A Picture Fuzzy Approach to Solving Transportation Problem

S. Sathya Geetha[1], K. Selvakumari[2*]

[1]*Research Scholar, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies, Chennai, Tamil Nadu, India.*

[2*]*Professor, Department of Mathematics, Vels Institute of Science, Technology and Advanced Studies, Chennai, Tamil Nadu, India.*

[1]*geethuk911@gmail.com*

[2*]*selvafeb6@gmail.com*

Abstract: In this paper, we introduce the concept of picture fuzzy sets (PFS), which are the direct extensions of fuzzy sets and intuitionistic fuzzy sets. The picture fuzzy sets are more suitable for capturing imprecise, inconsistent and uncertain information. We applied three types ranking using proposed critical path method in which supply, demand and unit costs are taken as picture fuzzy numbers and to find the solution of transportation cost is minimum to compare with different ranking methods. A comparison table is established with results of existing technique and proposed method.

Keywords: Fuzzy Set, Intuitionistic Fuzzy Set, Picture Fuzzy Set, Distance between Picture Fuzzy Sets.

1. INTRODUCTION

Transportation problem helps to solve problems on distribution and transportation of resources move from one place to another. A lot of transport decisions take place under imprecision, uncertainity and partial truth in real life situation because of unpredictable factors such as human factors, vehicle factors, climate etc. To overcome this uncertain situation Zadeh [10] introduced fuzzy set theory. Many researchers give more attention to the generalization of fuzzy set theory. In 1986, Atanassov [1] studied about intuitionistic fuzzy sets. It is an extension of fuzzy sets. Dinagar and Palanivel [4] intimated to solve transportation problem under fuzzy environment. In 2010, Pandian and Natarajan [7] studied a new algorithm to solve a fuzzy optimal solution of fuzzy transportation problem. In 2013, Nagoor gani and Abbas [6] solved transportation problems with intuitionistic fuzzy supply and demands. Atanassov and Gargov [2] presented the concept of interval valued intuitionistic fuzzy set which is characterized by a membership function, a non-membership function and a hesitancy function whose values are intervals.

Sometimes, Fuzzy set and Intuitionistic fuzzy set face it hard to express the situations when the human thoughts involve more opinions like ‘yes’, ‘abstain’, ’no’, and ‘refusal’. The general election of a country is one of the good example for such situations where the voters give their opinions such as ‘vote for’, ‘abstain’, ‘vote against’ and the refusal of the election’. To handle this type of situations Cuong [3] introduced picture fuzzy set is an extension of fuzzy set and intuitionistic fuzzy set. It is characterized by three functions such as the degree of membership, the degree of neutral membership and the degree of non- membership of an element in a given set satisfying the condition that the sum of these three degrees is equal to or less than one.

In 2015, Singh [8] proposed correlation coefficient of picture fuzzy set and applied it to clustering analysis problem. In 2016, Guiwu [5] introduced picture fuzzy set in decision-making problem and proposed cross entropy of picture fuzzy sets. Similarly, Wang [9] introduced picture fuzzy sets based geometric aggregation operators and compared two picture fuzzy numbers using score and accuracy functions. We applied three types ranking using proposed critical path method in which supply, demand and unit costs are taken as picture fuzzy numbers and to find the solution of transportation cost is minimum to compare with different ranking methods.

2. PRELIMINARIES

2.1 Fuzzy Set

Let X be an universal set. Then a fuzzy set $\tilde{A}$ in X is defined by $\tilde{A} = \{(x_i, \mu_{\tilde{A}}(x_i)): x_i \in X\}$, where $\mu_{\tilde{A}}: X \to [0,1]$. Where $\mu_{\tilde{A}}(x_i)$ is said to be the membership function.

2.2 Intuitionistic Fuzzy Set

An Intuitionistic fuzzy set A^I on a universe X is an object of the form $A^I = \{(x_i, \mu_{A^I}(x_i), \gamma_{A^I}(x_i) \ / x_i \in X)\}$. Where the function $\mu_{A^I}(x_i)$ and $\gamma_{A^I}(x_i)$ denotes the degree of membership and the degree of non-membership of the element $x_i \in X$ to the set A^I respectively with the condition: $0 \le \mu_{A^I}(x_i) \le 1$, $0 \le \gamma_{A^I}(x_i) \le 1$, $0 \le \mu_{A^I}(x_i) + \gamma_{A^I}(x_i) \le 1$ and $\rho_{A^I}(x_i) = 1 - \mu_{A^I}(x_i) - \gamma_{A^I}(x_i)$ is usually called the degree of refusal membership of x to A^I.

A generalization of fuzzy sets and intuitionistic fuzzy sets are the following notion of picture fuzzy sets.

2.3 Picture Fuzzy Set

A picture fuzzy set (PFS) A^P on a universe X is an object of the form $A^P = \{(x_i, \mu_{A^P}(x_i), \eta_{A^P}(x_i), \gamma_{A^P}(x_i) \ / x_i \in X)\}$. Where the function $\mu_{A^P}(x_i)$, $\eta_{A^P}(x_i)$ and $\gamma_{A^P}(x_i)$ denotes the degree of positive membership, the degree of neutral membership and the degree of negative membership of the element $x_i \in X$ to the set A^P respectively with the condition: $0 \le \mu_{A^P}(x_i) \le 1$, $0 \le \eta_{A^P}(x_i), \le 1$, $0 \le \gamma_{A^P}(x_i) \le 1$, $0 \le \mu_{A^P}(x_i) + \eta_{A^P}(x_i), + \gamma_{A^P}(x_i) \le 1$ and $\rho_{A^P}(x_i) = 1 - \mu_{A^P}(x_i) - \eta_{A^P}(x_i) - \gamma_{A^P}(x_i)$ is usually called the degree of refusal membership of x to A^P. When $\eta_{A^P}(x_i) = 0$, for every $x_i \in X$, then the picture fuzzy set reduces into intuitionistic fuzzy set.

For a fixed set $x_i \in A^P$, ($\mu_{A^P}(x_i)$, $\eta_{A^P}(x_i)$, $\gamma_{A^P}(x_i)$, $\rho_{A^P}(x_i)$) is called picture fuzzy number, where $\mu_{A^P}(x_i) \in [0,1]$, $\eta_{A^P}(x_i) \in [0,1]$, $\gamma_{A^P}(x_i) \in [0,1]$, $\rho_{A^P}(x_i) \in [0,1]$ and $\mu_{A^P}(x_i) + \eta_{A^P}(x_i), + \gamma_{A^P}(x_i) + \rho_{A^P}(x_i) = 1$. Simply, we represent the picture fuzzy number as ($\mu_{A^P}(x_i)$, $\eta_{A^P}(x_i)$, $\gamma_{A^P}(x_i)$).

3. DISTANCE BETWEEN PICTURE FUZZY SETS

The distance between two picture fuzzy sets $A = (\mu_{a^P}, \eta_{a^P}, \gamma_{a^P})$ and $\boldsymbol{B} = (\mu_{b^P}, \eta_{b^P}, \gamma_{b^P})$ in $X = \{x_1, x_2, \ldots\ldots, x_n\}$ is calculated as follows.

(i)Normalized hamming distance

$$d_H(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left(\left|\mu_{a^P}(x_i) - \mu_{b^P}(x_i)\right| + \left|\eta_{a^P}(x_i) - \eta_{b^P}(x_i)\right| + \left|\gamma_{a^P}(x_i) - \gamma_{b^P}(x_i)\right|\right)$$

(ii)Normalized Euclidean distance

$$d_E(A,B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\mu_{a^P}(x_i) - \mu_{b^P}(x_i))^2 + (\eta_{a^P}(x_i) - \eta_{b^P}(x_i))^2 + (\gamma_{a^P}(x_i) - \gamma_{b^P}(x_i))^2}$$

4. SCORE AND ACCURACY FUNCTION OF PICTURE FUZZY SETS

Let $A = (\mu_{A^P}, \eta_{A^P}, \gamma_{A^P}, \rho_{A^P})$ be a picture fuzzy numbers, then a score function $S(A)$ is defined as $S(A) = \mu_{A^P} - \gamma_{A^P}$, where $S(A) \in [-1,1]$ and the accuracy function H(A) is defined as $H(A) = \mu_{A^P} + \eta_{A^P} + \gamma_{A^P}$, where $H(A) \in [0,1]$. Then, for two picture fuzzy numbers A and B

Case(i) If $S(A) > S(B)$, then A>B.

Case(ii) If $S(A) = S(B)$ and $H(A) > H(B)$ then $A > B$

Case(iii) If $S(A) = S(B)$ and $H(A) = H(B)$ then $A = B$

5. PROPOSED CRITICAL PATH METHOD

Step (1) We construct the given transportation problem into three types namely, type-1, type-2 and type-3. At first, we choose the types one by one to solve picture fuzzy transportation problem.

Step (1a) If it is type-1 picture fuzzy transportation problem, then calculate the score function of each PFN to convert all PFN into crisp value, we obtain the classical transportation problem.

Step (1b) If it is type-2 picture fuzzy transportation problem, then calculate the accuracy function of each PFN to convert all PFN into crisp value, we obtain the classical transportation problem.

Step (1c) If it is type-3 picture fuzzy transportation problem, then calculate the Euclidean distance measure function of each PFN to convert all PFN into crisp value, we obtain the classical transportation problem.

Step (2) To examine the balanced of given transportation problem. If $\sum_{i=0}^{m} a_i = \sum_{j=0}^{n} b_j$

Then, we go to step 3. If $\sum_{i=0}^{m} a_i \neq \sum_{j=0}^{n} b_j$ then, add dummy row or column and make it balance and proceed with balanced transportation problem.

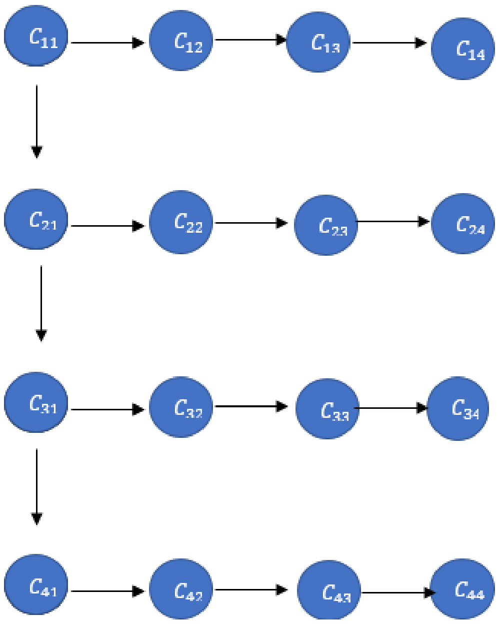
Step (3) We apply new method namely, proposed critical path method to all three types of picture fuzzy transportation problem to obtain the IBFS.

Step (4) First**,** we consider each row of transportation table

Step (4a) We choose first element from north west corner cell (NWC), adding all elements of the first row from NWC and mark it by right side of that corresponding row.

Step (4b) we start from the same NWC, adding all elements of the second row from NWC and mark it by right side of that corresponding row.

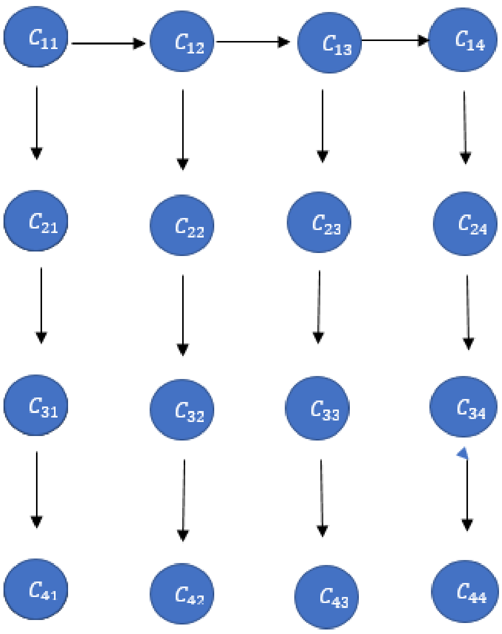
In the same way we add and mark it all the rows. For example, we plot the paths under the following each row



Step (5) First**,** we consider each column of transportation table

Step (5a) We choose first element from north west corner cell (NWC), adding all elements of the first column from NWC and mark it by straight down of that corresponding column.

Step (5b) we start from the same NWC, adding all elements of the second column from NWC and mark it by straight down of that corresponding column. In the same way we add and mark it all the columns. For example, we plot the paths under the following each column



Step (6) We find the longest path containing maximum resultant values of each row and each column and find the corresponding minimum cost value and do the allocation of that particular cost cell of the given table. Suppose we have more than one longest path choose any one.

Step (7) We repeat the procedures 4, 5 and 6 until all the allocations are completed.

6. NUMERICAL EXAMPLE

Consider a four dairy market A, B, C, D in a town with four dairies D1, D2, D3 and D4. The number of units available at the market is (0.6,0.3,0.1), (0.8,0.1,0.1), (0.5,0.3,0.1), (0.4,0.4,0.1**)** and the Demand at the dairies D1, D2, D3, and D4 is (0.6,0.2,0.1), (0.7,0.2,0.1), (0.4,0.3,0.2), (0.6,0.4,0) respectively. The unit cost of the transportation quoted in terms of Picture fuzzy numbers which are given in the matrix below. Find the transportation plan such that the total transportation cost is minimum.

	D1	**D2**	**D3**	**D4**	**Capacity**
A	(0.8,0.1,0.1)	(0.4,0.3,0.3)	(0.5,0.3,0.2)	(0.7,0.1,0.1)	**(0.6,0.3,0.1)**
B	(0.2,0.6,0.1)	(0.5,0.3,0.1)	(0.8,0.1,0)	(0.4,0.5,0.1)	**(0.8,0.1,0.1),**
C	(0.6,0.3,0.1)	(0.7,0.3,0)	(0.3,0.4,0.1)	(0.6,0.2,0.2)	**(0.5,0.3,0.1)**
D	(0.4,0.4,0.2)	(0.8,0.2,0)	(0.4,0.6,0)	(0.7,0.1,0.1)	**(0.4,0.4,0.1)**
Demand	**(0.6,0.2,0.1)**	**(0.7,0.2,0.1)**	**(0.4,0.3,0.2)**	**(0.6,0.4,0)**	

Solution: Since the given problem is a balanced Picture transportation problem. Applying the proposed critical path algorithm, the solution of problem is as follows:

Type-1

We compute the PFN by using score function to convert all PFN into crisp value, we obtain the classical transportation problem

	D1	**D2**	**D3**	**D4**	**capacity**
A	0.7	0.1	0.3	0.6	**0.5**
B	0.1	0.4	0.8	0.3	**0.7**
C	0.5	0.7	0.2	0.4	**0.4**
D	0.2	0.8	0.4	0.6	**0.3**
Demand	**0.5**	**0.6**	**0.2**	**0.6**	

By applying proposed critical path method to the given transportation problem, first we consider each row of the transportation table

We choose first element from north west corner cell (NWC), adding all elements of the first row from NWC and mark it by right side of that corresponding row. Similarly,

We start from the same NWC, adding all elements of the second row from NWC and mark it by right side of that corresponding row. In the same way we add and mark it all the rows. We do the same for column also, we get

	D1	**D2**	**D3**	**D4**	**capacity**	**Row summation**
A	0.7	0.1	0.3	0.6	**0.5**	**1.7**
B	0.1	0.4	0.8	0.3	**0.7**	**2.3**
C	0.5	0.7	0.2	0.4	**0.4**	**2.6**
D	0.2	0.8	0.4	0.6	**0.3**	**3.3**
Demand	**0.5**	**0.6**	**0.2**	**0.6**		
Column summation	**1.5**	**2.7**	**2.5**	**3**		

We find the longest path containing summation values of each row and each column and find the corresponding minimum cost value and do the allocation of that particular cost cell of the given table. Suppose we have more than one longest path choose anyone.

ISSN 2515-8260 Volume 07, Issue 02, 2020

	D1	D2	D3	D4	Capacity	Row summation
A	0.7	0.1	0.3	0.6	**0.5**	1.7
B	0.1	0.4	0.8	0.3	**0.7**	2.3
C	0.5	0.7	0.2	0.4	**0.4**	2.6
D	**0.3** 0.2	0.8	0.4	0.6	**0.3**	**3.3←**
Demand	**0.5**	**0.6**	**0.2**	**0.6**		
Column summation	1.5	2.7	2.5	3		

We apply the same technique repeatedly, the final allocation is

	D1	D2	D3	D4	capacity
A	0.7	**0.5** 0.1	0.3	0.6	**0.5**
B	**0.2** 0.1	**0.1** 0.4	0.8	**0.4** 0.3	**0.7**
C	0.5	0.7	**0.2** 0.2	**0.2** 0.4	**0.4**
D	**0.3** 0.2	0.8	0.4	0.6	**0.3**
Demand	**0.5**	**0.6**	**0.2**	**0.6**	

The Transportation Cost

$$Z = 0.1* 0.5+ 0.1* 0.2 + 0.4 * 0.1 + 0.3 * 0.4 + 0.2 * 0.2 + 0.4 * 0.2 + 0.2 * 0.3$$

$$Z = 0.41$$

Type-2

We compute the PFN by using accuracy function to convert all PFN into crisp value, we obtain the classical transportation problem.

	D1	D2	D3	D4	capacity
A	1	1	1	9	**1**
B	9	9	9	1	**1**
C	1	1	8	1	**9**
D	1	1	1	9	**9**
Demand	**9**	**1**	**9**	**1**	

By applying proposed critical path method to the given transportation problem, first we consider each row of the transportation table.

We choose first element from north west corner cell (NWC), adding all elements of the first row from NWC and mark it by right side of that corresponding row. Similarly,

We start from the same NWC, adding all elements of the second row from NWC and mark it by right side of that corresponding row. In the same way we add and mark it all the rows. We do the same for column also, we get.

	D1	D2	D3	D4	capacity	Row summation
A	1	1	1	9	**1**	**12**
B	9	9	9	1	**1**	**29**
C	1	1	8	1	**9**	**21**

D	1	1	1	9	**9**	**23**
Demand	**9**	**1**	**9**	**1**		
Column summation	**12**	**13**	**21**	**23**		

We find the longest path containing summation values of each row and each column and find the corresponding minimum cost value and do the allocation of that particular cost cell of the given table. Suppose we have more than one longest path choose anyone.

	D1	**D2**	**D3**	**D4**	**capacity**	**Row summation**
A	1	1	1	9	**1**	12
B	9	9	9	**1** / 1	**1**	29←
C	1	1	8	1	**9**	21
D	1	1	1	9	**9**	**23**
Demand	**9**	**1**	**9**	**1**		
Column summation	12	13	21	23		

We apply the same technique repeatedly, the final allocation is

	D1	**D2**	**D3**	**D4**	**Capacity**
A	1	1	**1** / 1	9	**1**
B	9	9	9	**1** / 1	**1**
C	**9** / 1	**0** / 1	8	**0** / 1	**9**
D	1	1 / 1	**8** / 1	9	**9**
Demand	**9**	**1**	**9**	**1**	

The Transportation Cost

$$Z = 1*1 + 1*1 + 1*9 + 1*0 + 1*0 + 1*8 + 1*1$$

$$Z = 20$$

Type-3

We compute the PFN by using Euclidean distance measure function to convert all PFN into crisp value, we obtain the classical transportation problem

	D1	**D2**	**D3**	**D4**	**capacity**
A	0.82	0.58	0.63	0.72	**0.70**
B	0.57	0.60	0.81	0.66	**0.82**
C	0.70	079	0.47	0.68	**0.60**
D	0.60	0.84	0.76	0.72	**0.57**
Demand	**0.65**	**0.75**	**0.53**	**0.76**	

By applying proposed critical path method to the given transportation problem, first we consider each row of the transportation table

We choose first element from north west corner cell (NWC), adding all elements of the first row from NWC and mark it by right side of that corresponding row. Similarly,

We start from the same NWC, adding all elements of the second row from NWC and mark it by right side of that corresponding row. In the same way we add and mark it all the rows. We do the same for column also, we get

	D1	**D2**	**D3**	**D4**	**capacity**	**Row summation**
A	0.82	0.58	0.63	0.72	**0.70**	**2.75**
B	0.57	0.60	0.81	0.66	**0.82**	**3.46**
C	0.70	079	0.47	0.68	**0.60**	**4.03**
D	0.60	0.84	0.76	0.72	**0.57**	**5.01**
Demand	**0.65**	**0.75**	**0.53**	**0.76**		
Column summation	**2.69**	**3.63**	**4.07**	**4.81**		

We find the longest path containing summation values of each row and each column and find the corresponding minimum cost value and do the allocation of that particular cost cell of the given table.

Suppose we have more than one longest path choose anyone.

	D1	**D2**	**D3**	**D4**	**capacity**	**Row summation**
A	0.82	0.58	0.63	0.72	**0.70**	2.75
B	0.57	0.60	0.81	0.66	**0.82**	3.46
C	0.70	079	0.47	0.68	**0.60**	4.03
D	**0.57** 0.60	0.84	0.76	0.72	**0.57**	**5.01←**
Demand	**0.65**	**0.75**	**0.53**	**0.76**		
Column summation	2.69	3.63	4.07	4.81		

We apply the same technique repeatedly, the final allocation is

	D1	**D2**	**D3**	**D4**	**Capacity**
A	0.82	**0.70** 0.58	0.63	0.72	**0.70**
B	**0.01** 0.57	**0.05** 0.60	0.81	**0.76** 0.66	**0.82**
C	**0.07** **0.70**	**0.79**	**0.53** 0.47	0.68	**0.60**
D	**0.57** 0.60	0.84	0.76	0.72	**0.57**
Demand	**0.65**	**0.75**	**0.53**	**0.76**	

The Transportation Cost

$$Z = 0.60 * 0.57 + 0.66 * 0.76 + 0.47 * 0.53 + 0.70 * 0.07 + 0.58 * 0.70 + 0.57 * 0.01 + 0.60 * 0.05$$

Z = 1.5834 = 1.58

Comparison with Three Types of Ranking

ISSN 2515-8260 Volume 07, Issue 02, 2020

The comparison of the proposed method with different ranking function is tabulated below, in which it is clearly shown that the proposed method provides the optimal results.

Ranking function	proposed method	VAM method
Type-1	**0.41**	0.47
Type-2	**20**	76
Type-3	**1.58**	1.58

7. CONCLUSION

In this paper, we proposed a new path method to solve picture fuzzy transportation problem. The proposed critical path method differs from the previous approaches for solving fuzzy transportation problem using picture fuzzy numbers. We applied three types of ranking and construct the same proposed method to solve picture fuzzy transportation problem to obtain the minimum transportation cost to compare with different ranking functions. Finally, a practical example for picture fuzzy transportation problem is given to verify our proposed critical path method is more effective and reliable.

8. REFERENCES

[1] K. Atanassov, Intuitionistic fuzzy sets and systems, 20 (1986): 87-96.

[2] K. Atanassov and G. Gargov, Interval-valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems,* 31(1989): 343-349.

[3] B. Cuong, Picture fuzzy sets - first results. Part 1, In: Seminar "Neuro – Fuzzy Systems with Applications", *Institute of Mathematics, Hanoi*, 2013.

[4] D.S. Dinagar and K. Palanivel, The transportation problem in fuzzy environment. *International Journal of Algorithm, Computing and Mathematics,* 12(3), (2009): 93-106.

[5] W. Guiwu, Picture fuzzy cross – entropy for multiple attribute decision making problems. *Journal of Business Economics and Management*, 17, (2016): 491-502.

[6] A. Nagoorgani and S. Abbas, A new method for solving Intuitionistic fuzzy transportation problem. *Applied Mathematical Sciences,* 7(28), (2013): 1357-1365.

[7] P. Pandian and G. Natarajan, A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problem. *Applied Mathematical Sciences,* 4(2), (2010): 79-90.

[8] P. Singh, Correlation coefficients for picture fuzzy sets. *Journal of Intelligent and Fuzzy Systems,* 28(2), (2015): 591-604.

[9] C. Wang, X. Zhou, S. Tao, Some geometric aggregations operators based on picture fuzzy sets and their application in multiple attribute decision making. *Italian Journal of Pure and Applied Mathematics,* 37, (2017): 477-492.

[10] L.A Zadeh, Fuzzy sets. *Information and control,* 8(1965): 338-353.