K-DIMENSIONAL INTUITIONISTIC MULTI FUZZY SUBNEAR-RING

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ABSTRACT: In this paper we introduce the concept of k-dimensional intuitionistic multi fuzzy subnear-ring , k-dimensional intuitionistic multi fuzzy ideals and k-dimensional anti intuitionistic multi fuzzy ideals. We give some proposition related to the above ideas.

Key words: Fuzzy subring, Multi Fuzzy Subring, Intuitionistic Multi Fuzzy Subring,K-Dimensional Intuitionistic Multi Fuzzy Subring

1. INTRODUCTION

The theory of fuzzy set was introduced by Zadeh [10], applying which Rosenfeld [9] in 1971 defined fuzzy subgroups. Salah Abou Zaid [11] introduced the theory of a fuzzy subnear ring and fuzzy ideals of a near-ring. Fuzzy ideals of a ring and a characterization of a regular ring studied by Lui [8] . The notion of fuzzy ideals of near rings with interval valued membership functions introduced by B.Davvaz [3] in 2001. In 2001, Kyung Ho Kim and Young B.Jun [7] in paper entitled "Normal fuzzy R-subgroups in near-ring" introduced the concept of a normal fuzzy R-subgroup in near-ring and explored some related properties . In 2005, Syam Prasad Kuncham and Satyanarayana Bhavanari in paper entitled "Fuzzy Prime ideal of a Gamm-near-ring"introduced fuzzy prime ideal in Γ -near rings. The anti-fuzzy ideals of near-ring defined by F.A.Azam, A.A.Mamun and F.Nasrin. T.L.Dewangan, Prof.M.M.Singh[4]. In this paper entitled K-dimensional multi fuzzy subnear-ring and anti-fuzzy ideals of near-ring and developed some related properties.

2. K-DIMENSIONAL INTUITIONISTIC MULTI FUZZY SUBNEAR RING OF A RING

Definition:2.1

Let R be a near ring . Let A be a k-dimensional intuitionistic multi-fuzzy subset of a near ring of a ring R . We say a k-dimensional intuitionistic multi fuzzy subnear-ring of R if

(i) $\mu_{A_i}(x-y) \geq \min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$

$\Upsilon_{A_i}\tau(x-y) \leq \max\{\Upsilon_{A_i}\tau(x), \Upsilon_{A_i}\tau(y)\}$

(ii) $\mu_{A_i}(xy) \geq \min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$

$\Upsilon_{A_i}\tau(xy) \leq \max\{\Upsilon_{A_i}\tau(x), \Upsilon_{A_i}\tau(y)\}$ $\forall\, x, y \in R$ and i=1,2……k

Definition:2.2

Let R be a near ring and A be a k-dimensional intuitionistic multi fuzzy subset of a ring R . A is called a k-dimensional intuitionistic multi fuzzy left ideal of R if A is a k-dimensional intuitionistic multi fuzzy subnear-ring of R and satisfies for all $x, y \in R$ and i=1,2,……….k.

(i) $\mu_{A_i}(x-y) \geq \min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$

$\Upsilon_{A_i}\tau(x-y) \leq \max\{\Upsilon_{A_i}\tau(x), \Upsilon_{A_i}\tau(y)\}$

(ii) $\mu_{A_i}(y+x-y) \geq \mu_{A_i}(x)$

$\Upsilon_{A_i}\tau(x+y-x) \leq \Upsilon_{A_i}\tau(y)\}$

(iii) $\mu_{A_i}(xy) \geq \mu_{A_i}(y)$ or $\mu_{A_i}(xy) \geq \mu_{A_i}(x)$

$\Upsilon_{A_i}\tau(xy) \leq \Upsilon_{A_i}\tau(y)$ or $\Upsilon_{A_i}\tau(xy) \leq \Upsilon_{A_i}\tau(x)$

Definition: 2.3

Let R be a near ring and A be a k-dimensional intuitionistic multi fuzzy subset of a ring R . A is called a k-dimensional intuitionistic multi fuzzy right ideal of R if A is a k-dimensional intuitionistic multi fuzzy subnear-ring of R and satisfies for all $x, y \in R$,i=1,2,…….k.

(i) $\mu_{A_i}(x-y) \geq \min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$

$\Upsilon_{A_i}\tau(x-y) \leq \max\{\Upsilon_{A_i}\tau(x), \Upsilon_{A_i}\tau(y)\}$

(ii) $\mu_{A_i}(xy) \geq \min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$

$\Upsilon_{A_i}\tau(xy) \leq \max\{\Upsilon_{A_i}\tau(x), \Upsilon_{A_i}\tau(y)\}$

(iii) $\mu_{A_i}((x+i)y-xy) \geq \mu_{A_i}(i)$

$\Upsilon_{A_i}\tau((x+i)y-xy) \leq \Upsilon_{A_i}\tau(i)$

(iv) $\mu_{A_i}(y+x-y) \geq \mu_{A_i}(x)$

$\Upsilon_{A_i}\tau(x+y-x) \leq \Upsilon_{A_i}\tau(y)\}$

Proposition:2.4

If a k-dimensional intuitionistic multi fuzzy subsets $\mu_{A_i}, \Upsilon_{A_i}$ of R satisfies the properties

$\mu_{A_i}(x-y) \geq \min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$, $\Upsilon_{A_i}\tau(x-y) \leq \max\{\Upsilon_{A_i}\tau(x), \Upsilon_{A_i}\tau(y)\}$ then

1. $\mu_{A_i}(0_R) \geq \mu_{A_i}(x)$ $\quad \Upsilon_{A_i}\tau(0_R) \leq \Upsilon_{A_i}\tau(x)$
2. $\mu_{A_i}(-x) = \mu_{A_i}(x)$ $\quad \Upsilon_{A_i}\tau(-x) = \Upsilon_{A_i}\tau(x)$ for all $x, y \in R$ and i=1,2,….k

Proof:

We have that for any $x \in R$

1. $\mu_{A_i}(0_R) = \mu_{A_i}(x-x)$

$\geq \min\{\mu_{A_i}(x), \mu_{A_i}(x)\}$

$= \mu_{A_i}(x)$

Hence, $\mu_{A_i}(0_R) \geq \mu_{A_i}(x)$

$\Upsilon_{A_i}\tau(0_R) = \Upsilon_{A_i}\tau(x-x)$

$\leq \max\{\Upsilon_{A_i}\tau(x), \Upsilon_{A_i}\tau(x)\}$

$= \Upsilon_{A_i}\tau(x)$

Hence, $\Upsilon_{A_i}\tau(0_R) \leq \Upsilon_{A_i}\tau(x)$

2. $\mu_{A_i}(-x) = \mu_{A_i}(0_R - x)$

$\geq \min\{\mu_{A_i}(0_R), \mu_{A_i}(x)\}$

$= \mu_{A_i}(x)$

Hence, $\mu_{A_i}(-x) = \mu_{A_i}(x)$

$$\Upsilon_{A_i}\tau(-x) = \Upsilon_{A_i}\tau(0_R - x)$$
$$\leq \max\{\Upsilon_{A_i}\tau(0_R), \Upsilon_{A_i}\tau(x)\}$$
$$= \Upsilon_{A_i}\tau(x)$$

Hence, $\Upsilon_{A_i}\tau(-x) = \Upsilon_{A_i}\tau(x)$

Proposition:2.5

Let μ_{A_i} and Υ_{A_i} be k-dimensional intuitionistic multi fuzzy ideals of R . If $\mu_{A_i}(x-y) = \mu_{A_i}(0_R)$ then $\mu_{A_i}(x) = \mu_{A_i}(y)$ and If $\Upsilon_{A_i}(x-y) = \Upsilon_{A_i}(0_R)$ then $\Upsilon_{A_i}\tau(x) = \Upsilon_{A_i}\tau(y)$.

Proof:

Assume that $\mu_{A_i}(x-y) = \mu_{A_i}(0_R)$ for all $x, y \in R$ and i=1,2,…….k

Then $\mu_{A_i}(x) = \mu_{A_i}(x-y+y)$
$$\geq \min\{\mu_{A_i}(x-y), \mu_{A_i}(y)\}$$
$$= \min\{\mu_{A_i}(0_R), \mu_{A_i}(y)$$
$$= \mu_{A_i}(y)$$

Thus, $\mu_{A_i}(x) \geq \mu_{A_i}(y)$ (1)

Also , $\mu_{A_i}(y) = \mu_{A_i}(y-x+x)$
$$\geq \min\{\mu_{A_i}(y-x), \mu_{A_i}(x)\}$$
$$= \min\{\mu_{A_i}(0_R), \mu_{A_i}(x)$$
$$= \mu_{A_i}(x)$$

Hence, $\mu_{A_i}(y) \geq \mu_{A_i}(x)$ (2)

From (1) and(2) $\mu_{A_i}(x) = \mu_{A_i}(y)$.

Assume that $\Upsilon_{A_i}\tau(x-y) = \Upsilon_{A_i}\tau(0_R)$ for all $x, y \in R$,i=1,2,…..k

$$\Upsilon_{A_i}\tau(x) = \Upsilon_{A_i}\tau(x-y+y)$$
$$\leq \max\{\Upsilon_{A_i}\tau(x-y), \Upsilon_{A_i}\tau(y)\}$$
$$= \Upsilon_{A_i}\tau(y)$$

Hence, $\Upsilon_{A_i}\tau(x) = \Upsilon_{A_i}\tau(y)$ (3)

We have $\Upsilon_{A_i}\tau(y) = \Upsilon_{A_i}\tau(y-x+x)$
$$\leq \max\{\Upsilon_{A_i}\tau(y-x), \Upsilon_{A_i}\tau(x)\}$$
$$= \max\{\Upsilon_{A_i}\tau(0_R), \Upsilon_{A_i}\tau(x)\}$$
$$= \Upsilon_{A_i}\tau(x)$$

Hence, $\Upsilon_{A_i}\tau(y) \leq \Upsilon_{A_i}\tau(x)$ (4)

From (3) and (4) $\Upsilon_{A_i}\tau(x) = \Upsilon_{A_i}\tau(y)$.

Proposition : 2.6

If μ_{A_i}:R→[0,1]and Υ_{A_i}:R→[0,1] are k-dimensional intuitionistic multi fuzzy ideals of near-ring R with multiplicative identity 1_R . Then $\mu_{A_i}(0_R) \geq \mu_{A_i}(x) \geq \mu_{A_i}(1_R)$ and $\Upsilon_{A_i}\tau(0_R) \leq \Upsilon_{A_i}\tau(x) \leq \Upsilon_{A_i}\tau(1_R)$ for all $x \in R$ i=1,2,……k .

Proof :

We know that , $\mu_{A_i}(x) = \mu_{A_i}(-x)$

And now , $\mu_{A_i}(0_R) = \mu_{A_i}(x-x)$

$= \mu_{A_i}(x+(-x))$

$\geq \min\{\mu_{A_i}(x), \mu_{A_i}(-x)\}$

$= \mu_{A_i}(x)$

$\therefore \mu_{A_i}(0_R) \geq \mu_{A_i}(x)$ (1)

Also , $\mu_{A_i}(x) = \mu_{A_i}(x, 1_R)$

$\geq \min\{\mu_{A_i}(x), \mu_{A_i}(1_R)\}$

$= \mu_{A_i}(1_R)$

$\therefore \mu_{A_i}(x) \geq \mu_{A_i}(1_R)$ (2)

From (1) and (2) $\mu_{A_i}(0_R) \geq \mu_{A_i}(x) \geq \mu_{A_i}(1_R)$, for all $x \in R$, i=1,2,……k .

We know that $\Upsilon_{A_i}\tau(x) = \Upsilon_{A_i}\tau(-x)$

And now $\Upsilon_{A_i}\tau(0_R) = \Upsilon_{A_i}\tau(x-x)$

$= \Upsilon_{A_i}\tau(x+(-x))$

$\leq \max\{\Upsilon_{A_i}\tau(x), \Upsilon_{A_i}\tau(-x)\}$

$= \Upsilon_{A_i}\tau(x)$

$\Upsilon_{A_i}\tau(0_R) \leq \Upsilon_{A_i}\tau(x)$ (3)

Also , $\Upsilon_{A_i}\tau(x) = \Upsilon_{A_i}\tau(x, 1_R)$

$\leq \max\{\Upsilon_{A_i}\tau(x), \Upsilon_{A_i}\tau(1_R)\}$

$= \Upsilon_{A_i}\tau(1_R)$

$\Upsilon_{A_i}\tau(x) \leq \Upsilon_{A_i}\tau(1_R)$ (4)

From (3) and (4) $\Upsilon_{A_i}\tau(0_R) \leq \Upsilon_{A_i}\tau(x) \leq \Upsilon_{A_i}\tau(1_R)$ for all $\in R$, i=1,2,…..k .

Definition: 2.7

Let R be a near ring and A be a k-dimensional intuitionistic multi fuzzy subset of R. A is called a k-dimensional Anti intuitionistic multi fuzzy left ideal of R if A is a k-dimensional intuitionistic multi fuzzy subnear ring of R and satisfies for all $x, y \in R$, i=1,2,……k .

(i) $\mu_{A_i}(x-y) \leq \max\{\mu_{A_i}(x), \mu_{A_i}(y)\}$

$\Upsilon_{A_i}\tau(x-y) \geq \min\{\Upsilon_{A_i}\tau(x), \Upsilon_{A_i}\tau(y)\}$.

(ii) $\mu_{A_i}(y+x-y) \leq \mu_{A_i}(x)$

$\Upsilon_{A_i}\tau(x+y-x) \geq \Upsilon_{A_i}\tau(y)$.

(ii) $\mu_{A_i}(xy) \leq \mu_{A_i}(y)$ or $\mu_{A_i}(xy) \leq \mu_{A_i}(x)$

$\Upsilon_{A_i}\tau(xy) \geq \Upsilon_{A_i}\tau(y)$ or $\Upsilon_{A_i}\tau(xy) \geq \Upsilon_{A_i}\tau(x)$.

Definition:2.8

Let R be a near ring and A be a k-dimensional intuitionistic multi fuzzy subset of R. A is called a k-dimensional Anti intuitionistic multi fuzzy left ideal of R if A is a k-dimensional intuitionistic multi fuzzy subnear ring of R and satisfies for all $x, y \in R$,i=1,2,……..k .

(i) $\mu_{A_i}(x-y) \leq \max\{\mu_{A_i}(x), \mu_{A_i}(y)\}$

$\Upsilon_{A_i}\tau(x-y) \geq \min\{\Upsilon_{A_i}\tau(x), \Upsilon_{A_i}\tau(y)\}$.

(ii) $\mu_{A_i}(xy) \leq \max\{\mu_{A_i}(x), \mu_{A_i}(y)\}$

$\Upsilon_{A_i}\tau(xy) \geq \min\{\Upsilon_{A_i}\tau(x), \Upsilon_{A_i}\tau(y)\}$.

(iii) $\mu_{A_i}(y+x-y) \leq \mu_{A_i}(x)$

$\Upsilon_{A_i}\tau(x+y-x) \geq \Upsilon_{A_i}\tau(y)$.

(iv) $\mu_{A_i}((x+i)y-xy) \leq \mu_{A_i}(i)$

$\Upsilon_{A_i}\tau((x+i)y-xy) \geq \Upsilon_{A_i}\tau(i)$.

3. FOR EVERY K-DIMENSIONAL ANTI INTUITIONISTIC MULTI FUZZY IDEALS

Properties 3.1

μ_{A_i} and Υ_{A_i} of R ,

(1) $\mu_{A_i}(0_R) \leq \mu_{A_i}(x)$

$\Upsilon_{A_i}\tau(0_R) \geq \Upsilon_{A_i}\tau(x)$ for all $x \in R$,i=1,2,……..k

(2) $\mu_{A_i}(x) = \mu_{A_i}(-x)$

$\Upsilon_{A_i}\tau(x) = \Upsilon_{A_i}\tau(-x)$ for all $x \in R$, i=1,2,……..k

(3) $\mu_{A_i}(x-y) = \mu_{A_i}(0_R) \Rightarrow \mu_{A_i}(x) = \mu_{A_i}(y)$

$\Upsilon_{A_i}\tau(x-y) = \Upsilon_{A_i}\tau(0_R) \Rightarrow \Upsilon_{A_i}\tau(x) = \Upsilon_{A_i}\tau(y)$ for all $x, y \in R$, i=1,2,……..k .

Proof :

(1) $\mu_{A_i}(0_R) = \mu_{A_i}(x-x)$

$\leq \max\{\mu_{A_i}(x), \mu_{A_i}(x)\}$

$= \mu_{A_i}(x)$

Hence $\mu_{A_i}(0_R) \leq \mu_{A_i}(x)$

$\Upsilon_{A_i}\tau(0_R) = \Upsilon_{A_i}\tau(x-x)$

$\geq \min\{\Upsilon_{A_i}\tau(x), \Upsilon_{A_i}\tau(x)\}$

$= \Upsilon_{A_i}\tau(x)$

Hence, $\Upsilon_{A_i}\tau(0_R) \geq \Upsilon_{A_i}\tau(x)$

(2) $\mu_{A_i}(-x) = \mu_{A_i}(0_R - x)$

$\leq \max\{\mu_{A_i}(0_R), \mu_{A_i}(x)\}$

$= \mu_{A_i}(x)$

$\Upsilon_{A_i}\tau(-x) = \Upsilon_{A_i}\tau(0_R - x)$

$\geq \min\{\Upsilon_{A_i}\tau(0_R), \Upsilon_{A_i}\tau(x)\}$

$= \Upsilon_{A_i}\tau(x)$

Hence, $\mu_{A_i}(-x) = \mu_{A_i}(x)$

$\Upsilon_{A_i}\tau(-x) = \Upsilon_{A_i}\tau(x)$

(3). Assume that $\mu_{A_i}(x-y) = \mu_{A_i}(0_R)$ and $\Upsilon_{A_i}\tau(x-y) = \Upsilon_{A_i}\tau(0_R)$ for all $x, y \in R$

Then , $\mu_{A_i}(x) = \mu_{A_i}(x-y+y)$

$\leq \max\{\mu_{A_i}(x-y), \mu_{A_i}(y)\}$

$= \max\{\mu_{A_i}(0_R), \mu_{A_i}(y)\}$

$= \mu_{A_i}(y)$ (1)

Also , $\mu_{A_i}(y) = \mu_{A_i}(y-x+x)$

$\leq \max\{\mu_{A_i}(y-x), \mu_{A_i}(x)\}$

$= \max\{\mu_{A_i}(0_R), \mu_{A_i}(x)\}$

$= \mu_{A_i}(x)$

$\mu_{A_i}(y) \leq \mu_{A_i}(x)$ (2)

From (1) and (2) $\mu_{A_i}(x) = \mu_{A_i}(y)$

Assume that $\Upsilon_{A_i}\tau(x-y) = \Upsilon_{A_i}\tau(0_R)$ for all $x, y \in R$ and i=1,2,……k .

Then , $\Upsilon_{A_i}\tau(x) = \Upsilon_{A_i}\tau(x-y+y)$

$\geq \min\{\Upsilon_{A_i}\tau(x-y), \Upsilon_{A_i}\tau(y)\}$

$\geq \min\{\Upsilon_{A_i}\tau(0_R), \Upsilon_{A_i}\tau(y)\}$

$= \Upsilon_{A_i}\tau(y)$

Hence , $\Upsilon_{A_i}\tau(x) \geq \Upsilon_{A_i}\tau(y)$ (3)

$\Upsilon_{A_i}\tau(y) = \Upsilon_{A_i}\tau(y-x+x)$

$\geq \min\{\Upsilon_{A_i}\tau(y-x), \Upsilon_{A_i}\tau(x)\}$

$\geq \min\{\Upsilon_{A_i}\tau(0_R), \Upsilon_{A_i}\tau(x)\}$

$= \Upsilon_{A_i}\tau(x)$

Hence, $\Upsilon_{A_i}\tau(y) \geq \Upsilon_{A_i}\tau(x)$ (4)

From (3) and (4) $\Upsilon_{A_i}\tau(x) = \Upsilon_{A_i}\tau(y)$.

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