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Stability of Neutral Delay Differential Equation using Spectral Approximations

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Abstract: This paper estimates the Spectral approximation by Spectral Tau and Spectral Least Squares Method for the first order Neutral Delay Differential Equation (NDDE). Here the NDDE is reduced into advection Equation to determine Spectral solutions. The effects on different types of basis are compared with numerical solution of NDDE. The Legendre and Chebyshev Basis generate much superior results than the Mixed Fourier Basis.

Keywords: Neutral Delay Differential Equation; Spectrum; Spectral Least Squares Method; Spectral-Tau Method.

1. INTRODUCTION

Ordinary differential equations (ODEs) and delay differential equations (DDEs) are accustomed to represent many phenomena of physical appearance. While ODEs comprise of derivatives that depend upon the outcome at the current value of the time, whereas DDEs depend upon the outcome at previous times. Various practical challenges are able to be resolved by delay differential equations (DDE), since delays are intrinsic to practical systems. Further more, DDEs appear in manufacturing processes including machine tool vibrations, dynamics, optics, chemical kinetics, robotics, controls, acoustics, biology, ecology, economics and many areas. These delays can be fixed value or variable that takes part in a significant task in physical environment. NDDEs are a special category of DDEs, where the delay appear in the highest derivative of the DDEs, while in retarded delay differential equations (RDDEs), the delay is absent in the highest derivative of the DDEs.

Delay differential equations of neutral type are the differential equations which engross either single delay or else several delays in the order of highest power of derivative, and which are engaged in diverse engineering models [1–7]. For instance the vibration of the beam is monitored by a delayed resonator and which is been suggested as the function of delayed acceleration feedback [3]. The present approach employ acceleration sensor for its gain in high frequency-low amplitude circumstances, the delayed signal of acceleration sensor resulting to NDDE. It can be formed as Delayed nonlinear controller [4, 5] which is utilized to reduce the effect of container crane. When it is planted on a massive industrial container crane, the swing of cargo is reduced as well as results a significant improvement in the production of the crane. It is moreover experienced in the applications of actual dynamic sub structuring approach [6]. In exploring the dynamic reaction to complex configurations, the technique replaces the branch with original model to a numerical representation. Time delay appear while merging together the actual substructure and numerical by applying actuators. Numerical solution of NDDE with infinite delay [8] is shown by converging the solution to actual solution.

To design oscillator for transmission line that generates chaotic elevated frequency and exhibit strong dynamics triggered with delay [9] which is designed by Nonlinear NDDE. Infinite Eigen values of characteristic equations represents DDEs of the dimensional systems. Universal approach for estimating the stability is to convert the partial differential equation (PDE) as DDE [10–13] of numerous delays. Then, PDE approximation is transformed by ordinary differential equations (ODEs) by applying spatial discretization technique, such as Spectral Least Squares [14], Spectral Tau methods [10, 15] and etc., this impact on the spectrum (All the Eigen values of the characteristic equation) is shown.

Here to estimate the Eigen values of NDDEs, we correlate the Spectral-Tau and Spectral Least Squares method. This Spectral methods gives favorable result since it converges exponentially to the exact solution [16]. Earlier, mixed Fourier basis was applied on construction of Spectral (Galerkin) methods, to obtain Eigen values of the NDDEs solution [14,15]. We learn the effect of the discretion of Spectral basis [16, 17] i.e, Mixed Fourier basis, Shifted Chebyshev basis and Shifted Legendre basis for a few sample problems on the convergence of the Eigen values.

2. FORMULATION AND COMPUTATION OF SOLUTION OF A NEUTRAL DELAY EQUATION INTO AN ADVECTION EQUATION

Consider the first order linear Neutral delay equation with *m* delays.

$$\dot{x}(t) + ax(t) + \sum_{q=1}^{m} b_q x(t-\tau_q) + \sum_{q=1}^{m} c_q \dot{x}(t-\tau_q) = 0, \tau_q > 0. \tag{1}$$

$$x(t) = \theta(t), -\tau_q \leq t \leq 0, \quad q = 1\ to\ m$$

where $\tau = \max(\tau_{1,}\tau_{2,}, \ldots., \tau_m)$.

Shifting of time is applied by $y(s,t) = x(t+s), s \in [-\tau, 0)$,

The given NDDE in Equation (1) can be transformed into advection Equation

$$\frac{\partial y(s,t)}{\partial t} = \frac{\partial y(s,t)}{\partial s}, \qquad s \in [-\tau, 0] \tag{2}$$

$$\left.\frac{\partial y(s,t)}{\partial t}\right|_{s=0} = -ay(0,t) - \sum_{q=1}^{m} b_q y(-\tau_q, t) - \sum_{q=1}^{m} c_q \frac{\partial y(-\tau_q,t)}{\partial t} \tag{3}$$

$$y(s,0) = \theta(s), s \in [-\tau, 0] \tag{4}$$

To approximate solution of Eqs. (2–4).are determined using the Spectral-Tau technique and Spectral-Least Squares technique as follows.

Spectral-Tau Method

The solution of the PDE (Eq.2) using Spectral –Tau method is denoted by means of

$$y(s,t) = \sum_{i=1}^{\infty} \emptyset_i(s)\eta_i(t) \tag{5}$$

where $\eta_i(t)$ *and* $\phi_i(s)$ are the time dependent co-ordinates and the basis functions.

Consider the summation for N terms

$$y(s,t) = \phi(s)^T \eta(t) \tag{6}$$

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$\phi(s) = [\phi_1(s), \phi_2(s), \dots \dots, \phi_N(s)]^T$ and $\eta(t) = [\eta_1(t), \eta_2(t), \dots, \eta_N(t)]^T$.

Substituting equation (6) in Equation (4) we get

$$\phi(s)^T \dot{\eta}(t) = \phi'(s)^T \eta(t) \tag{7}$$

Pre-multiplying equation (7) with $\phi(s)$ and integrating over the domain we get:

$$\int_{-\tau}^{0} \phi(s)\phi(s)^T \, ds \dot{\eta}(t) = \int_{-\tau}^{0} \phi(s)\phi'(s)^T \, ds \eta(t) \tag{8}$$

In matrix form

$$A\dot{\eta}(t) = B\eta(t) \tag{9}$$

$$\text{with A} = \int_{-\tau}^{0} \phi(s)\phi(s)^T \, ds, \tag{10}$$

$$\text{B} = \int_{-\tau}^{0} \phi(s)\phi'(s)^T \, ds \tag{11}$$

Substituting Equation (5) in equation (3) we get the scalar equation

$$\left. \frac{\partial(\phi(s)^T \eta(t))}{\partial t} \right|_{s=0} = -a\phi(0)^T \eta(t) - \sum_{q=1}^{m} b_q \phi(-\tau_q)^T \eta(t) - \sum_{q=1}^{m} c_q \frac{\partial \phi(-\tau_q)^T \eta(t)}{\partial t}$$

$$\phi(0)^T \dot{\eta}(t) = -a\phi(0)^T \eta(t) - \sum_{q=1}^{m} b_q \phi(-\tau_q)^T \eta(t) - \sum_{q=1}^{m} c_q \phi(-\tau_q)^T \dot{\eta}(t)$$

$$\phi(0)^T \dot{\eta}(t) = -a\phi(0)^T \eta(t) - \sum_{q=1}^{m} b_q \phi(-\tau_q)^T \eta(t) - \sum_{q=1}^{m} c_q \phi'(-\tau_q)^T \eta(t)$$

$$\phi(0)^T \dot{\eta}(t) = [-a\phi(0)^T - \textstyle\sum_{q=1}^{m} b_q \phi(-\tau_q)^T \eta(t) - \sum_{q=1}^{m} c_q \phi'(-\tau_q)^T]\eta(t) \tag{12}$$

From equation (9, 12) we get N+1 independent Equation.

Truncate the system (9) and augment it with equation (12), we get an determinate system of the form

$$M_{Tau}\dot{\eta}(t) = K_{Tau}\eta(t) \tag{13}$$

where

$$M_{Tau} = \begin{bmatrix} \bar{A} \\ \phi(0)^T \end{bmatrix} \tag{14}$$

$$K_{Tau} = \begin{bmatrix} \bar{B} \\ -a\phi(0)^T - \sum_{q=1}^{m} b_q \phi(-\tau_q)^T - \sum_{q=1}^{m} c_q \phi'(-\tau_q)^T \eta(t) \end{bmatrix} \tag{15}$$

and the matrices $\bar{A}, \bar{B}$ are evaluated by eliminating the end row of the given matrix A and B correspondingly.

The initial state for equation (13) is $\eta(0) = M^{-1} \int_{-\tau}^{0} \emptyset(s)\theta(s)ds$

The outcome of the NDDE is calculated by

$$x(t) = y(0,t) = \emptyset(0)^T \eta(t).$$

An approximation of the equation (1) is represented by the finite dimensional system (13).

Spectral Least-Squares Method

The replacement of the truncated approximate solution $y(s,t) = \sum_{i=1}^{\infty} \emptyset_i(s)\eta_i(t)$, causes error in the PDE (2) which is denoted by

$$e(s,t) = \phi(s)^T \dot{\eta}(t) - \phi'(s)^T \eta(t) \tag{16}$$

An excellent approximation is described by a "minute" error *e(s, t)* from the given boundary condition Equation (12).we aspire to estimate the subsequent constrained optimization problem to reduce the given error:

$$\min_{\dot{\eta}(t)} \frac{1}{2} \int_{-\tau}^{0} e(s,t)^2 \, ds = \min_{\dot{\eta}(t)} \frac{1}{2} \int_{-\tau}^{0} [\phi(s)^T \dot{\eta}(t) - \phi'(s)^T \eta(t)]^2 \, ds \tag{17}$$

$$[\phi(0)^T = [-a\phi(0)^T - \textstyle\sum_{q=1}^{m} b_q \phi(-\tau_q)^T - \sum_{q=1}^{m} c_q \phi'(-\tau_q)^T]\eta(t) \tag{18}$$

the integral square of the error function for the field is reduced by constructing a Lagrange multiplier Λ, to calculate $\dot{\eta}(t)$ is given by

$$\mathrm{L}(\dot{\eta}(t),\Lambda) = \frac{1}{2}\int_{-\tau}^{0}[\phi(s)^T\dot{\eta}(t) - \phi'(s)^T\eta(t)]^2\,ds \;-\; \Lambda[[\phi(0)^{\mathrm{T}}\dot{\eta}(t) + \;a\phi(0)^T\eta(t) \;+ \sum_{q=1}^{m} b_q\phi(-\tau_q)^T\eta(t) + \sum_{q=1}^{m} c_q\phi'(-\tau_q)^T\eta(t)] \qquad (19)$$

To minimize L, the first order optimality condition is as follows

$$\frac{\partial L}{\partial \dot{\eta}(t)} = 0 \qquad (20)$$

$$\frac{\partial L}{\partial \Lambda} = 0 \qquad (21)$$

equation (20) and (21) are Substituted by equation (19) we get:

$$A\dot{\eta}(t) = B\eta(t) + \phi(0)\Lambda, \qquad (22)$$

$$\phi(0)^{\mathrm{T}}\dot{\eta}(t) = -a\phi(0)^T\eta(t) - \sum_{q=1}^{m} b_q\phi(-\tau_q)^T\eta(t) - \sum_{q=1}^{m} c_q\phi'(-\tau_q)^T\eta(t) \qquad (23)$$

Where A and B are the basis-dependent matrices defined in equation (10) and equation (11) respectively.

Applying Lagrange multiplier to solve Equation (22) and Equation (23) we get

$$\frac{\dot{\eta}(t)}{B\left([-\sum_{q=1}^{m} b_q\phi(-\tau_q)^T\eta(t) - \sum_{q=1}^{m} c_q\phi^{'(-\tau_q)^T}\eta(t)]\right) + \phi(0)^T\phi(0)\Lambda}$$
$$= \frac{\eta(t)}{-a\phi(0)\Lambda\phi(0)^T + A\sum_{q=1}^{m} b_q\phi(-\tau_q)^T\eta(t) + A\sum_{q=1}^{m} c_q\phi^{'(-\tau_q)^T}\eta(t)}$$
$$= \frac{1}{-Aa\phi(0)^T - B\phi(0)^{\mathrm{T}}}$$

Cross multiplying we get

$$\eta(t)\{-Aa\phi(0)^T - \phi(0)^{\mathrm{T}}B\} = +\phi(0)^T\phi(0)\Lambda + A\sum_{q=1}^{m} b_q\phi(-\tau_q)^T\eta(t) + A\sum_{q=1}^{m} c_q\phi^{'(-\tau_q)^T}\eta(t)$$

$$\eta(t)\{-Aa\phi(0)^T - \phi(0)^{\mathrm{T}}B\} - A\sum_{q=1}^{m} b_q\phi(-\tau_q)^T\eta(t) - A\sum_{q=1}^{m} c_q\phi^{'(-\tau_q)^T}\eta(t) = +\phi(0)^T\phi(0)\Lambda$$

$$\{-A\left\{a\phi(0)^T + \sum_{q=1}^{m} b_q\phi(-\tau_q)^T + \sum_{q=1}^{m} c_q\phi^{'(-\tau_q)^T}\} - \phi(0)^{\mathrm{T}}B\right\}\eta(t) = +\phi(0)^T\phi(0)\Lambda$$

$$\frac{-[a\phi(0)^T + \sum_{q=1}^{m} b_q\phi(-\tau_q)^T + \sum_{q=1}^{m} c_q\phi'(-\tau_q)^T]\eta(t)}{+a\phi(0)^TA^{-1}\phi(0)} + \frac{-A^{-1}B}{a\phi(0)^{\mathrm{T}}A^{-1}\phi(0)} = \phi(0)\Lambda$$

Substituting the value of $\Lambda\,\phi(0)$ in Eq.(22)

$$A\dot{\eta}(t) = B\eta(t) + \left[\frac{-[a\phi(0)^T + \sum_{q=1}^{m} b_q\phi(-\tau_q)^T + \sum_{q=1}^{m} c_q\phi^{'(-\tau_q)^T}]\eta(t)}{+a\phi(0)^TA^{-1}\phi(0)} + \frac{-A^{-1}B}{a\phi(0)^{\mathrm{T}}A^{-1}\phi(0)}\right]\eta(t) \qquad (24)$$

and simplifying we get:

$$A\dot{\eta}(t) = K_{LS}\eta(t), \qquad (25)$$

where $K_{LS} = B - [\frac{[a\phi(0)\phi(0)^T + \sum_{q=1}^{m} b_q\phi(0)\phi(-\tau_q)^T + \sum_{q=1}^{m} c_q\phi(0)\phi^{'(-\tau_q)^T}]}{+a\phi(0)^TA^{-1}\phi(0)} - \frac{A^{-1}B}{a\phi(0)^{\mathrm{T}}A^{-1}\phi(0)}]$ (26)

3. COMPUTATION OF SPECTRUM OF NDDE

The characteristic equation of (1) is given by $x(t) = ce^{\lambda t}$

$$C(\lambda) = \lambda + a + \sum_{q=1}^{m} b_q e^{-\lambda\tau_q} - \lambda \sum_{q=1}^{m} c_q e^{-\lambda\tau_q} = 0 \qquad (27)$$

where the complex eigen value λ of the characteristic equation (1) can be described as

$$\lambda = \alpha + i\beta \qquad (28)$$

Applying Euler's identity in equation (27), we get

$$\alpha + i\beta + a + \sum_{q=1}^{m} b_q e^{-(\alpha+i\beta)\tau_q} - (\alpha + i\beta)\sum_{q=1}^{m} c_q e^{-(\alpha+i\beta)\tau_q} = 0$$

$$\alpha + i\beta + a + \sum_{q=1}^{m} b_q e^{-\alpha\tau_q} e^{-i\beta\tau_q} - (\alpha + i\beta)\sum_{q=1}^{m} c_q e^{-\alpha\tau_q} e^{-i\beta\tau_q} = 0$$

$$\alpha + i\beta + a + \sum_{q=1}^{m} b_q e^{-\alpha\tau_q}(cos\beta\tau_q - isin\beta\tau_q)$$
$$- \alpha \sum_{q=1}^{m} c_q e^{-\alpha\tau_q}(cos\beta\tau_q - isin\beta\tau_q) - i\beta \sum_{q=1}^{m} c_q e^{-\alpha\tau_q}(cos\beta\tau_q - isin\beta\tau_q)$$
$$= 0$$

$$\alpha + i\beta + a + \sum_{q=1}^{m} b_q e^{-\alpha\tau_q} cos\beta\tau_q - i\sum_{q=1}^{m} b_q e^{-\alpha\tau_q} sin\beta\tau_q$$
$$- \alpha \sum_{q=1}^{m} c_q e^{-\alpha\tau_q} cos\beta\tau_q + i\alpha \sum_{q=1}^{m} c_q e^{-\alpha\tau_q} sin\beta\tau_q - i\beta \sum_{q=1}^{m} c_q e^{-\alpha\tau_q} cos\beta\tau_q$$
$$- \beta \sum_{q=1}^{m} c_q e^{-\alpha\tau_q} sin\beta\tau_q = 0$$

$$\alpha + a + \sum_{q=1}^{m} b_q e^{-\alpha\tau_q} cos\beta\tau_q - \alpha\sum_{q=1}^{m} c_q e^{-\alpha\tau_q} cos\beta\tau_q - \beta\sum_{q=1}^{m} c_q e^{-\alpha\tau_q} sin\beta\tau_q = 0 \quad (29)$$

$$\beta - \sum_{q=1}^{m} b_q e^{-\alpha\tau_q} sin\beta\tau_q + \alpha\sum_{q=1}^{m} c_q e^{-\alpha\tau_q} sin\beta\tau_q - \beta\sum_{q=1}^{m} c_q e^{-\alpha\tau_q} cos\beta\tau_q = 0 \qquad (30)$$

Equation (29) and (30) are transcendental equation having infinite number of Eigen values.

We describe the set of the characteristic equation by spectrum of equation (1) as given below

$$S = \{\lambda_i | C(\lambda_i) = 0, Re\lambda_1 \geq Re\lambda_2 \geq \cdots\}. \qquad (31)$$

The characteristic roots of the $N \times N$ system of Equation (13) and Equation (25) is estimated to get approximate spectrum which is given by

$$\hat{S}_{Tau} = \{\widehat{\lambda_\iota} \,|\, \det(M_{Tau}\widehat{\lambda_\iota} - K_{Tau}) = 0, Re\widehat{\lambda_1} \geq Re\widehat{\lambda_2} \geq \cdots\} \qquad (32)$$

$$\hat{S}_{LS} = \{\widehat{\lambda_\iota} \,|\, \det(A\widehat{\lambda_\iota} - K_{LS}) = 0, Re\widehat{\lambda_1} \geq Re\widehat{\lambda_2} \geq \cdots\} \quad (33)$$

Here the different choice of the basis functions $\emptyset(s)$ in equation (6) acts as an significant role in the convergence of the characteristic equation. To experiment this assumption let us consider the three different basis functions.

i) Mixed Fourier Basis[15]: $\emptyset(s) = [1, s, \sin\left(\frac{\pi}{\tau}s\right), \sin\left(2\frac{\pi}{\tau}s\right), \ldots]^T$ (34)

ii) Shifted Legendre Polynomial[16]:

$$\emptyset_1(S) = 1, \emptyset_2(S) = 1 + \frac{2s}{\tau},\ \emptyset_i(S) = \frac{(2i-3)\emptyset_2(S)\emptyset_{i-1}(S) - (i-2)\emptyset_{i-2}(S)}{i-1}, \text{ i=3, 4….} \quad (35)$$

iii) Shifted Chebyshev Polynomial [16]:

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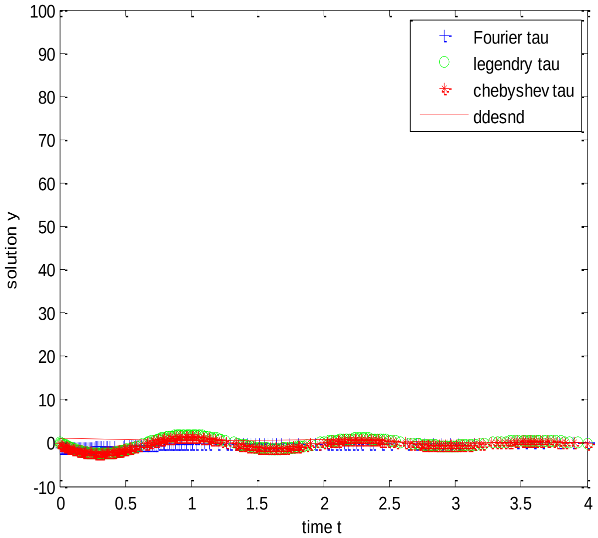
$$\emptyset_1(S) = 1, \emptyset_2(S) = 1 + \frac{2s}{\tau}, \emptyset_i(S) = 2\emptyset_2(S)\emptyset_{i-1}(S) - \emptyset_{i-2}(S), i = 3,4, \ldots \ldots \quad (36)$$

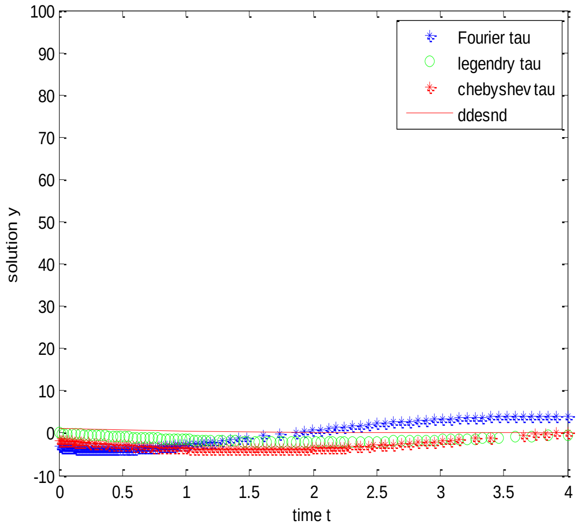
4. NUMERICAL STUDIES

We illustrate the above constructed algorithm using the following examples. The numerical solutions of the NDDEs (Eq. (1)) are computed and compared using ddeNsd solver, the Spectral Tau Method (Eq. 13) and Spectral Least Squares methods (Eq.25). To solve the ODEs (Eq. (13) and (Eq.25) resulting from the Spectral approximation, we use the "ode15s" integrator in Mat lab. The NDDEs (Eq.(1)) were solved using "ddeNsd" solver in Mat lab and are based on a dissipative approximation [8] of NDDEs. It is noted that we may require a large number of terms in the series solution (Eq. (10)) to get a good approximation for the non-smooth solutions of NDDEs (Eq. (1)).

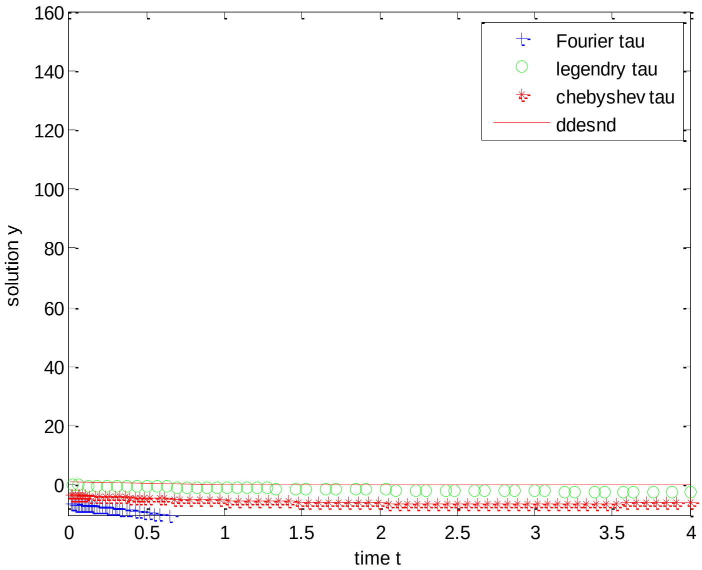
Consider the following linear scalar NDDE [18]: when a=1, b=1, c=1 in general equation $\dot{x}(t) + ax(t) + \sum_{q=1}^{m} b_q x(t-\tau_q) + \sum_{q=1}^{m} c_q \dot{x}(t-\tau_q) = 0, \tau_q > 0.$ we have $x'(t) + x(t) + x(t-1) + x'(t-1) = 0, t > 0$ with initial condition $x(t) = 1$ for $t \leq 0.$



$\tau = 1$



$\tau = 5$



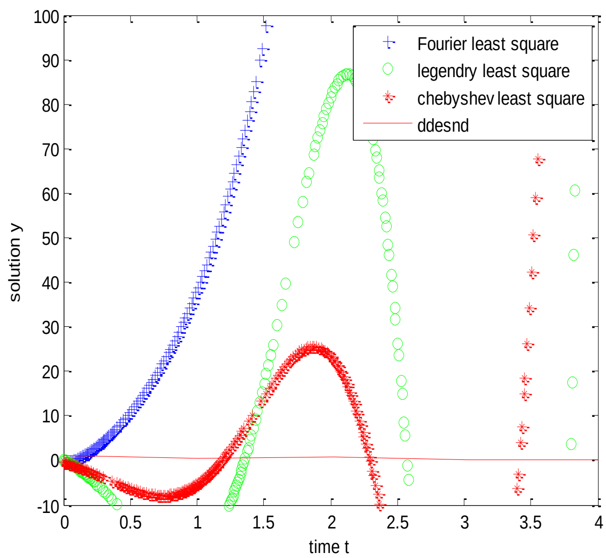
$\tau = 10$

Figure 4.1 Solution of NDDE by Spectral Tau Method for N=3

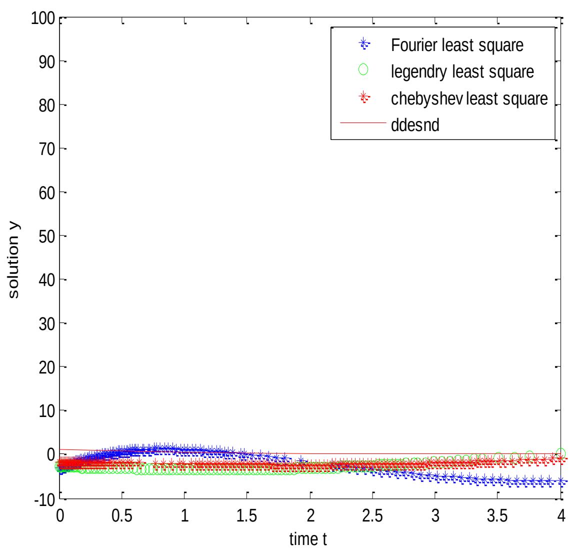
While we study the stability analysis of equation (1), it is found that greater stability occurs when N>2 also we infer the study of stability in the choice of Legendre Tau over the Chebyshev Tau and Fourier Tau over the ddeNsd solver.

When τ take different values, say $\tau = 1$, 5 and 10, it is shown that the convergence rate is higher for Spectral Tau method when delay $\tau \leq 1$ whereas the Spectral Least Square method converge for $\tau > 1$, over the choice of legendre basis over the chebyshev Tau and Fourier Tau.

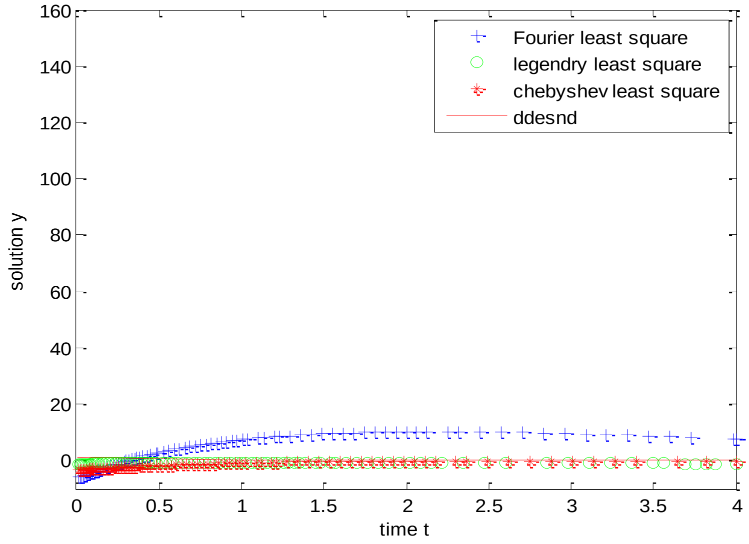
Which is inferred in figure 1.1and 1.2, for all three values of τ($\tau = 1$, 5 and 10) in Spectral Tau methods it is clear that Legendre Tau Methods and Legendre Least Square Methods almost coincide with all the three types.



$\tau = 1$



$\tau = 5$



$\tau = 10$

Figure 4.2 Solution of NDDE by Spectral Least Square Method for N=3

Table 4.1 Spectrum of NDDE by Spectral Tau and Spectral Least Square Method

	N=2			N=3		
	$\tau = 1$	$\tau = 5$	$\tau = 10$	$\tau = 1$	$\tau = 5$	$\tau = 10$
Fourier Tau	**-2.0 -2.0**	**- 0.4 + 0.8*i - 0.4 - 0.8*i**	**- 0.2 + 0.6*i - 0.2 - 0.6*i**	-1.099 - 0.9831 + 4.569*i - 0.9831 - 4.569*i	-1.973 - 0.12 + 0.6871*i - 0.12 - 0.6871*i	**-2.047 - 0.02998 + 0.3411*i - 0.02998 - 0.3411*i**
Legendre Tau	**-2.0 -2.0**	**- 0.4 + 0.8*i - 0.4 - 0.8*i**	**-2.0 -2.0**	**-1.043 - 0.4783 + 4.772*i - 0.4783 - 4.772*i**	**-1.764 - 0.1179 + 0.7282*i - 0.1179 - 0.7282*i**	**-1.94 - 0.02997 + 0.3504*i - 0.02997 - 0.3504*i**
Chebyshev Tau	**-2.0 -2.0**	**- 0.4 + 0.8*i - 0.4 - 0.8*i**	**-2.0 -2.0**	**-1.043 - 0.4783 + 4.772*i - 0.4783 - 4.772*i**	**-1.764 - 0.1179 + 0.7282*i - 0.1179 - 0.7282*i**	**-1.94 - 0.02997 + 0.3504*i - 0.02997 - 0.3504*i**
Fourier Least Square spectrum	**-2.0 -1.5**	**- 0.55 + 0.5454*i - 0.55 - 0.5454*i**	**- 0.4 + 0.3742*i - 0.4 - 0.3742*i**	**1.578 - 1.819e-12*i - 3.961 - 1.819e-12*i - 0.8918 + 3.638e-12*i**	-1.372 - 0.1518 + 0.5637*i - 0.1518 - 0.5637*i	**-1.344 - 0.06556 + 0.2944*i - 0.06556 - 0.2944*i**
Legendre Least Square spectrum	**- 1.0 + 1.118*i - 1.0 - 1.118*i**	**- 0.4 + 0.5385*i - 0.4 - 0.5385*i**	- 0.325 + 0.3455*i - 0.325 - 0.3455*i	**-0.9945 1.164 + 2.567*i 1.164 - 2.567*i**	**-0.7847 - 0.007667 + 0.6346*i - 0.007667 - 0.6346*i**	**-0.9827 - 0.04201 + 0.2804*i - 0.04201 - 0.2804*i**
chebyshev Least Square spectrum	**- 1.0 + 1.118*i - 1.0 - 1.118*i**	**- 0.4 + 0.5385*i - 0.4 - 0.5385*i**	- 0.325 + 0.3455*i - 0.325 - 0.3455*i	**-0.8949 1.114 + 2.804*i 1.114 - 2.804*i**	**-0.7715 - 0.01425 + 0.6498*i - 0.01425 - 0.6498*i**	**-0.9871 - 0.0398 + 0.2845*i - 0.0398 - 0.2845*i**

Spectrum of equation (1) using Spectral Tau and Spectral Least Square method are computed and shown in table1above, which is inferred in figure 1.3 and figure 1.4.

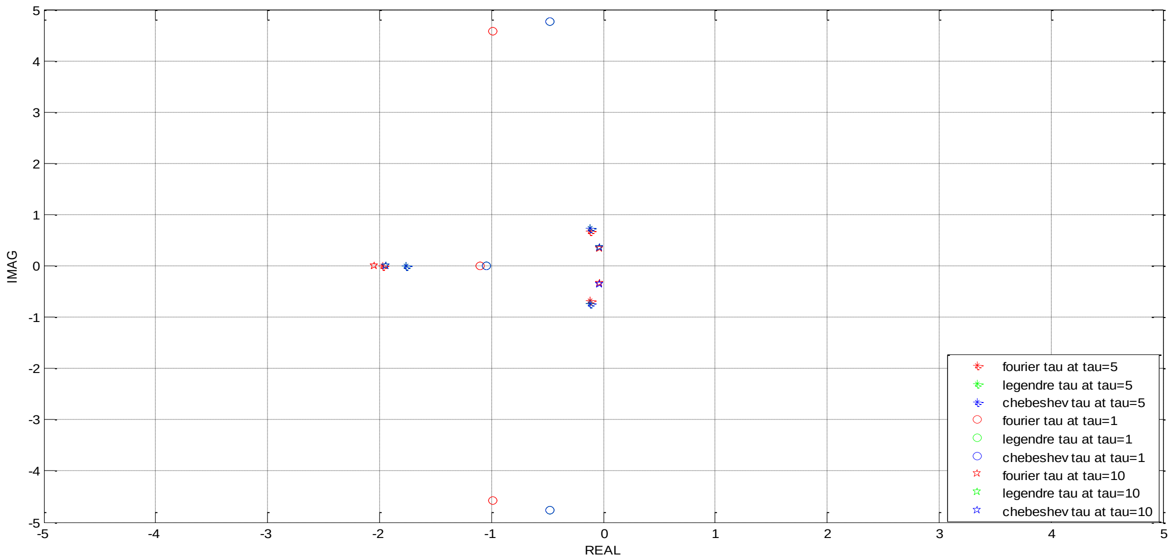


Figure 4.3 Spectrum of NDDE by Spectral Tau Method

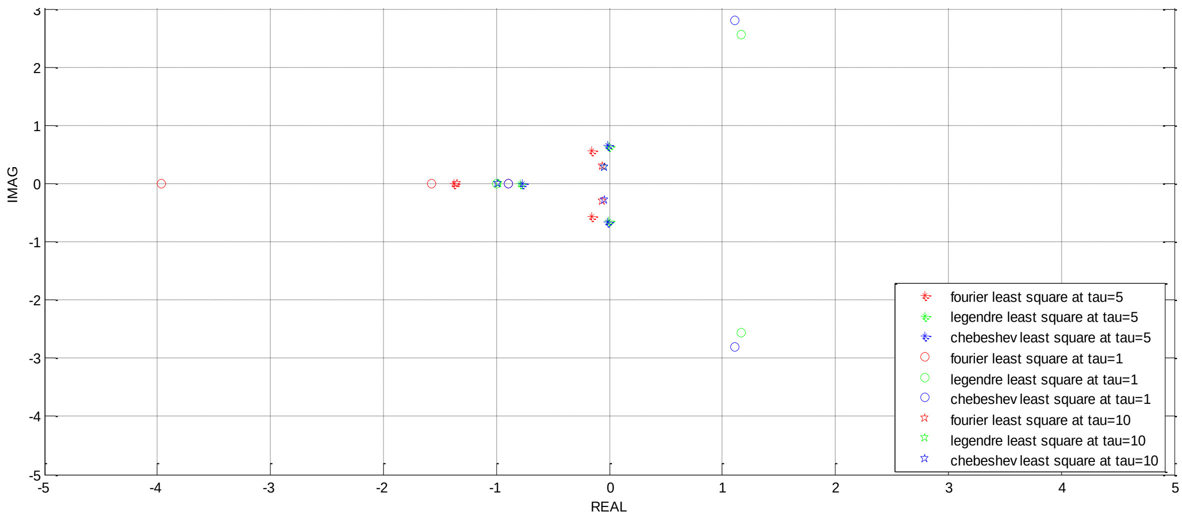


Figure 4.4 Spectrum of NDDE by Spectral Least Square Method

5. DISCUSSIONS AND CONCLUSION

In this paper the first order linear Neutral Delay Differential Equation is reconstructed as Advection Equation and solutions are computed using Spectral Tau and Spectral Least Square Method. To study the stability analysis. Spectrum was found for different choice of basis function and Legendre Basis guarantees the stability of NDDE as all the Eigen values lies in the left half of the real plane compared to Fourier and Chebeshev method. But for $\boldsymbol{\tau \leq 1}$, the stability fails for all the basis of Spectral Least Square method.

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