation problem of type-2
Arora	2018	Interval-valued fuzzy fractional transportation problem
Kumar	2018	Intuitionistic fuzzy problem
Ahamed and Adhami	2019	Neutrosophic programming approach to multi-objective nonlinear transportation problem with fuzzy parameters
Kumar etial	2019	Pythogorean fuzzy approach to the transportation problem
Roy etial	2019	Multi-objective fixed-charge transportation problem under two-fold uncertainty
Ghosh etial	2021	Multi –objective fully intuitionistic fuzzy fixed –charge solid transportation problem
Ghosh and Roy	2021	Fuzzy–rough multi-objective product blending fixed-charge transportation problem with truck load constraints through transportation.
Midya etial	2021	Intuitionstic fuzzy multi-stage multi-objective fixed charge solid transportation problem in green supply chain
M.K.Sharma et.al	2022	A Fermatean fuzzy ranking function in optimization of intuitionistic fuzzy transportation problems

(iv) A solution methodology is proposed to solve FFTP.

(v) Finding optimal solution, we use Excel solver the rest of the paper is organized as follows:

in section-2 some basic definitions about PFS's and FFS's are presented. The mathematical formulation of TP is presented in section-3. In section-4, proposed solution methodologies of geometric mean are provided and the result of Fermatean fuzzy transportation (FFTP) are discussed in section-5.The conclusions are drawn in section-6.

2.Preliminaries

Several basic definitions regarding FFSS Senapati and Yager (2020), Senapati&Yager (2019),Sahoo(2021) are provided in this section in a modified form.

2.1Definition: [Zadeh]A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0,1]. (i,e) A = {(x, $\mu_A(x)$; x ε X}, Here μ_A: X →[0,1] is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of x ε X in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

2.2 Example: Consider U = { a, b, c ,d } and A : U→ [0,1] defined by A(a)=0, A(b)=0.7, A(c)=0.4, A(d)=1.

2.3 Definition : [K.T.Attanassov]An Intuitionistic fuzzy set A in the universe of discourse U is characterized by two membership functions given by

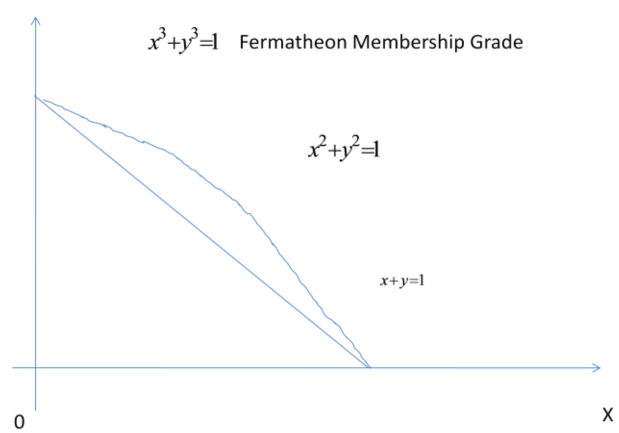
1. A truth Membership function t_A : U → [0,1]
2. A false membership function.f_A : U → [0.1]

where t_A (x) is a Lower bound of the grade of membership of x derived from the evidence for x and f_A (x) is a Lower bound on the negation of x derived from the evidence against x and t_A (x)+ f_A (x)≤ 1. The intuitionistic fuzzy set A is written as Ã = {(x, (t_A (x) , f_A (x))} / x ∈U } where the interval [t_A (x) ,1- f_A (x)] is called intuitionistic values of x in A and denoted by I_A (x). In an intuitionistic fuzzy sets are independently proposed by the decision maker but they are mathematically not independent. This makes a major difference in the judgement about the grade of membership.

2.4 Definition : [Yagar et.al]Let us consider a universal set Uˆ. A PFS on set Uˆ is denoted and defined as = {κ,μ(κ), ψ (κ) | κ ∈ Uˆ}, where μ : Uˆ → [0, 1] represents the membership degree and ψ : Uˆ → [0, 1] represents the non-member ship degree of κ ∈ Uˆ to the set , satisfying that $0 \leq (\mu(\kappa))^2 + (\psi(\kappa))^2 \leq 1$. Here $\pi(\kappa) = 1 - \mu(\kappa))^2 + (\psi(\kappa))^2$ represents the indeterminacy of an object κ ∈ Uˆ. The collection of all Pythagorean fuzzy subsets of Uˆ is represented by PFS(Uˆ).

2.5 Definition : [Senapati and Yager, 2019a] Let 'X' be a universe of discourse A. Fermatean uncertainty set "F" in X is an object having the form $F = \{\langle x, m_F(x),\ n_F(x)\rangle / x \in X\}$,where $m_F(x): X \to [0,1]$ and $n_F(x): X \to [0,1]$, including the condition $0 \leq \left(m_F(x)\right)^3 + \left(m_F(x)\right)^3 \leq 1, for\ all\ x \in X$.The numbers $m3_F(x)$ signifies the level (degree) ofmembership and $n_F(x)$indicate the non-membership of the element 'x' in the set F.All through this paper, we will indicate a fermatean uncertainty set is FUS.

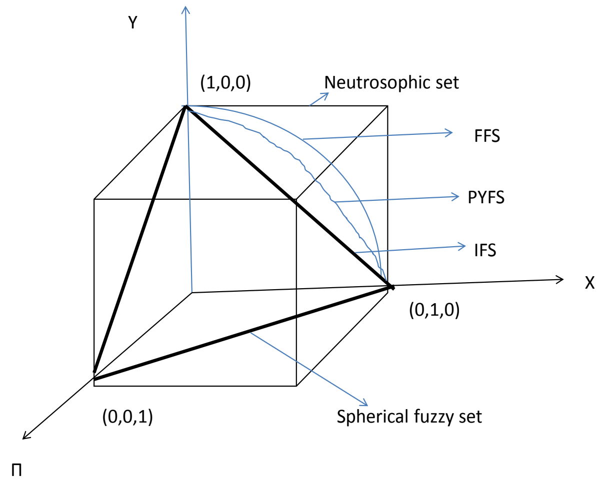
For any FUS 'F' and $x \in X$, $\pi_F(x) = \sqrt[3]{1 - \left(m_F(x)\right)^2 - \left(n_F(x)\right)^2}$ is to find out as the degree of indeterminacy of 'x' to F. For convenience, Senapathi and Yager[9] called $\left(m_F(x),\ n_F(x)\right)$ a fermatean uncertaintynumber (FUN) denoted by $F = (m_F,\ n_F)$.



We will explain the membership grades (MG's) related Fermatean uncertainty collections as Fermatean membership grades.

2.6 Theorem: [Senapati and Yager, 2019a] The collections of FMG's is higher than the set of Pythagorean membership grades (PMG's) and bi uncertainty membership grades (BMG's).

Proof: This improvement can be evidently approved in the following figure.



Here we find that BMG's are all points beneath the line $x + y \leq 1$, the PMG's are all points with $x^2 + y^2 \leq 1$. We see that the BMG's enable the presentation of a bigger body of non-standard membership grades than BMG's and PMG's. Based on fermateanuncertainty membership grades,we study interval-valued fermatean uncertainty soft set in matrix aspects.

2.7 Operations on Fermatean fuzzy set

Let us consider three FFs D= (α_D,β_D), $D_1=(\alpha_{D1},B_{D1})$ and $D_2=(\alpha_{D2},B_{D2})$ on Universal set X and $\lambda > 0$. The elementary operations on the FFS are defined as follows.

1.Addition $D_1 \oplus D_2 = (\alpha_{D1})^3 + (\alpha_{D2})^3 - (B_{D1})^3 + (B_{D2})^3$

2.Multiplication $D_1 x\, D_2 = \sqrt[3]{(\alpha_{D1},\alpha_{D2}(B_{D1})^3 + (B_{D2})^3 - (B_{D1})^3}$

3.Scalar Multiplication: $\lambda \bullet D = \sqrt[3]{1-(1-\alpha_D{}^3)^\lambda, (B_D)^\lambda}$

4. Exponent: $D^\lambda = ((\alpha)^\lambda, \sqrt{1-(1-B_D{}^3)^\lambda}$

5. Union: $D_1 \cup D_2 = (\max(\alpha_{D1},\alpha_{D2}) , \min(B_{D1},B_{D2}))$

6. Intersection: $D_1 \cap D_2 = (\min(\alpha_{D1},\alpha_{D2}) ,\max(B_{D1},B_{D2})$

7. Complement: $D_1{}^c = <B_D,\alpha_D>$

2.8 Example : Let D = (0.7,0.4), D_1=(0.3,0.6), D_3=(0.6,0.5) be the FFS and λ=2 be a scalar. Then,

1.Addition $D_1 \oplus D_2 = 0.152$

2.Multiplication $D_1 x\, D_2 = 0.007$

3.Scalar Multiplication: $\lambda \odot D = 0.006$

4. Exponent: $D^\lambda = 0.034$

5. Union: $D_1 \cup D_2 = 0.6$

6. Intersection: $D_1 \cap D_2 = 0.3$

7. Complement: $D_1{}^c = < 0.4,0.7 >$.

2.9Definition :(Score function of FFS): Let us consider an D=(α_D, β_D), then the score function for D is symbolized as $S_D(D)$ and described in the following manner

$$S_D(D) = (\alpha_D{}^3 - \beta_D{}^3) \text{ ------------------------------------- } (1)$$

2.10 Definition :(Accurracy function of FFS): Let us consider an D=(α_D, β_D), then the accuracy function for D is symbolized as S_D(D) and described in the following manner

$A_D(D) = (\alpha_D^3 + \beta_D^3)$ ------------------------------------- (2)

2.11 Proposed Fermatean fuzzy score function:

In this section, we developed a ranking function for the ordering of the FFS in decision making problems.

Let us consider an FFS D=(α_D, β_D). Then the score function for D is symbolized as follows :

Type-1: $S^*_{1D}(D) = (1 - \alpha_D^2 - \beta_D^2) /2$

Type-2: $S^*_{2D}(D) = (1 + 2\alpha_D^3 - \beta_D^3) /3$

Type-3: $S^*_{3D}(D) = [(1 + \alpha_D^2 - \beta_D^2) \; |\alpha_D - \beta_D| \,] /2$

2.12 Definition : The geometric mean is a mean average, which indicates the central tendency or typical value of a set of numbers by using the product of their values (as opposed the AM which uses their sum).

In general the geometric mean is defined as the nth root of the product on n numbers, i,e., for a set of numbers x_1 ,x_2, ……….. x_n, the geometric mean is defined as

$(\prod xi)^{1/n} = (x_1x_2x_3……x_n)^{1/n}$ ------------------------------- (3)

2.13:Theorem : (i) Consider an FFS D= (α_D,β_D), then S*(D) ∈ [0,1].

(ii) Consider an FFS D= $1+ \alpha_D^2 - \beta_D^{2,}$ then $S^*_D(D) \in [0,1]$.

Proof:

(i) By the definition of an orthopair α_D, $\beta_D \in [0,1]$. .Then $S^*_D(D) \in [0,1]$.

(ii) Also $\alpha_D^2 \geq 0, \beta_D^2 \geq 0$, $\alpha_D^2 \leq 1$ and $\beta_D \leq 1$ which implies $1- \alpha_D^2 \geq 0$

$1+ \alpha_D^2 - \beta_D^2 \geq 0$, $1/2(1+ \alpha_D^2 - \beta_D^2) \geq 0$.

Also,

$\alpha_D^2 - \beta_D^2 \leq 1$ Implies $1+ \alpha_D^2 - \beta_D^2 \geq 1/2$ $(1+ \alpha_D^2 - \beta_D^2) \geq 1$. Hence $S_D^*(D) \in [0,1]$.

3. MODEL OF FERMATEAN FUZZY TRANSPORTATION PROBLEM (FFTP)

The balanced fermatean fuzzy transportation problem in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, may be formulated as follows:

Fermatean fuzzy transportation problem given by

Minimize $< \alpha_D, \beta_D > = \sum_{i=1}^{m} \sum_{j=1}^{n} < \alpha C_{ij}, \beta C_{ij} >$

Subject to $\sum_{j=1}^{n} xij \leq < \alpha_{ai}, \beta_{ai}$, i=1,2,…m

$\sum_{j=1}^{n} x_{ij} \leq < \alpha_{ai}, \beta_{ai} >$, i=1,2,….n

Where $0 \leq (\alpha_{Ď})^3 + (\beta_{Ď})^3 \leq 1$

$0 \leq (\alpha_{ai})^3 + (\beta_{ai})^3 \leq 1$, i=1,2,3….m

$0 \leq (\alpha_{bj})^3 + (\beta_{bj})^3 \leq 1$ j=1,2,3….n

$x_{ij} \geq 0$ and $0 \leq (\alpha_{cij})^3 + (\beta_{cij})^3 \leq 1$

where (α_{ai},β_{aj}) is the total fermatean fuzzy availability of the item and product at i^{th}orgin (α_{bj}, β_{bj}) is the total fermatean fuzzy demand of the item/product at the j^{th} destination ,< α_{cij}, β_{cij}> is unit fermatean fuzzy transportation cost from i^{th} origin to j^{th} destination.

Now problem(1) is a mathematical model of a TP in fermatean environment called as FFTP.

It is noted that if

$\sum_{i=1}^{m}\oplus(\alpha_{ai},\beta_{ai})=\sum_{j=1}^{n}\oplus(\alpha_{bj},\beta_{bj})$, then FFTP is said to balanced. Otherwise it is called unbalanced FFTP. The symbol $\sum\oplus$ denoted as summation in terms of fermatean addition sense.

4. ALGORTHIM USING GEOMETRIC MEAN TO SOLVE FFTP

Step:1 Make sure whether the TP is balanced or not, if not, make it balanced.

Step:2 Acquire the Geometric mean using for every row and column.

Step:3 Choose the utmost GM value from step-2, and assign the min(supply or demand) at the place of lowest value of consequent row or column.

Step:4 Repeat step-2 and step-3 till the demand and supply are fatigued.

Step:5 Compute the total transportation cost of the FFTP.

5. Numerical Example:

The input data for fermatean fuzzy transportation problem is given below. The optimal aim of the process is to minimize the transportation cost and maximize the profit. The same problem used in Laxminarayanansahoo is taken for verification.

Table-1 Fermatean fuzzy numbers

	D1	D2	D3	D4	Supply
D1	(0.1,0.9)	(0.2,0.8)	(0.1,0.8)	(0.2,0.9)	**(0.7,0.1)**
D2	(0.01,0.99)	(0.3,0.9)	(0.3,0.8)	(0.1,0.7)	**(0.8,0.1)**
D3	(0.1,0.8)	(0.4,0.8)	(0.4,0.9)	(0.1,0.9)	**(0.7,0.1)**
Demand	**(0.4,0.7)**	**(0.7,0.3)**	**(0.3,0.9)**	**(0.6,0.4)**	

Applying the solve function to all values, we convert all the fermatean fuzzy numbers the defuzzified fermatean fuzzy TP is given below.

Table-2 Defuzzification of Fermatean fuzzy numbers

	D1	D2	D3	D4	Supply
D1	0.09	0.16	0.125	0.075	0.25
D2	0.198	0.05	0.135	0.25	0.175
D3	0.175	0.1	0.015	0.18	0.25
Demand	0.175	0.21	0.05	0.24	-

Step-2: Find the Geometric mean using definition for every row and column and write it below the corresponding rows and columns.

Table:3 Geometric mean cost values

	D1	D2	D3	D4	Supply	GM
D1	0.09	0.16	0.135	0.075	0.25	**0.1357**
D2	0.198	0.05	0.135	0.25	0.175	**0.1347**
D3	0.175	0.1	0.015	0.18	0.25	**0.0660**
Demand	0.175	0.21	0.05	0.24	--	---
GM	**0.1486**	**0.0950**	**0.0664**	**0.1517**	---	--

Step-3: Choose the maximum geometric mean value from table-3 and assign the minimum (supply or demand) at the place of lowest value of consequent row or column. The maximum geometric mean value at the first column and the minimum allocation is at the cell (2,4).

Table:4 First allocation by GM

	D1	D2	D3	D4	Supply	GM
D1	0.090	0.16	0.135	0.075 **0.24**	0.25 ***(0.01)***	**0.1357**
D2	0.198	0.05	0.135	0.25	0.175	**0.1347**
D3	0.175	0.1	0.015	0.18	0.25	**0.0660**
Demand	0.175	0.21	0.05	0.24 ***(0)***	----	-----
GM	**0.1486**	**0.0950**	**0.0664**	**0.1517**	---	----

Step-4: Repeating the procedure until all the requirements is satisfied

Table-5 Optimal solution of fermatean fuzzy transportation problem

	D1	D2	D3	D4	Supply	GM
D1	0.090 **0.01**	0.16	0.135	0.075 **0.24**	0.25	**0.1357**
D2	0.198	0.05 **0.175**	0.135	0.25	0.175	**0.1347**
D3	0.175 **0.165**	0.1 **0.035**	0.015 **0.05**	0.18	0.25	**0.0660**
Demand	0.175	0.21	0.05	0.24	---	-----
GM	**0.1486**	**0.0950**	**0.0664**	**0.1517**	**-------**	**------**

The above table satisfies the rim conditions with (m+n-1) non negative allocations at independent positions. Thus the optimal solution is:

The transportation cost according to the VAM's method is:

$x_{11} = 0.01$, $x_{14} = 0.24$, $x_{22} = 0.175$, $x_{31} = 0.165$, $x_{32} = 0.035$, $x_{33} = 0.05$

Total cost = (0.090 x 0.01) + (0.075 x 0.24) + (0.05 x 0.175) +(0.175 x 0.165) +(0.1 x 0.035) + (0.015 x 0.05) = 0.060.

Total minimum cost will be Rs : 0.060

By solving the above fermatean fuzzy transportation problem using type -2 and type-3 of definition(2.11) the minimum transportation cost is 0.261 and 0.268 respectively. In order to show the efficiency of the proposed method, the same problem is solved with various existing methods like NWCR and LCM. we get the following results after solving the same problem.

The input data for the above fermatean fuzzy transportation problem without geometric mean is given below and the optimal solutions is obtained by using 2.11 of type-I, type-2 and type-3 respectively. The optimal aim of the process is to minimize the transportation cost and maximize the profit.

Table-6 Input data for transportation cost, supply and demand

	D1	D2	D3	D4	Supply
D1	(0.1,0.9)	(0.2,0.8)	(0.1,0.8)	(0.2,0.9)	**(0.7,0.1)**
D2	(0.01,0.99)	(0.3,0.9)	(0.3,0.8)	(0.1,0.7)	**(0.8,0.1)**
D3	(0.1,0.8)	(0.4,0.8)	(0.4,0.9)	(0.1,0.9)	**(0.7,0.1)**
Demand	**(0.4,0.7)**	**(0.7,0.3)**	**(0.3,0.9)**	**(0.6,0.4)**	

In the given table the total supply is equal to the total demand . Hence the transportation problem is balanced one.

Step-1: Determine the cost table from the given problem. Hence total supply equals to demand, hence we can proceed to step 2. By the definition 2.11 of type-1 is given by,

$S^*_{1D}(D)= ½(\alpha_D^2-\beta_D^2)$ by (i) of definition 2.11

$S(C_{11}) = (1 - (0.1)^2- (0.9)^2) /2 = 0.09$ by type-1

Step-2: Applying the score function to all the values , we convert all the fermatean fuzzy numbers into crisp numbers.

Table-7 Defuzzification of Fermatean fuzzy numbers

	D1	D2	D3	D4	Supply
D1	0.09	0.16	0.125	0.075	0.25
D2	0.198	0.05	0.135	0.25	0.175
D3	0.175	0.1	0.015	0.18	0.25
Demand	0.175	0.21	0.05	0.24	-

The optimal solutions is obtained by using type-1 of definition 2.11 by various existing methods like NWCR,LCM and VAM without using Geometric mean.

Table-8 Northwest corner rule (NWCR)

	D1	D2	D3	D4	Supply
D1	0.09 **0.175**	0.16	0.125	0.075 **0.075**	0.25

D2	0.198	0.05 **0.125**	0.135 **0.05**	0.25	0.175
D3	0.175	0.1 **0.085**	0.015	0.18 **0.165**	0.25
Demand	0.175	0.21	0.05	0.24	-

The initial basic feasible solutions are x_{11}=0.175, x_{14}=0.075, x_{22}=0.125, x_{23}=0.05, x_{32}=0.085, x_{34}=0.165.

The minimum optimal solution is Minz = 0.0725

Table-9 Least cost method (LCM)

	D1	D2	D3	D4	Supply
D1	0.09 **0.175**	0.16	0.125	0.075 **0.075**	0.25
D2	0.198	0.05 **0.21**	0.135	0.25 **0.035**	0.175
D3	0.175	0.1	0.015 **0.05**	0.18 **0.2**	0.25
Demand	0.175	0.21	0.05	0.24	-

The initial basic feasible solutions ax_{11}=0.175, x_{14}=0.075**, x_{22}=0.21, x_{24}=0.035, x_{33}=0.05, x_{34}=0.2.**

The minimum optimal solution is Minz = 0.077

Table-10 Vogels approximation Method (VAM)

	D1	D2	D3	D4	Supply
D1	0.09	0.16	0.125	0.075 **0.15**	0.25
D2	0.198	0.05 **0.2**	0.135	0.25	0.175
D3	0.175 **0.085**	0.1 **0.06**	0.015 **0.11**	0.18 **0.16**	0.25
Demand	0.175	0.21	0.05	0.24	-

The initial basic feasible solutions are x_{14}=0.15, x_{12}=0.2, x_{22}=0.2, x_{31}=0.085, x_{32}=0.06, x_{33}=0.11, x_{33}=0.16.

The minimum optimal solution is Minz = 0.072.

Similarly definition 2.11 of type-2 $S^*_{2D}(D)= (1 + 2\alpha_D^3-\beta_D^3) / 3$ and type-3 $S^*_{3D}(D)= [(1 + \alpha_D^2- \beta_D^2) \left|\alpha_D - \beta_D\right|] /2$ respectively given by optimal solutions

0.274 (by NWCR) , 0.159 (LCM),0.147 (VAM) and

0.125 (by NWCR) , 0.118 (LCM),0.0364 (VAM)

The above results satisfies the conditions with (m+n-1) non-negative allocations at independent positions.

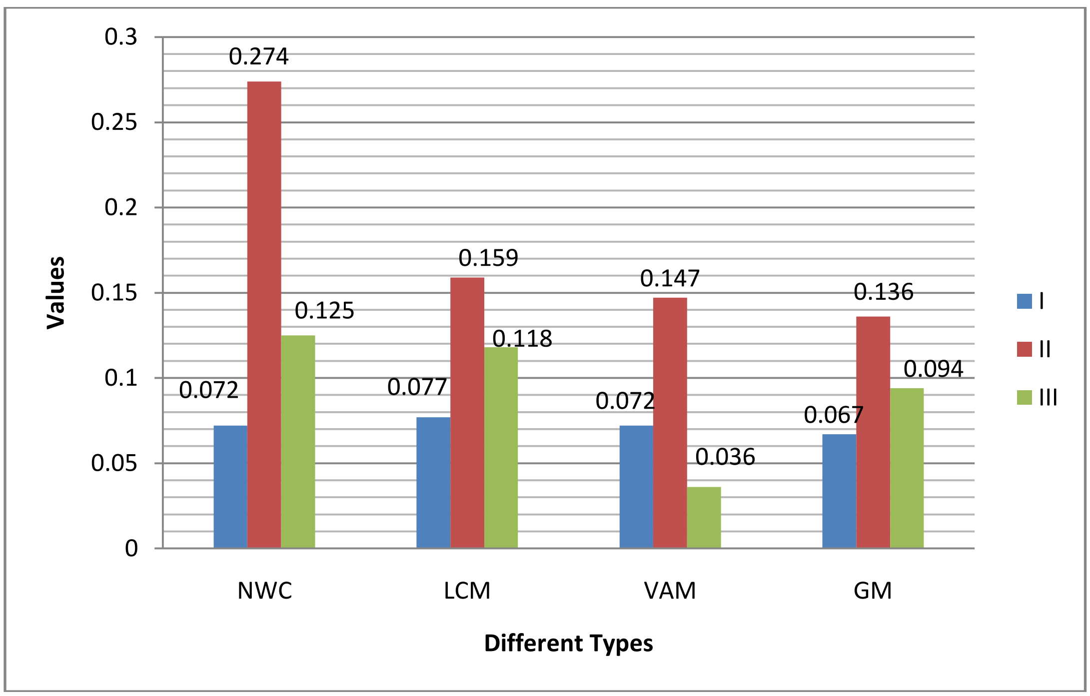
In order to show that the efficiency of the proposed method, the same problem is solved with various methods like NWC,LCM, VAM Method . We set the following results after solving the problems .

Table-11 Comparison Table

Type of Score function	NWC	LCM	VAM	GM (Proposed method)
I	0.072	0.077	0.072	**0.0677**
II	0.274	0.159	0.147	**0.136**
III	0.125	0.118	0.0364	**0.094**

By comparing the proposed method with the other existing methods, the proposed method (Geometric mean) gives the better optimum solution . This method is more efficient to reduce to transportation cost than the other existing methods.

Comparison chart:



Conclusion: Based on the earlier discussion transportation problem and recently available several research articles on TP, there are no existing methodologies which are available on TP under fermatean fuzzy environment. Hence, there is an essential urgency to introduce a new solution methodology for solving TP in the light of fermatean fuzzy environment. A modified algorithm using geometric mean to unravel the fermatean fuzzy transportation problem is suggested in this work.

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