REGULAR DOMINATION IN INTUITIONISTICFUZZY GRAPH

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Abstract: ***In this paper we define regulardomination set and regular dominating number in Intuitionisticfuzzy graph and investigate some properties and bounds of regular domination number in variousIntuitionistic fuzzy graphs.***

Keywords: ***Intuitionisticfuzzy graph, regular dominating set and regular dominating number.***

1. INTRODUCTION:

ThefirstdefinitionoffuzzygraphswasproposedbyKafmann,from thefuzzyrelations introduced by Zadeh. Although Rosenfeld introduced another elaborated definition, including fuzzyvertexandfuzzyedges,andseveralfuzzyanalogsofgraphtheoreticconceptssuchas paths, cycles, connectedness and etc. The concept of domination in fuzzy graphs was investigatedbyA.Somasundaram,S.SomasundaramandA.Somasundarampresentthe conceptsofindependentdomination,totaldomination,connecteddominationoffuzzygraphs. C. Natarajanand S.K. Ayyaswamy introduce the strong (weak) domination in fuzzy graph. The first definition of intuitionistic fuzzy graphs was proposed by Atanassov. The concept of dominationinIntuitionisticfuzzygraphs was investigated by R.parvathi and G.Thamizhendhi.

In this paper we define regular domination set and regular dominating number in Intuitionisticfuzzy graph and investigate some properties and bounds of regular domination number in variousIntuitionistic fuzzy graphs.

2. PRELIMINARIES

This section deals the some basic definitions of Intuitionisticfuzzy graphs. It is useful to construct the next section.

An Intuitionisticfuzzy graph (IFG) is of the form G=(V,E) , where

$V=\{v_1,v_2,...,v_n\}$ suchthat $\mu_1:V\to[0,1]$, $\gamma_1:V\to[0,1]$ denotethedegreeofmembershipand non-member ship of the element $v_i\in V$ respectively and $0\le\mu_1(v_i)+\gamma_1(v_i)\le 1$ for every $v_i\in V,(i=1,2,...n)$. $E\subseteq V\times V$ where $\mu_2:V\times V\to[0,1]$, *and* $\gamma_2:V\times V\to[0,1]$ aresuchthat

$$\mu_2(v_i,v_j)\le\mu_1(v_i)\wedge\mu_1(v_j),$$

$$\gamma_2(v_i,v_j)\le\gamma_1(v_i)\vee\gamma_1(v_j) \text{ and } 0\le\mu_2(v_i,v_j)+\gamma_2(v_i,v_j)\le 1$$

.An arc (v_i,v_j) of an IFG G is called an strong arcif

$$\mu_2(v_i,v_j)=\mu_1(v_i)\wedge\mu_1(v_j),$$

$$\gamma_2(v_i,v_j)=\gamma_1(v_i)\wedge\gamma_1(v_j).$$

Let G =(V,E) be an IFG. The vertex cardinality of Gisdefinedtobe $|v_i|=\left|\sum_{v_i\in V}\left[\frac{(1+\mu_1(v_i)-\gamma_1(v_i))}{2}\right]\right|$ for all $v_i\in V,(i=1,2,...n)$.

Let G=(V, E) beanIFG. A set $D\subseteq V$ is said to be a dominating set of G if every $v\in V-D$ there exist $u\in D$ such that u dominates v.

An Intuitionisticfuzzy dominating set D of an IFG, G is called minimal dominating set of G if every node $u\in D$, $D-\{u\}$ is not a dominating set in G. An Intuitionisticfuzzy domination number $\gamma_{if}(G)$ of an IFG, G is the minimum vertex cardinality over all minimal dominating sets in G.

3. REGULAR DOMINATING SET

In this section the idea of regular domination in Intuitionisticfuzzy graphs and also discusses some properties and bounds of a regular domination number in Intuitionisticfuzzy graphs.

Definition 3.1A set $S\subseteq V$ is said to be a regular dominating set in Intuitionisticfuzzy graphs $G(V,E)$ if

i) Every vertex $u\in V-S$ is adjacent to some vertex in S.
ii) Every vertex in $S\subseteq V$ has the same degree.

Minimum cardinality among all the regular dominating sets is called the regular domination number $\gamma_R(G)$ of $G(V,E)$.

Theorem3.1: In a regular Intuitionisticfuzzy graph $G(V,E)$ then every dominating set is a regular dominating set of $G(V,E)$.

Proof: Let $G(V,E)$ be a regular Intuitionisticfuzzy graph. Therefore degree of every vertex in $G(V,E)$ are unique. This implies every dominating set is a regular dominating set of $G(V,E)$.

Theorem 3.2: Let $d_N(v) = \Delta_N(G)$ in an Intuitionisticfuzzy graph $G(V,E)$, then the degree of every vertexin $\gamma_R(G)$ set is equal to $\Delta_N(G)$.

Proof: Let $G(V,E)$ be an Intuitionisticfuzzy graph. Let $d_N(v) = \Delta_N(G)$ in $G(V,E)$. Therefore the vertex v belongs to $\gamma_R(G)$ set. This implies the degree of every vertex in $\gamma_R(G)$ set is equal to $\Delta_N(G)$. Hence proved.

Example 3.1

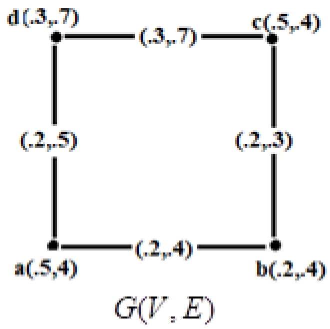


Figure 3.1

In the figure 3.1, the degree of the vertices in $G(V,E)$ are $d(a) = 0.4$, $d(b) = 0.55$ $d(c) = 0.3$, $d(d) = 0.55$, and $\Delta_N(G) = 0.55$. The regular dominating set of $G(V,E)$ is $D_1 = \{b,d\}$.

Definition 3.2: Let $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ be intuitionistic fuzzy graphs on V_1,V_2 respectively with $V_1 \cap V_2 = \phi$. The union of $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ denoted by $G_1 + G_2$ is the intuitionistic fuzzy graph $G(V,E)$ on $V_1 \cup V_2$ defined by $G = (G_1 \cup G_2) = ((\mu_1 \cup \mu_1^{'}),(\gamma_1 \cup \gamma_1^{'}),(\mu_2 \cup \mu_2^{'}),(\gamma_2 \cup \gamma_2^{'}))$ where

$$(\mu_1 \cup \mu_1^{'})(u) = \begin{cases} \mu_1(u) \ if \ u \in V_1 \\ \mu_1^{'}(u) \ if \ u \in V_2 \end{cases} \quad (\gamma_1 \cup \gamma_1^{'})(u) = \begin{cases} \gamma_1(u) \ if \ u \in V_1 \\ \gamma_1^{'}(u) \ if \ u \in V_2 \end{cases}$$

$$(\mu_2 \cup \mu_2^{'})(uv) = \begin{cases} \mu_2(uv) \ if \ \ uv \in E_1 \\ \mu_2^{'}(uv) \ if \ \ uv \in E_2 \\ 0 \ otherwise \end{cases} \& (\gamma_2 \cup \gamma_2^{'})(uv) = \begin{cases} \gamma_2(uv) \ if \ \ uv \in E_1 \\ \gamma_2^{'}(uv) \ if \ \ uv \in E_2 \\ 0 \ otherwise \end{cases}$$

Theorem 3.2: Let $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ are two IFG. Let $D_1 \ and \ D_2$ be the minimal regular dominating sets of $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ respectively. Then the regular dominating number of $G_1 \cup G_2$ is $\gamma_R(G_1 \cup G_2) = |D_1| + |D_2|$.

Proof: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are two IFG. Assume D_1 *and* D_2 be the minimal regulardominating sets of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. If every vertex $u \in G_1 \cup G_2$ this implies $u \in G_1$ or $u \in G_2$ therefore there is a vertex $v \in D_1$ or $v \in D_2$ such that 'v' regularly dominates $u \in G_1 \cup G_2$. Since D_1 *and* D_2 be the regular dominating sets of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ respectively. The regular dominating number of $G_1 \cup G_2$ is $\gamma_R(G_1 \cup G_2) = |D_1| + |D_2|$. Hence proved.

Example 3.2

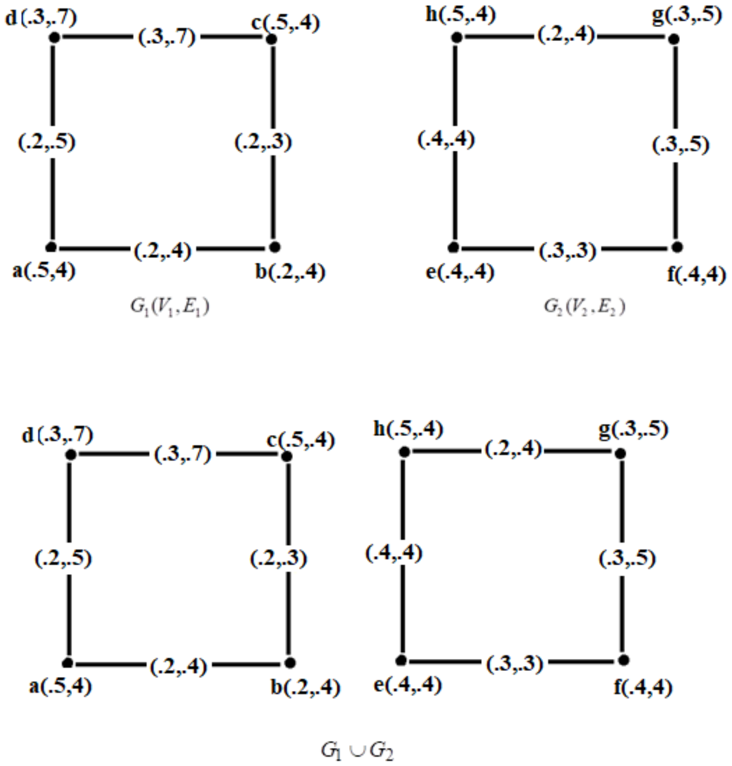


Figure 3.2

In the figure 3.2, the degree of the vertices in $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $d(a) = 0.4$, $d(b) = 0.55$ $d(c) = 0.3$, $d(d) = 0.55$, and $d(e) = 0.55$, $d(f) = 0.4$, $d(h) = 0.5$. The regular dominating set of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $D_1 = \{b, d\}$ and $D_2 = \{g, h\}$.. The regular dominating set of $(G_1 \cup G_2)$ is $D = \{b, d\}$ and the minimal dominating number of the graph $(G_1 \cup G_2)$ is $\gamma_{RF}(G_1 \cup G_2) = 1.35$.

Definition 3.3: Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be intuitionistic fuzzy graphs on V1,V2respectively with $V_1 \cap V_2 = \phi$. The join of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ is the intuitionistic fuzzy graph G on $V_1 \cup V_2$ defined by $G = (G_1 + G_2) = ((\mu_1 + \mu_1^{'}), (\gamma_1 + \gamma_1^{'}), (\mu_2 + \mu_2^{'}), (\gamma_2 + \gamma_2^{'}))$ where

$$(\mu_1+\mu_1^{'})(u)=\begin{cases}\mu_1(u)\ if\ u\in V_1\\ \mu_1^{'}(u)\ if\ u\in V_2\end{cases}\quad(\gamma_1+\gamma_1^{'})(u)=\begin{cases}\gamma_1(u)\ if\ u\in V_1\\ \gamma_1^{'}(u)\ if\ u\in V_2\end{cases}$$

$$(\mu_2+\mu_2^{'})(uv)=\begin{cases}\mu_2(uv)\ if\ \ uv\in E_1\\ \mu_2^{'}(uv)\ if\ \ uv\in E_2\\ \mu_1(u)\wedge\mu_1^{'}(v)\ if\ \ u\in V_1\,\&v\in V_2\end{cases}$$

$$(\gamma_2+\gamma_2^{'})(uv)=\begin{cases}\gamma_2(uv)\ if\ \ uv\in E_1\\ \gamma_2^{'}(uv)\ if\ \ uv\in E_2\\ \gamma_1(u)\vee\gamma_1^{'}(v)\ if\ \ u\in V_1\,\&v\in V\end{cases}$$

Theorem 3.3: The sets D_1,D_2 be a regular dominating set of theIntuitionistic fuzzy graphs $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ respectively. Then $\gamma_R(G_1+G_2)=\min\{|D_1|,|D_2|\}$.

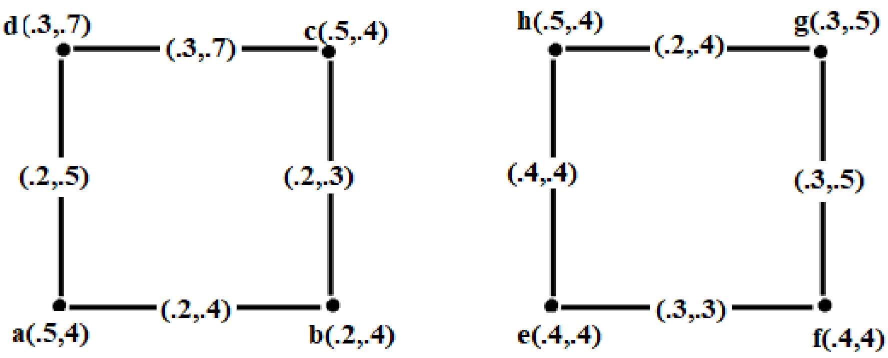
Proof: Let $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ be Intuitionisticfuzzy graphs and The sets D_1,D_2 be regular dominating sets of the fuzzy graphs $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$ respectively. In G_1+G_2 every vertex in $G_1(V_1,E_1)$ is adjacent to every vertices in $G_2(V_2,E_2)$ and vise-versa. Thisimplies the sets D_1,D_2 are dominating sets of G_1+G_2 and degree of the vertices in G_1+G_2 are

$$d(v)=\begin{cases}(d_{G_1}(v)+O(G_2)),\ if\ \ v\in G_1\\ (d_{G_2}(v)+O(G_1)),\ if\ \ v\in G_2\end{cases}$$

Thisimplies $d(u)=d(v)$, $\forall u,v\in D_1\ or\, u,v\in D_2$ in G_1+G_2. Hence $D_1\ or\, D_2$ be a regular dominating sets of G_1+G_2. Therefore the minimal dominating number of G_1+G_2 is

$$\gamma_R(G_1+G_2)=\min\{|D_1|,|D_2|\}$$

Example 3.2:



$G_1(V_1,E_1)$ $G_2(V_2,E_2)$

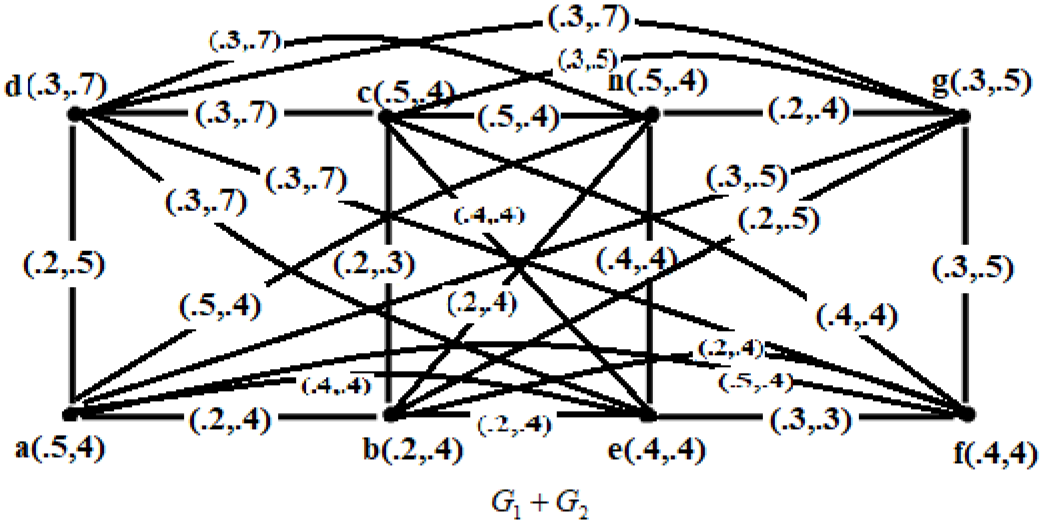


Figure 3.3

In the figure 3.3, the degree of the vertices in $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $d(a) = 0.4$, $d(b) = 0.55$ $d(c) = 0.3$, $d(d) = 0.55$, and $d(e) = 0.55$, $d(f) = 0.4$, $d(g) = 0.5$, $d(h) = 0.5$. The regular dominating set of $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are $D_1 = \{b, d\}$ and $D_2 = \{g, h\}$. The degree of the vertices in $(G_1 + G_2)$ are $d(a) = 2.35, d(b) = 2.5, d(c) = 2.25$, $d(d) = 2.5$, $d(e) = 2.35$, $d(f) = 2.2, d(g) = 2.3, d(h) = 2.3$. The regular dominating set of $(G_1 + G_2)$ is $D = \{b, d\}$ and the minimal dominating number of the graph $(G_1 + G_2)$ is $\gamma_{RF}(G_1 + G_2) = 0.7$.

Conclusion:

In this paper we define regular domination set and regular domination number in Intuitionisticfuzzy graph and investigate some properties and bounds of regular domination number in variousIntuitionistic fuzzy graphs.

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